

Aerodynamics Formula Overview

Equation of state

$$p = \rho RT \quad (1)$$

Relationship between gravity with geometric height

$$g = g_0 \left(\frac{r}{r + h_G} \right)^2 \quad (2)$$

Hydrostatic equation

Geometric altitude:

$$dp = -\rho g dh_G \quad (3)$$

Geopotential altitude:

$$dp = -\rho g_0 dh \quad (4)$$

Relationship geopotential and geometric altitudes

$$dh = \frac{r^2}{(r + h_G)^2} dh_G \Rightarrow h = \frac{r}{r + h_G} h_G \quad (5)$$

Standard atmosphere

Isothermal layer

$$\frac{p}{p_1} = \frac{\rho}{\rho_1} = e^{-\frac{g_0(h-h_1)}{RT}} \quad (6)$$

Gradient layer

$$\frac{p}{p_1} = \left(\frac{T}{T_1} \right)^{-\frac{g_0}{\alpha R}} \quad (7)$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1} \right)^{-\left(\frac{g_0}{\alpha R} + 1\right)} \quad (8)$$

$$T = T_1 + \alpha (h - h_1) \quad (9)$$

Euler equation

$$dp = -\rho V dV \quad (10)$$

Steady frictionless incompressible flow

Simplified continuity equation:

$$A_1 V_1 = A_2 V_2 \quad (11)$$

Bernoulli equation:

$$p_1 + \frac{1}{2} \rho V_1^2 = p_2 + \frac{1}{2} \rho V_2^2 \quad (12)$$

Curved flow

$$\frac{dp}{dr} = \rho \frac{V^2}{r} \quad (13)$$

Thermodynamics

Enthalpy:

$$h = e + pv = e + RT \quad (14)$$

First law of thermodynamics:

$$\delta q + \delta w = de \quad (15)$$

At constant pressure:

$$\delta q = de + pdv \quad (16)$$

At constant volume:

$$\delta q = dh - v dp \quad (17)$$

Specific heat at constant volume:

$$c_v = \frac{\delta q}{dT} \quad (18)$$

Specific heat at constant pressure:

$$c_p = \frac{\delta q}{dT} \quad (19)$$

$$de = c_v dT \Rightarrow e = c_v T \quad (20)$$

$$dh = c_p dT \Rightarrow h = c_p T \quad (21)$$

$$R = c_p - c_v \quad (22)$$

$$\gamma = \frac{c_p}{c_v} \quad (23)$$

Mach number

$$M = \frac{V}{a} \quad (24)$$

$$a = \sqrt{\gamma RT} \quad (25)$$

Isentropic flow

Continuity equation:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad (26)$$

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1} \right)^\gamma = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} \quad (27)$$

$$\frac{T_0}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2 \quad (28)$$

$$\frac{p_0}{p_1} = \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{\frac{\gamma}{\gamma-1}} \quad (29)$$

$$\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{\frac{1}{\gamma-1}} \quad (30)$$

$$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_2 + \frac{1}{2} V_2^2 \quad (31)$$

Measurement of airspeed

Incompressible flow ($M < 0.3$):

$$V_{true} = \sqrt{\frac{2(p_0 - p)}{\rho}} \quad (32)$$

$$V_e = \sqrt{\frac{2(p_0 - p)}{\rho_s}} \quad (33)$$

Subsonic compressible flow ($0.3 < M < 1$):

$$V_1^2 = \frac{2a_1^2}{\gamma - 1} \left[\left(\frac{p_0 - p_1}{p_1} + 1 \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad (34)$$

$$V_{cal}^2 = \frac{2a_1^2}{\gamma - 1} \left[\left(\frac{p_0 - p_1}{p_s} + 1 \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad (35)$$

Relationship true airspeed and equivalent airspeed:

$$V_e = V \sqrt{\frac{\rho}{\rho_s}} \quad (36)$$

Area velocity relation

$$\frac{dA}{A} = (M^2 - 1) \frac{dV}{V} \quad (37)$$

Conclusions:

$M < 1$: For the velocity to increase, the area must decrease

$M > 1$: For the velocity to increase, the area must increase

$M = 1$: The velocity will be sonic always at the throat

Reynold's number

$$Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty} \quad (38)$$

Viscous flow

$$\tau_w = \mu \left(\frac{dV}{dy} \right)_{y=0} \quad (39)$$

$$c_{fx} = \frac{\tau_w}{\frac{1}{2} \rho_\infty V_\infty^2} = \frac{\tau_w}{q_\infty} \quad (40)$$

Laminar boundary layer:

$$\delta = \frac{5.2x}{\sqrt{Re_x}} \quad (41)$$

$$c_{fx} = \frac{0.664}{\sqrt{Re_L}} \quad (42)$$

$$C_f = \frac{D_f}{q_\infty S} = \frac{1.328}{\sqrt{Re_L}} \quad (43)$$

Turbulent boundary layer

$$\delta = \frac{0.37x}{Re_x^{0.2}} \quad (44)$$

$$c_{fx} = \frac{0.0592}{Re_L^{0.2}} \quad (45)$$

$$C_f = \frac{D_f}{q_\infty S} = \frac{0.074}{Re_L^{0.2}} \quad (46)$$

Drag due to viscous effects:

$$D_{profile} = D_{friction} + D_{pressure} \quad (47)$$

Airfoil nomenclature

Relation lift and drag force with normal and axial force:

$$L = N \cos \alpha - A \sin \alpha \quad (48)$$

$$D = N \sin \alpha + A \cos \alpha \quad (49)$$

AC, CP and the pitching moment

$$C_{m,CP} = 0 \quad (50)$$

$$\frac{dC_{m,AC}}{d\alpha} = 0 \quad (51)$$

$$C_{mQ_2} = C_{mQ_1} + C_n \left(\frac{x_{Q_2} - x_{Q_1}}{c} \right) \quad (52)$$

Lift, drag and moment for 2-dimensional wings

$$L = q_\infty S c_l \quad (53)$$

$$D = q_\infty S c_d \quad (54)$$

$$M = q_\infty S c c_m \quad (55)$$

Pressure coefficient:

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{1}{2} \rho_\infty V_\infty^2} \quad (56)$$

Prandtl-glauert rule:

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}} \quad (57)$$

$$c_n = \frac{1}{c} \int_0^c (C_{p,l} - C_{p,u}) dx \quad (58)$$

Induced drag for 3-dimensional wings:

$$D_i = L\alpha_i = L \frac{C_L}{\pi A} \quad (59)$$

$$C_{D,i} = \frac{C_L^2}{\pi e A} \quad (60)$$

($e = 1$ for elliptical shapes)

$$A = \frac{b}{c} = \frac{b^2}{S} \quad (61)$$

$$\frac{dC_L}{d\alpha} = a = \frac{a_0}{1 + \frac{57.3a_0}{\pi e_1 A}} \quad (62)$$

$$D_{total} = D_{profile} + D_{induced} \quad (63)$$

Mach waves

$$\mu = \arcsin \frac{1}{M} \quad (64)$$

$$M_{cr,swept} = \frac{M_{cr,airfoil}}{\cos \Omega} \quad (65)$$