Data and formulae for aircraft preliminary weight estimation and sizing

Typical fuel fractions for non-fuel intensive mission segments

aircraft type	engine start and warm-up	taxi	take-off	Climb and acceleration to cruise	descent	landing, taxi and shut-down
homebuilts	0.998	0.998	0.998	0.995	0.995	0.995
single engine piston props	0.995	0.997	0.998	0.992	0.993	0.993
twin engine props	0.992	0.996	0.996	0.990	0.992	0.992
agricultural	0.996	0.995	0.996	0.998	0.999	0.998
business jets	0.990	0.995	0.995	0.980	0.990	0.992
regional turboprops	0.990	0.995	0.995	0.985	0.985	0.995
transport jets	0.990	0.990	0.995	0.980	0.990	0.992
military trainers	0.990	0.990	0.990	0.980	0.990	0.995
fighters	0.990	0.990	0.990	0.96 - 0.9	0.990	0.995
military patrol, bombers and transport	0.990	0.990	0.995	0.980	0.990	0.992
flying boats, amphibians and float planes	0.992	0.990	0.996	0.985	0.990	0.990
supersonic aircraft	0.990	0.995	0.995	0.92 - 0.87	0.985	0.992

Breguet formulas for range (R) and endurance (E):

$$R_{prop} = \left(\frac{\eta_p}{g.c_p}\right)_{cruise} . \left(\frac{L}{D}\right)_{cruise} \ln\left(\frac{W_{start}}{W_{end}}\right)$$

$$E_{jet} = \left(\frac{1}{g.c_j}\right)_{loiter} . \left(\frac{L}{D}\right)_{loiter} \ln\left(\frac{W_{start}}{W_{finish}}\right)$$

$$R_{jet} = \left(\frac{V}{g.c_j}\right)_{cruise} . \left(\frac{L}{D}\right)_{cruise} \ln\left(\frac{W_{start}}{W_{end}}\right)$$

$$E_{prop} = \left(\frac{\eta_p}{Vg.c_j}\right)_{loiter} . \left(\frac{L}{D}\right)_{loiter} \ln\left(\frac{W_{start}}{W_{finish}}\right)$$

Reference data for Breguet formulas:

Cruise data

aircraft type	L/D	C_j [lbs/hr/lbs]	C_p [lbs/hr/hp]	η_p
homebuilt	8-10	-	0.6 - 0.8	0.7
single engine piston props	8-10	-	0.5 - 0.7	0.8
twin engine props	8-10	-	0.5 - 0.7	0.82
Agricultural	5-7	-	0.5 - 0.7	0.82
business jets	10-12	0.5 - 0.9	-	-
regional turboprops	11-13	-	0.4 - 0.6	0.85
transport jets	13-15	0.5 - 0.9	-	-
military trainers	8-10	0.5 - 0.9	0.4 - 0.6	0.82
fighters	4-7	0.6 - 1.4	0.5 - 0.7	0.82
military patrol, bombers and transport	13-15	0.5 - 0.9	0.4 - 0.7	0.82
flying boats, amphibians and floatplanes	10-12	0.5 - 0.9	0.5 - 0.7	0.82
supersonic aircraft	4-6	0.7 - 1.5	-	-

Loiter data

aircraft type	L/D	C_j [lbs/hr/lbs]	C_p [lbs/hr/hp]	η_p
homebuilt	10-12	-	0.5 - 0.7	0.6
single engine piston props	10-12	-	0.5 - 0.7	0.7
twin engine props	9-11	-	0.5 - 0.7	0.72
Agricultural	8-10	-	0.5 - 0.7	0.72
business jets	12-14	0.4 - 0.6	-	-
regional turboprops	14-16	-	0.5 - 0.7	0.77
transport jets	14-18	0.4 - 0.6	-	-
military trainers	10-14	0.4 - 0.6	0.5 - 0.7	0.77
fighters	6-9	0.6 - 0.8	0.5 - 0.7	0.77
military patrol, bombers and transport	14-18	0.4 - 0.6	0.5 - 0.7	0.77
flying boats, amphibians and floatplanes	13-15	0.4 - 0.6	0.5 - 0.7	0.77
supersonic aircraft	7-9	0.6 - 0.8	-	-

Maximum lift coefficient values for different a/c categories (clean configuration, take off and landing with deployed high-lift devices)

	CLmax clean		CLmax take-off		CLmax land	
aircraft type	min	max	min	max	min	max
homebuilts	1.2	1.8	1.2	1.8	1.2	2.0
single engine piston props	1.3	1.9	1.3	1.9	1.6	2.3
twin engine props	1.2	1.8	1.4	2.0	1.6	2.5
agricultural	1.3	1.9	1.3	1.9	1.3	1.9
business jets	1.4	1.8	1.6	2.2	1.6	2.6
regional turboprops	1.5	1.9	1.7	2.1	1.9	3.3
transport jets	1.2	1.8	1.6	2.2	1.8	2.8
military trainers	1.2	1.8	1.4	2.0	1.6	2.2
fighters	1.2	1.8	1.4	2.0	1.6	2.6
military patrol, bombers and transport	1.2	1.8	1.6	2.2	1.8	3.0
flying boats, amphibians and float planes	1.2	1.8	1.6	2.2	1.8	3.4
supersonic aircraft	1.2	1.8	1.6	2.0	1.8	2.2

Takeoff parameter definition for jet and propeller a/c

$$\mathrm{TOP_{jet}} = \left(\frac{W}{S}\right)_{TO} \cdot \left(\frac{W}{T}\right)_{TO} \cdot \frac{1}{C_{L_{max}}} \cdot \frac{1}{\sigma} \qquad \mathrm{TOP_{prop}} = \left(\frac{W}{S}\right)_{TO} \cdot \left(\frac{W}{P}\right)_{TO} \cdot \frac{1}{C_{L_{max}}} \cdot \frac{1}{\sigma}$$

Statistical relationship between landing distance and stall speed

$$s_L = 0.5915 * V_{s_{land}}^2 \rightarrow \mathrm{CS23}$$

$$s_L = 0.5847 * V_{s_{land}}^2 \rightarrow \mathrm{CS25}$$

Data and formula for preliminary polar drag estimation

Parasite drag definition as function of the equivalent skin friction coefficient and the wetted area/reference lifting surface area ratio:

$$C_{D0} = C_{fe} \frac{S_{wet}}{S}$$

Equivalent skin friction coefficient values for different aircraft categories

$C_{D0}=C_{fe}\,S_{wet}/S$	C_{fe} - subsonic
Civil transport	0.0030
Bomber	0.0030
Airforce fighter	0.0035
Navy fighter	0.0040
Clean supersonic cruise aircraft	0.0025
Light aircraft – single engine	0.0055
Light aircraft – twin engine	0.0045
Propeller seaplane	0.0065
Jet seaplane	0.0040

Correction factors for ΔC_{D0} and Oswald factor at take off and landing

	ΔC_{D0}	Δe
Clean configuration	0	0
Take-off flaps	0.010 - 0.020	0.05
Landing flaps	0.055 - 0.075	0.10
Undercarriage*	0.015 - 0.025	0

Climb rate formulas

Climb rate : $c = V(T-D)/W = P_a - P_r/W$

For propeller aircraft:

$$c = \frac{\eta_p \cdot P_{br}}{W} - \frac{\sqrt{\frac{W}{S}} \cdot \sqrt{2}}{\frac{C_L^{3/2}}{C_D} \cdot \sqrt{\rho}}$$

For jet aircraft:

$$c = \left(\frac{T}{W} - \frac{C_D}{C_L}\right) \cdot \sqrt{\frac{W}{S}\,\frac{2}{\rho}\,\frac{1}{C_L}}$$

$\frac{C_L^{3/2}}{C_D}$ maximum for $C_L = \sqrt{3C_{D_0}\pi A e}$ and $C_D = 4C_{D_0}$

Climb gradient formulas

Climb gradient : G = (T-D)/W

For propeller aircraft:

$$G = \frac{c}{V} = \eta_p . \frac{P_{br}}{W} . \frac{1}{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}} - \frac{C_D}{C_L}$$

Optimizing this expression (for best climb gradient) leads to a value for C_L which is very close to the maximum lift coefficient. This would result in a dangerous flight condition; thereby a safety margin of 0.2 on the maximum lift coefficient should be used to construct the curves for W/P versus W/S.

For jet aircraft:

$$G = \frac{c}{V} = \frac{T}{W} - \frac{C_D}{C_L}$$

For maximum aerodynamic efficiency:

$$C_D = 2.C_{D_0} \qquad C_L = \sqrt{C_{D_0}.\pi.A.e}$$

Formulae for Space Vehicle Design and Sizing

General

Mean

$$\mu = \sum_{i=1}^{i=n} \frac{x_i}{n}$$

Sample Standard Deviation (SSD)

$$SSD = \sqrt{\sigma^2} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{i=n} \left(x_i - \mu\right)^2}$$

SSD of sum (Indep. Var.)

$$SSD = \sqrt{\sum SSD_i^2}$$

Standard Error of Estimate (SEE)

$$SEE = \sqrt{\frac{1}{n-m} \cdot \sum_{i=1}^{i=n} \left(\frac{y_i}{f(x_i)} - 1 \right)^2}$$

Space Vehicle/Launcher Sizing

Rocket Equation

$$\Delta V = V_e \cdot \ln\left(\frac{M_o}{M_e}\right) \; ; \; V_e = w$$

Initial Mass

$$M_o = M_e + M_F$$

Empty Mass

$$M_e = M_P + M_S$$

Vehicle Mass Ratio

$$R = \Lambda = \frac{M_o}{M_e} = 1 + \frac{M_F}{M_e} = \frac{1+\sigma}{\lambda+\sigma}$$

Spacecraft Launch Mass

$$M_L = M_{SC} + M_{KM} + M_{LVA}$$

Body Volume

$$V = \frac{M}{\rho}$$

Reliability Spacecraft

$$R_{SC} = R_{Payload} \cdot R_{Bus}$$

Reliability

$$R = e^{(-\lambda \cdot t)}$$

Risk

$$Risk = F \cdot Severity$$

Failure probability

$$F = 1 - R$$

Design Margin

$$DM = TC - CBE$$ *[1]

Launcher Mass ratios/fractions

Payload Ratio

$$\lambda = \frac{M_P}{M_o}$$

Propellant Mass Fraction

$$\mu = \frac{M_F}{M_o}$$

Structural Mass Fraction

$$s = \frac{M_S}{M_o}$$

(Stage) Structural Coefficient / Efficiency

$$\sigma = \frac{M_S}{M_F} = \frac{1-\mu'}{\mu'}$$

Stage Propellant Mass Fraction

$$\mu' = \frac{M_F}{M_F + M_S} = \frac{1}{1+\sigma}$$

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[1] DM = Design Margin, TC = Total Capability, CBE = Current Best Estimate

Disturbance Forces & torques

Aerodynamic Drag

$$F_a = \frac{1}{2} \cdot C_D \cdot \rho \cdot V^2 \cdot S$$

Solar Radiation Pressure Force

$$F_s = (1+\rho) \cdot P_s \cdot S$$

Solar Pressure

$$P_s = \frac{J_s}{c}$$

Gravity Gradient Torque

$$T \cong 3 \cdot n^2 \begin{bmatrix} (I_{zz} - I_{yy}) \cdot \phi \\ (I_{zz} - I_{xx}) \cdot \theta \\ 0 \end{bmatrix}$$

Mean Motion

$$n = \frac{2 \cdot \pi}{\tau} = \sqrt{\frac{\mu}{a^3}}$$

Aerodynamic Torque Vector

$$\underline{T} = \underline{r} \times \underline{F}_a$$

Solar Radiation Torque

$$\underline{T} = \underline{r} \times \underline{F}_s$$

Magnetic Torque

$$\underline{T} = \underline{M} \times \underline{B}$$

Internal Torque

$$\underline{T} = \underline{r} \times \underline{F}$$

Structures – Beam Approximation (one end fixed)

Natural Frequency

$$f_n = \sqrt{\frac{k}{M}}$$

Stiffness (Long. and Lat. Direction)

$$k_x = \frac{E \cdot A}{L} \qquad k_y = \frac{3 \cdot E \cdot I}{L^3}$$

Stress at Clamp

$$\sigma_{tot} = \frac{g_y \cdot M \cdot L \cdot c}{I} + \frac{g_x \cdot M}{A}$$

Critical Buckling Load[2]

$$P_{cr} = C \cdot \frac{\pi^2 \cdot E \cdot I}{L^2}$$

Critical Buckling Stress

$$\sigma_{cr} = \frac{C \cdot \pi^2 \cdot E}{(L/\rho)^2} = \frac{P}{A}$$

Radius of Gyration

$$\rho^2 = \frac{I}{A}$$

Critical Axial Stress Cylinder

$$\sigma_c = E\left(9\left(\frac{t}{R}\right)^{1.6} + 0.16\left(\frac{t}{L}\right)^{1.3}\right)$$

Critical Stress with internal pressure

$$\sigma_c = (K_o + K_p) \cdot \frac{E \cdot t}{R} \qquad K_o = 9 \cdot \left(\frac{t}{R}\right)^{0.6} + 0.16 \cdot \left(\frac{R}{L}\right)^{1.3} \cdot \left(\frac{t}{R}\right)^{0.3} \qquad K_p = 0.191 \cdot \left(\frac{p}{E}\right) \cdot \left(\frac{R}{t}\right)^2$$

Sizing Tank structures (Pressurized Thin Walled Cylindrical Tank)

Tank Mass

$$M_{tank} = K \cdot \rho \cdot S \cdot t$$

Cylinder Hoop Stress

$$\sigma_{hoop} = \frac{p \cdot R}{t}$$

Cylinder Axial Stress

$$\sigma_{axial} = \frac{p \cdot R}{2 \cdot t}$$

Spherical End Cap Stress

$$\sigma_{hoop} = \sigma_{axial} = \frac{p \cdot R}{2 \cdot t}$$

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[2] For one end fixed and the other end free to move laterally: C = 0.25

Spacecraft Thermal Control

Heat Balance

$$Q_{in} = Q_{out}$$

Heat Flow

$$q = \frac{Q}{A}$$

Spectral Abs., Transm. and Refl.

$$\alpha + \tau + \rho = 1$$

Radiation Heat

$$Q = \varepsilon \cdot \sigma \cdot A \cdot T^4$$

Heat Conduction

$$Q = \frac{k \cdot A}{l} \cdot \left(T_2 - T_1\right)$$

Heat Convection

$$Q = h_c \cdot \left(T_2 - T_1\right) \cdot A$$

Heat Balance

$$\alpha_s \cdot A_s \cdot J_s + \alpha_s \cdot a \cdot A_a \cdot J_s + \alpha_{IR} \cdot A_{IR} \cdot J_{IR} + Q_{int} = \varepsilon \cdot A_{ext} \cdot \sigma \cdot T^4$$

Solar Flux

$$J_s = \frac{P}{4 \cdot \pi \cdot d^2}$$

Planet Flux

$$q_{IR} = \sigma \cdot T_{IR}^4$$

Kirchhoff's Law

$$\alpha = \varepsilon$$

Power

Energy

$$E = P \cdot t$$

Power Solar Cell

$$P_{cell} = J_s \cdot \eta \cdot A_{cell}$$

Power Solar Array

$$P_\delta = J_s \cdot \eta \cdot I_d \cdot L_d \cdot \cos\left(\theta\right)$$

Area Solar Array

$$A_a = \frac{P}{P_\delta}$$

Mass Solar Array

$$M_a = \frac{P}{P_{sp}}$$

Mass Fuel Cell

$$M_{fc} = \frac{P}{P_{sp}}$$

Mass Battery

$$M_{bat} = \frac{E}{E_{sp}}$$

Volume Battery

$$V_{bat} = \frac{E}{E_\delta}$$

Propulsion

Thrust

$$F_T = \dot{m} \cdot w = \dot{m} \cdot V_e$$

Burn time

$$t_b = \frac{M_F}{\dot{m}}$$

Specific Impulse

$$I_{sp} = \frac{F_T \cdot t_b}{M_F \cdot g_o} = \frac{w}{g_o} = \frac{V_e}{g_o}$$ [3]

Power Required

$$P = \eta_T \cdot P_j$$

Power Req. Ext. Power Source

$$P_W = \frac{P_j}{\eta}$$

Mass External Power Source

$$M_W = \alpha_W \cdot P_W$$

Fuel volume

$$V_{fuel} = \frac{1}{1 + O/F} \cdot \frac{M_F}{\rho_{fuel}}$$

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[3] Note that propellant mass is indicated as M_p in the text on spacecraft design, whereas M_F is used in the text on launcher design.

Attitude Determination and Control

Angular Moment Rigid Body

$$H = I \cdot \omega$$

External Torque

$$T = I \cdot \alpha$$

Rotation Angle Spacecraft

$$\Delta\theta = \omega \cdot t = \frac{1}{2} \cdot \alpha \cdot t^2 + \omega_o \cdot t$$

Torque Magneto-torquer

$$\underline{T}_m = \underline{a} \cdot N \cdot I \cdot A \times \underline{B} = \underline{D} \times \underline{B}$$

Torque Thruster Pair

$$\underline{T}_{thrust} = 2 \cdot \underline{F}_T \cdot \underline{L}$$

C&DH

Digitizing Analogue Signal

$$DR_{analogue} = f \cdot SR \cdot n_{bits}$$

Digitizing an image

$$DR_{image} = N_{images} \cdot S_{pixel} \cdot n_{bits}$$

TT&C

Wave Speed

$$v = \lambda \cdot f$$

Travel Time (Space)

$$t = \frac{d}{c}$$

Required Bandwidth

$$B = \frac{DR}{\text{Spectrum Utilization}}$$

Eff. Isotropic Rad. Power

$$EIRP = P \cdot L_l \cdot G_t$$

Power Flux Density Antenna

$$W_f = \frac{P \cdot G_t}{4 \cdot \pi \cdot r^2} = \frac{EIRP}{4 \cdot \pi \cdot r^2}$$

Received Power

$$C = W_f \cdot A_{ant} \cdot \eta_{ant}$$

Received Energy per Bit

$$E_b = \frac{C}{R}$$

Received Noise Power

$$N = k \cdot T_s \cdot B = No \cdot B$$

Navigation

Measured Frequency

$$f_w = f_b \cdot \sqrt{\frac{c+v}{c-v}}$$