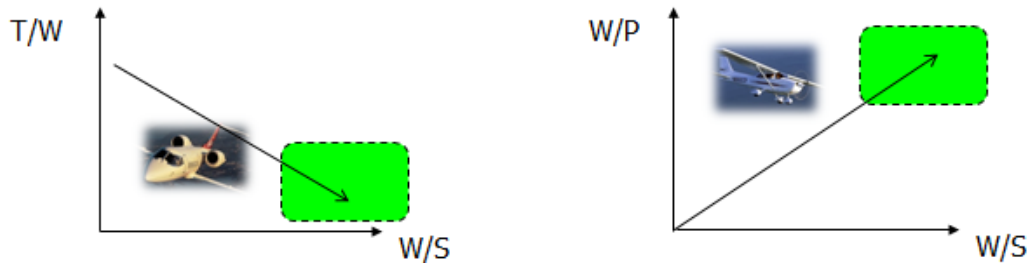# T/W - W/S and W/P - W/S diagrams

Among all combinations that allow to meet the mission performance requirements, the designer should choose the design point that yield the aircraft with the lowest cost and weight. That is the point with:

1. the **highest possible wing loading**, AND
2. the **lowest possible thrust-to-weight ratio** (or highest possible power loading)

When W_{TO} is determined*, also T (and P) and S can be computed

These graphs are built up by collecting specific requirements and plot them as a relationship. See below a short list of all requirements that are considered:

Sizing for: Stall speeds

Take-off

Cruise performance

Landing

Cruise performance

Climb performance — Climb rate
                  — Climb gradient

Manouvring

In the following text it shall be demonstrated how those graphs are built up and what requirements need to be considered. We shall first focus on Jet aircrafts, and then head on to the Props.

Jet aircrafts:

Stall speeds:

In order to size for stall speeds, all you need is the formula shown below:

$$\frac{W}{S} = \frac{1}{2} \cdot \rho \cdot V_S^2 \cdot C_{L_{max}}$$

The stall speed [Vs] will usually be given to you. If not, then the stall speed can be derived by dividing the approach speed by 1.2 or 1.3.

CLmax will depend on whether you use a clean, takeoff or landing configuration. It is fairly obvious that most of the times its the landing configuration we are interested here, however some aircrafts do not have HLDs (high lift devices) and therefore will land using the clean configuration.
Either the CLmax values for all configurations will be given to you, or you must take a look at the table below:

aircraft type	CLmax clean		CLmax take-off		CLmax land	
	min	max	min	max	min	max
homebuilts	1.2	1.8	1.2	1.8	1.2	2.0
single engine piston props	1.3	1.9	1.3	1.9	1.6	2.3
twin engine props	1.2	1.8	1.4	2.0	1.6	2.5
agricultural	1.3	1.9	1.3	1.9	1.3	1.9
business jets	1.4	1.8	1.6	2.2	1.6	2.6
regional turboprops	1.5	1.9	1.7	2.1	1.9	3.3
transport jets	1.2	1.8	1.6	2.2	1.8	2.8
military trainers	1.2	1.8	1.4	2.0	1.6	2.2
fighters	1.2	1.8	1.4	2.0	1.6	2.6
military patrol, bombers and transport	1.2	1.8	1.6	2.2	1.8	3.0
flying boats, amphibians and float planes	1.2	1.8	1.6	2.2	1.8	3.4
supersonic aircraft	1.2	1.8	1.6	2.0	1.8	2.2

Density [p] will usually be 1.225, the standard zero altitude ISO atmosphere value, but they might give you a different reference altitude, so pay attention to that! Usually, to ensure the aircraft can operate also on airports at high altitudes on very hot days, a value of p = 0.974 is chosen.

Once you have all those numbers, you can calculate the wing-loading [W/S], which will give you a straight line on the graph, just as shown below for both clean and landing configurations:

$T/W - W/S$ and $W/P - W/S$ diagrams

T/W versus W/S diagram for a business jet

W/S < 2917 N/m^2

W/S < 3044 N/m^2

— stall clean conf.
— stall landing conf.
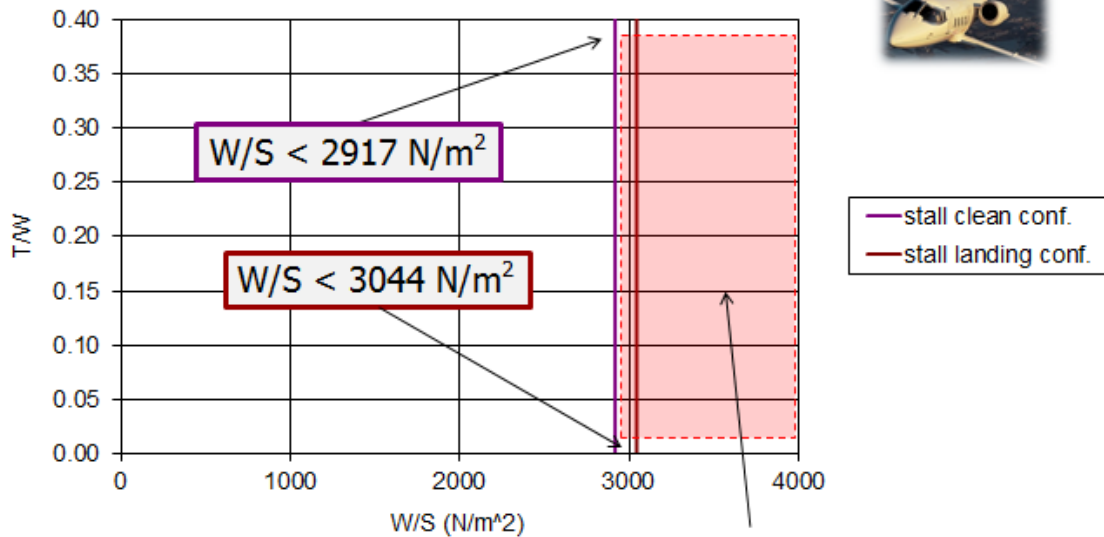
Zone de graphique

W/S is too high here → Maximum stall speed in clean configuration requirement not fulfilled!
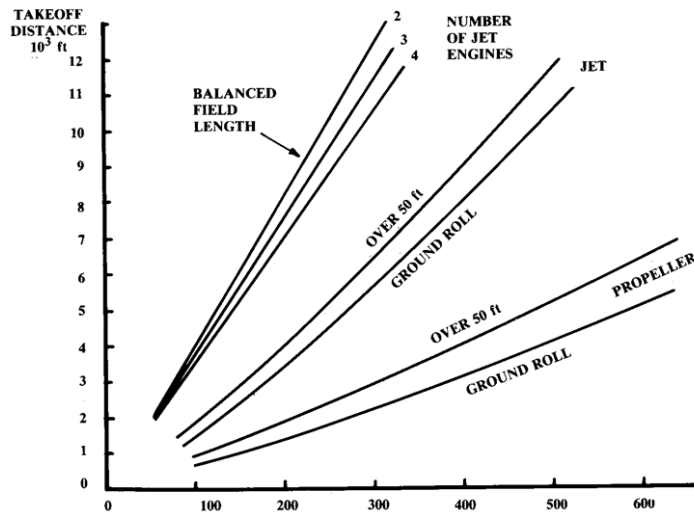
Takeoff

In the case of the takeoff requirement, you will need to focus on the take off parameter [TOP]:

$$TOP_{jet} = \left(\frac{W}{S}\right)_{TO} \cdot \left(\frac{W}{T}\right)_{TO} \cdot \frac{1}{C_{L_{max}}} \cdot \frac{1}{\sigma}$$

With this formula you will end up plotting a relationship between [W/S] and [W/T].

The TOP value might be given to you sometimes, but usually you would rather be given the landing distance, and from it you must find the TOP value. This you can do with the plot below:

CLmax might again be either given to you, or you must find it on the table (see Stall Speed). Obviously in this case we will choose the CLmax takeoff configuration, however, this is not the final value we use, since CLmax would be on the limit of stall and unsafe for all aircrafts! Instead they will tell you how the take off speed [Vto] refers to the stall speed [Vs]. From it you can then find the CL for take off. Below an example where Vto relates to Vs by a factor of 1.1:

$$V_{TO} = 1.1 V_s$$

$$C_{L_{TO}} = \frac{C_{L_{maxTO}}}{1.1^2}$$

σ is nothing more than the ratio of the density at the altitude the aircraft takes off to the density at zero altitude. $\sigma = \frac{\rho}{\rho_0}$ Therefore you must first know the altitude from which the aircraft must take off.
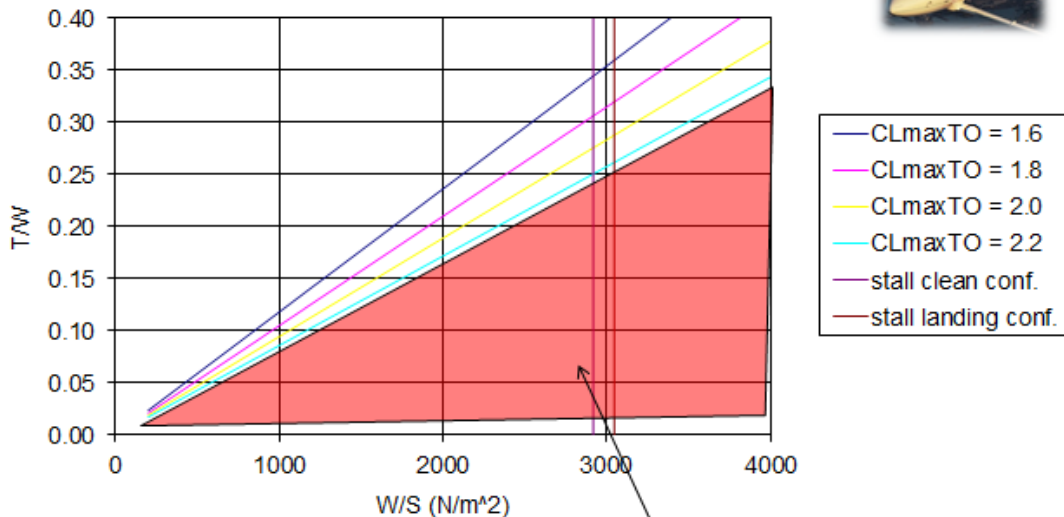
Once done that you can finally graph your relationship. We will replace TOP with [k]; W/S with [x] and T/W with [y]. We would then rearrange everything to [y] and like that we can easily plot a graph for different values of [x]. It will appear as a linear relationship. (See demo below)

$$TOP_{jet} = \left(\frac{W}{S}\right)_{TO} \cdot \left(\frac{W}{T}\right)_{TO} \cdot \frac{1}{C_{L_{maxTO}}} \cdot \frac{1}{\sigma} = k$$

$$\frac{x}{y} \cdot \frac{1}{C_{L_{maxTO}}} \cdot \frac{1}{\sigma} = k \quad \Rightarrow \quad y = \frac{x}{k} \cdot \frac{1}{C_{L_{maxTO}}} \cdot \frac{1}{\sigma}$$

T/W − W/S and W/P − W/S diagrams

T/W versus W/S diagram for a business jet

Legend:
- CLmaxTO = 1.6
- CLmaxTO = 1.8
- CLmaxTO = 2.0
- CLmaxTO = 2.2
- stall clean conf.
- stall landing conf.

The take off requirement is not satisfied for any of the T/W and W/S combinations in this plot area (TOP too large)

Landing

For the landing requirement you should focus on the following formula:

$$\frac{W}{S} = \frac{CL_{max} * \rho * \frac{S_{land}}{\blacksquare}}{2 * f}$$

CLmax here will of course be the value for the landing configuration which either will be given to you or you must find it in a table (see stall speed).

ρ specifies again the density at which altitude the aircraft will land. Therefore, you must know its landing altitude!

[sland] is nothing more than the landing distance for that specific aircraft and is usually given.

Now to the dot… this represents a constant number derived from experimental data, but in exams they might change that number, and you must be able to plug that new number in! The number is shown in the formula below, which is the one you have to pay special attention to!

$$s_L = 0.5847 * V^2_{s_{land}}$$

[f] represents the weight ratio Wl/Wto which you have derived from the weight estimation to build up the payload-range diagram. It usually is given to you.

Again, plug in all numbers and you should get one value for W/S, which appears as a straight vertical line in the graph similar to the one shown in the stall speeds.

Cruise performance (Cruise speed)

For the cruise performance you are given the formula shown below.

$$\frac{T_{TO}}{W} = \left(\frac{\rho_0}{\rho}\right)^{3/4} \left[\frac{C_{D_0}\frac{1}{2}\rho V^2}{(W/S)} + \left(\frac{W}{S}\right)\frac{1}{\pi Ae\frac{1}{2}\rho V^2}\right]$$

Thrust correction
$T = T_{to}(\rho_0/\rho)^{0.75}$

Since we are focusing now to set a requirement for the cruise speed, we need to take a specific moment during our cruise and set include them as factors into our formula. In the case shown below the aircraft is at a setting of 90% throttle and 80% of max. take off weight.

$$\frac{T_{TO}}{W} = \frac{1}{0.9}\left(\frac{\rho_0}{\rho}\right)^{3/4} \left[\frac{C_{D_0}\frac{1}{2}\rho V^2}{(0.8W/S)} + \left(0.8\frac{W}{S}\right)\frac{1}{\pi Ae\frac{1}{2}\rho V^2}\right]$$

The density [p] you must usually find out by the value of the cruise altitude you should be provided with.

The cruise speed is also a value you should be given.

CDo is a value also usually given to you, but it will appear, similar to the CL, as different values for the clean, takeoff, and landing settings. Sometimes they actually won't give you directly the CDo for takeoff and landing, but give you the change in CDo. In that case, just add that difference to the clean configuration. Of course, in this case we are just interested in the clean configuration.

[e] is very similar to the CDo in the sense that it might be given to you as the three standard values, or give you the value for the clean configuration and the change in them for take off and landing.

[A] is a value usually given to you.

Once done that, you can graph this formula as a function of T/W for different values of W/S. This should result in a curve on the graph.

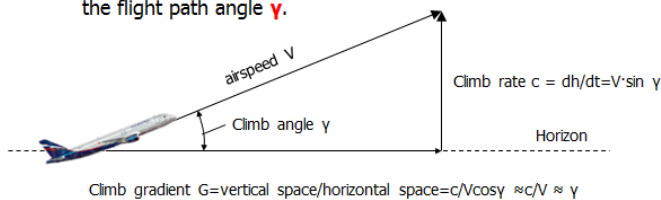## T/W versus W/S diagram for a business jet
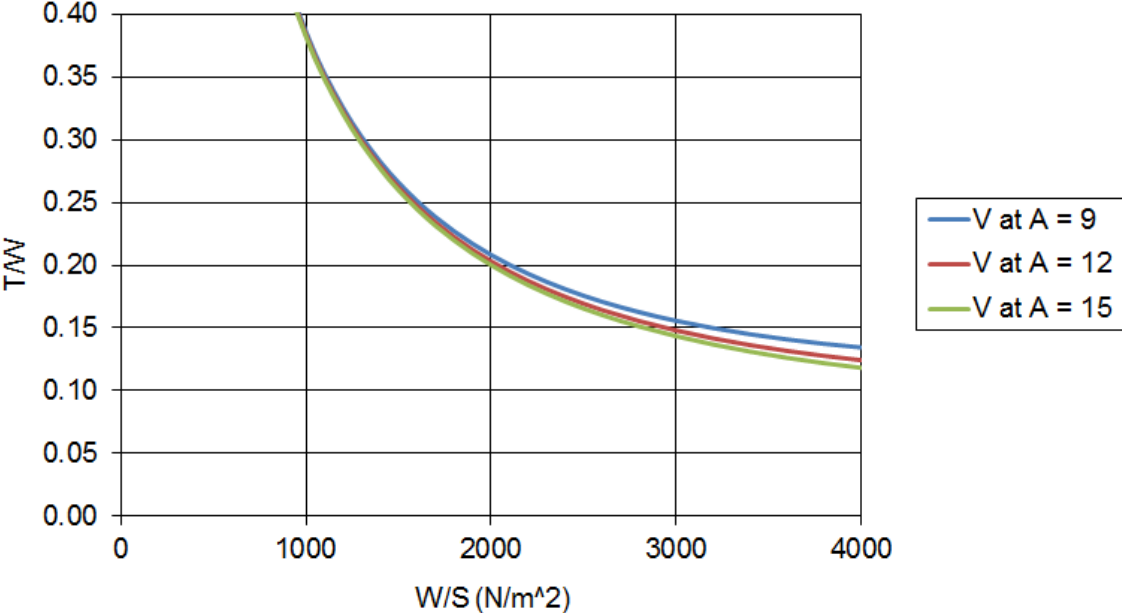
Climb performance - climb rate

It is very important to keep climb rate and climb gradient as two different things: They do NOT mean the same thing! See below the definition for both:

In order to size for the climb rate, one needs to focus on the following formula:

$$c = \left(\frac{T}{W} - \frac{C_D}{C_L} \right) \cdot \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}}$$

[c] is what your requirement is and therefore should be given to you as a value.

CL will as usual either be given to you, or you must find it on the table (see stall speed). Since this requirement is about climbing, we can assume a takeoff configuration for the wings.

In order to find the CD you are required to use the following formula:

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

CDo and [e] will be either given to you as three values, one for each wing configuration or only one value for the clean configuration and then the changes in their value for the other two configurations. In this case we are mostly interested in the takeoff configuration.

[A] is a given value, since it takes part in the design of your aircraft.(Basically you are trying to play around with values and dimensions in order to see what will give you your best aircraft.)

Going back to the previous formula, [p] will depend on what altitude your requirement is taken from. That is why, always have a big eye on the value for altitude!

Similar to what we did for the takeoff requirement (see end of takeoff) we replace T/W with [y] and W/S with [x]. We then rearrange everything to [y] and can then plot this on a graph for different values of [x]. See below a demonstration.

T/W – W/S and W/P – W/S diagrams

Making use of the previously derived expression:

$$c = \left(\frac{T}{W} - \frac{C_D}{C_L} \right) \cdot \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}}$$

Call x = W/S and y = T/W:

$$c = \left(y - \frac{C_D}{C_L} \right) \cdot \sqrt{x} \cdot \sqrt{\frac{2}{\rho} \frac{1}{C_L}} \implies y = \frac{c}{\sqrt{x} \cdot \sqrt{\frac{2}{\rho} \frac{1}{C_L}}} + \frac{C_D}{C_L}$$

Before though plotting the graph, one needs to make sure that one includes the Thrust correction! (Basically, include the thrust correction to all calculations that include thrust.)

Just add a factor of $(\frac{\rho_0}{\rho})^{0.75}$ to your equation and thats it. You should end up with a curve on your plot.

T/W – W/S and W/P – W/S diagrams

T/W versus W/S diagram for a business jet
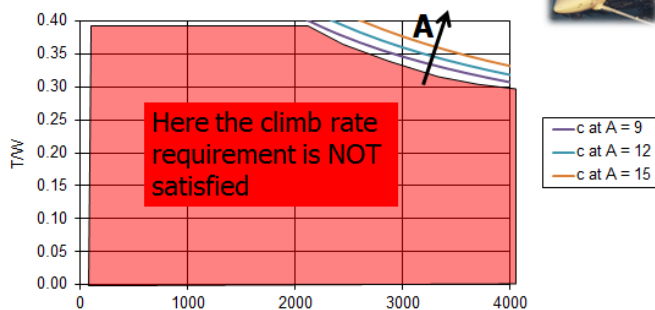
Note, the **negative** effect **high aspect ratio** on the required thrust ratio to meet a given climb rate. The effect of the aspect ratio (negative in this example) depends on the given value of climb rate*

Climb performance - climb gradient

For the climb gradient you will have to focus on the following formula:

$$\frac{T}{W} \geq \left(\frac{\rho_0}{\rho}\right)^{0.75} * \frac{N}{N-1} * \left(\frac{c}{V} + 2\sqrt{\frac{CDo}{Ae\pi}}\right)$$

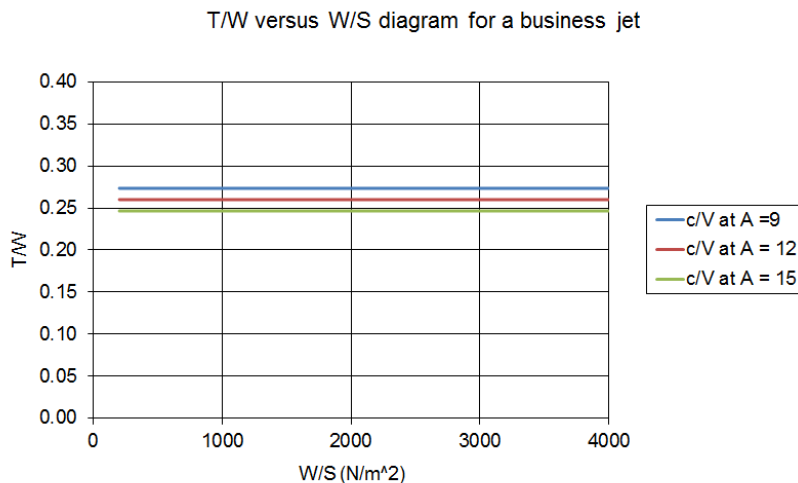⇧ **Thrust correction**   ⇧ **Engine correction**

[N] is the number of engines that operate on your aircraft.

[c/V] represents the gradient, which is a value usually given to you. Notice though that they might give it to you in %.

CDo and [e] will be either given to you as three values, one for each wing configuration or only one value for the clean configuration and then the changes in their value for the other two configurations. In this case we are mostly interested in the takeoff configuration.

[A] again is a value that must be given to you.

Once you have done that, you can plot the resultant T/W on the graph, which will result in a horizontal line, just like shown below:

T/W versus W/S diagram for a business jet

Manoeuvring

For this requirement just focus on the formula shown below:

$$\frac{T}{W} = \frac{C_{D_0}\frac{1}{2}\rho V^2}{(W/S)} + \left(\frac{W}{S}\right)\frac{n_{max}^2}{\pi A e \frac{1}{2}\rho V^2}$$

Again, since we are involved with calculating some thrust, we MUST include the thrust correction factor $(\frac{\rho_0}{\rho})^{0.75}$ to this formula! Dont forget that!

CDo and [e] will be either given to you as three values, one for each wing configuration or only one value for the clean configuration and then the changes in their value for the other two configurations. In this case however we are interested in the cruise conditions, therefore clean configurations unless specified otherwise!

[p] will as usual always depend on your altitude, so be careful with that.

[n] is the requirement value and represents the load factor. This is a given value.

Similar to story with [A] and [V]. They are both given values.

We can now make a plot the function as a relationship of T/W set by different values of W/S. You should end up with a curve again.

Props

to be continued…
I hate cliff hangers…

Stall speed

The formula used for sizing for stall speed is equal to that of jet aircraft

$$\frac{W}{S} = \frac{1}{2} \cdot \rho \cdot V_S^2 \cdot C_{L_{max}}$$

Take off

$$TOP_{prop} = \left(\frac{W}{S}\right)_{TO} \cdot \left(\frac{W}{P}\right)_{TO} \cdot \frac{1}{C_{L_{max}}} \cdot \frac{1}{\sigma}$$

Where most of the time a different formula is given to calculate the TOP in a different way, This way the formula can be rewritten so W/P is dependent on W/S.

Landing

Sizing for landing will not be different from that for jet aircraft

$$\frac{W}{S} = \frac{CL_{max} * \rho * \frac{S_{land}}{\blacksquare}}{2 * f}$$

Again for the dot, this represents a constant number derived from experimental data, but in exams they might change that number, and you must be able to plug that new number in! The number is shown in the formula below, which is the one you have to pay special attention to!

$$s_L = 0.5847 * V_{s_{land}}^2$$

Plug in all numbers and you should get one value for W/S, which appears as a straight vertical line in the graph similar to the one shown in the stall speeds.

Cruise speed

$$\frac{W}{P_{TO}} = \eta_p \left(\frac{\rho}{\rho_0}\right)^{3/4} \left[\frac{C_{D_0} \frac{1}{2}\rho V^3}{(W/S)} + \left(\frac{W}{S}\right)\frac{1}{\pi A e \frac{1}{2}\rho V}\right]^{-1}$$

↑ — Effect of altitude

With power setting and maximum take off corrections. In this case the power setting, 90%, and the maximum take-of weight setting, 80%

$$\frac{W}{P_{TO}} = 0.9\eta_p \left(\frac{\rho}{\rho_0}\right)^{3/4} \left[\frac{C_{D_0} \frac{1}{2}\rho V^3}{(0.8W/S)} + \left(0.8\frac{W}{S}\right)\frac{1}{\pi A e \frac{1}{2}\rho V}\right]^{-1}$$