Automatic Flight Control - Exam April 2008 - Problems

1 Automatic control (2 points)

Root locus plots are used to analyse graphically the influence of the open loop gain on the position of the closed loop poles. Alternatively, root contours can be used to analyse the influence of open loop zero positions on the closed loop poles.

Derive the expression for the modified open loop transfer function which can be used for this purpose. The open loop transfer function can be expressed as follows:

$$G(s)H(s) = (s - z_{var})P(s) \tag{1.1}$$

where the variable zero is situated in $s = z_{var}$ and $P(s)$ groups all the other non-varying terms.

Hint: include eq. (1.1) *in the closed loop transfer function and rewrite the latter one such that* z_{var} *appears as a common factor in the numerator for the modified open loop transfer function:* $GH_{OL_{mod}}(s) = z_{var}\frac{NUM(s)}{DEN(s)}$. *DEN(s) .*

2 Reduced short period equations of motion (2 points)

Consider the longitudinal dynamics of the McDonnell Douglas F-4 Phantom II military aircraft. For the purpose of the determination and analysis of the handling qualities it is sufficient to study the part of the linearized equations of motion for constant trim speed $V = U_0$, the short period dynamics of the aircraft given by the following equations:

$$\begin{aligned} \dot{w} &= Z_w w + U_0 q + Z_{\delta_e}\delta_e \\ \dot{q} &= M_w w + M_q q + M_{\delta_e}\delta_e \end{aligned} \tag{2.1}$$

Here w[m/s] and q[rad/s] represent the perturbations of the vertical body rate and pitch rate respectively, while δ_e[rad] is the elevator deflection.

Determine from the set of equations (2.1) the so-called characteristic polynomial (commonly defined as $\Delta(s)$), expressed in terms of the short period damping ζ_s and short period frequency ω_s. Show how the system dynamics vary with the dimensional stability derivatives.

Determine, given the expression of the so-called characteristic polynomial (commonly defined as $\Delta(s)$), determined from the set of equations (2.1), the eigenvalues λ_i for $i = 1, 2$. Express the real part and imaginary part of the eigenvalues in terms of the previously calculated terms ζ_s and ω_s. It is assumed that the system is stable and underdamped.

Determine from the set of equations (2.1) the following transfer functions expressed in T_{θ_2}, k_q, k_α and $\Delta(s)(\zeta_s, \omega_s)$, when further the trim airspeed $V = U_0$ and the acceleration of gravity g[m/s^2] are given, and the following assumptions can be made for use of simplifications:

$$\begin{aligned} M_w Z_{\delta_e} &<< M_{\delta_e} Z_w \\ M_q Z_{\delta_e} &<< U_0 M_{\delta_e}, \quad \alpha \approx \frac{w}{V} \end{aligned} \tag{2.2}$$

$$\left| -\frac{Z_{\delta_e}}{U_0} \right| << |-M_{\delta_e} Z_w| \tag{2.3}$$

- Show that the transfer function from elevator to pitch rate:

$$\frac{q(s)}{\delta_e(s)} = k_q \frac{1 + T_{\theta_2} s}{\Delta(s)} \tag{2.4}$$

when is given that $k_q = -M_{\delta_e} Z_w$ and $T_{\theta_2} = -\frac{1}{Z_w}$.

- Show that the transfer function from elevator to angle of attack:

$$\frac{\alpha(s)}{\delta_e(s)} = k_\alpha \frac{1 + T_\alpha s}{\Delta(s)} \tag{2.5}$$

when is given that $k_\alpha = M_{\delta_e}$ and $T_\alpha = \frac{Z_{\delta_e}}{U_0 M_{\delta_e}}$.

- Show that the transfer function from attitude to flight-path is given by:

$$\frac{\gamma(s)}{\theta(s)} = \frac{1}{1 + T_{\theta_2} s} \tag{2.6}$$

when is given that $T_{\theta_2} = -\frac{1}{Z_w}$.

3 Stability Augmentation System (1.5 point)

Give the three different kinds of dampers and explain in detail their purpose and in which channels they act. Give block diagrams of the closed loop structures and explain all blocks in the diagram. Give also transfer functions where possible and explain them.

4 Control Augmentation System (2.5 points)

Part A (1.5 point)

Explain in depth how a heading angle hold mode works in an aircraft. Your description should include at least the following topics enumerated below.

- Explain the purpose of a heading angle hold mode and describe which type of control device (surface) and type of feedback loop structure (sensor) are needed.
- Illustrate the working principle by drawing:
  1. a simple block diagram with the outer loop only.
  2. a comprehensive block diagram with all possible inner loops (the system can be considered as decoupled, i.e. you only have to consider the lateral situation).
- Explain the blocks of the outer loop in detail. Provide transfer functions and give drawings where applicable.

Part B (1.0 point)

How can the heading angle ψ be calculated from the roll angle ϕ? For this purpose, consider an aircraft in a flat turn with constant turn radius R_t and determine the equations of motion. Rewrite these expressions so that you obtain the result $\psi = \psi(\phi)$.

5 Autopilot navigational mode (2 points)

Describe the glideslope hold mode of a general autopilot in detail. Your description should include at least the following topics enumerated below.

- A clear picture of the situation in which the to-be-controlled parameter can be found.
- Assumptions which are made in this figure.
- A procedure which shows clearly how the control law is determined.
- A block diagram which represents the control law determined earlier.
- An explanation of the nature of each block in the diagram, and definitions of models or transfer functions of all these blocks.

After you described the glideslope hold mode in detail, explain the influence of the slant range R on the closed loop performance. Give three ways how this can be compensated.