University of the West of England BRISTOL

Bristol Institute of Technology

Academic Year: 2009-10
Examination Period: SUMMER

Module Leader: **Kevin Golden**
Module No: **UFQETH-10-1**
Title of Module: **Engineering Mathematics**

Examination Date: **19th May 2010**
Examination Start time: **14:00pm**
Duration of Examination: **2:00 Hour(s)**

Instructions to Students:

1. **Answer ALL FIVE questions**
2. **Each question is equally weighted.**
3. **Wherever numerical calculations are performed, <u>THREE</u> decimal places of accuracy are required, unless otherwise stated.**

Materials supplied to the student will be:

Number of Examination Booklets (+ any continuation booklets as required) per Examination	1
Number of Pre-printed OMR (Multiple Choice Answer Sheet)	0
Number of sheets of Graph Paper size G3 (Normal)	0

Additional Instruction to Invigilators:

Calculators May be used subject to University regulations	Yes
Students allowed to keep Examination Question Paper	No
Additional Specialised Material : *Mathematical Formula Sheet*	

Treasury tags & adhesive triangles will be supplied as standard

Question 1

(a) Express each of the following complex numbers in the polar form $r\angle\theta$ where $-\pi < \theta \le \pi$.

(i) $z = 5 + 4j$

(ii) $z = -j$

(iii) $z = \dfrac{1}{j}$

(iv) $z = (-2 + j)(4 - 3j)$

[6 marks]

(b) Given $w_1 = 6e^{0.25j}$ and $w_2 = 2e^{-0.75j}$, determine the magnitude (r) and argument (θ) of the following complex numbers, where $-\pi < \theta \le \pi$.

(i) $\dfrac{w_1^2}{w_2}$

(ii) $\dfrac{1}{w_1 w_2}$

[4 marks]

(c) Determine the poles and zeros of the rational function

$$f(x) = \frac{x + 4}{x^2 + x + 1}$$

[4 marks]

Question 2

(a) Solve the differential equation

$$\frac{dy}{dt} + 4y = 4t + 1$$

Given that when $t = 0, y = 1$.

[8 marks]

(b) Determine the form of the forced response of the linear dynamical system described by the second order differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 6\cos 2t$$

[6 marks]

Question 3

(a) Given the function

$$z = 4x^2 + e^{xy} + 2y^2$$

Determine $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial^2 z}{\partial y^2}$

[4 marks]

(b) For $u = \frac{3xy}{1+y^2}$ show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

[4 marks]

(c) For the function $V = \sqrt{x^2 + y^2}$ determine the partial derivatives $\frac{\partial^2 V}{\partial x^2}$ and $\frac{\partial^2 V}{\partial y^2}$.

Hence show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{1}{V}$$

[6 marks]

Question 4

(a) Given the vectors $\mathbf{a} = 5\mathbf{i} - \mathbf{j}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, carry out the following vector operations

(i) $3\mathbf{a} + \mathbf{b} - 2\mathbf{c}$

(ii) $\hat{\mathbf{a}}$

(iii) $\mathbf{a} \cdot \mathbf{b}$

(iv) $\mathbf{b} \times \mathbf{c}$

[5 marks]

(b) A flat plate has vertices A, B and C with co-ordinates $(2, 1, 5)$, $(1, 0, 3)$ and $(4, 1, 1)$ respectively.

Determine a unit vector that is perpendicular to the flat plate.

[5 marks]

(c) Given $\mathbf{a} = 3\mathbf{i} - 4t\,\mathbf{j} - 8\mathbf{k}$ and $\mathbf{b} = t^2\mathbf{i} - 5\,\mathbf{j} + 2\mathbf{k}$, determine the value of *t* for which the vectors **a** and **b** are at right angles to each other.

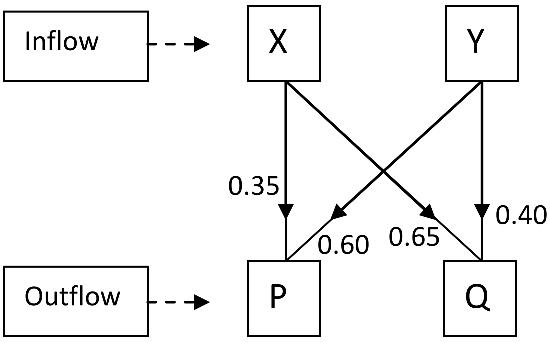
[4 marks]

Question 5

(a) Given $\mathbf{A} = \begin{pmatrix} 8 & 0 & 1 \\ 2 & 3 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & -2 \\ -1 & 2 \\ 3 & 2 \end{pmatrix}$, show that $\mathbf{AB} \neq \mathbf{BA}$

[3 marks]

(b) Consider the figure below which describes flow of water through a network of pipes



- Let x be the rate of flow from X and let y be the rate of flow from Y.
- Let p be the rate of flow arriving at P and let q be the rate of flow arriving at Q.
- Of the water leaving X, 35% arrives at P and 65% arrives at Q
- Of the water leaving Y, 60% arrives at P and 40% arrives at Q

(i) Express the above information as a matrix equation of the form

$$\mathbf{W} = \mathbf{MT}$$

where $\mathbf{W} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $\mathbf{T} = \begin{pmatrix} x \\ y \end{pmatrix}$

(ii) Determine the inverse matrix $\mathbf{M}^{-1}$, and hence find $\mathbf{T}$, when $\mathbf{W} = \begin{pmatrix} 155 \\ 145 \end{pmatrix}$.

[5 marks]

(c) The system of equations below describe the currents I_1, I_2 and I_3 flowing through an electrical network

$$\begin{aligned} 2I_1 + I_2 + 5I_3 &= 12 \\ 3I_1 + 15I_2 + 4I_3 &= 20 \\ I_1 + 12I_2 + 8I_3 &= 8 \end{aligned}$$

Solve the above system of equations using the method of Gaussian elimination.

[6 marks]