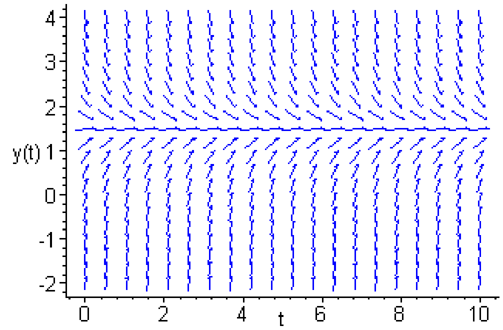
Chapter One

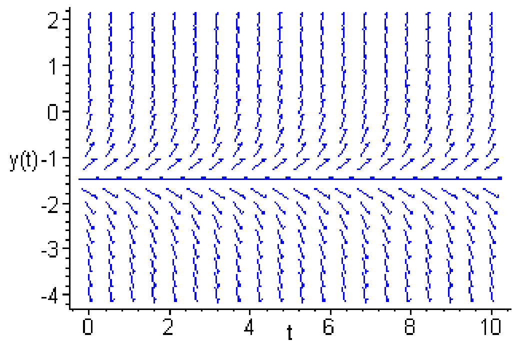
Section 1.1

1.



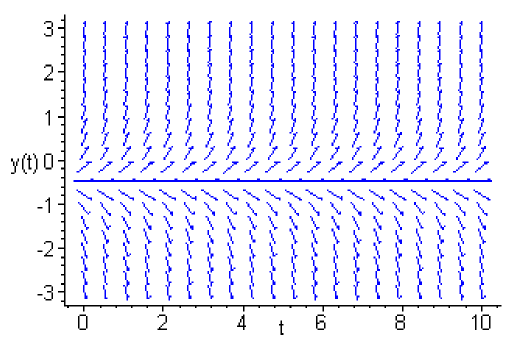
For $y > 1.5$, the slopes are *negative*, and hence the solutions decrease. For $y < 1.5$, the slopes are *positive*, and hence the solutions increase. The equilibrium solution appears to be $y(t) = 1.5$, to which all other solutions converge.

3.



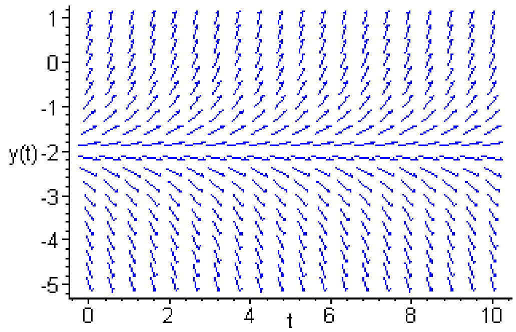
For $y > -1.5$, the slopes are *positive*, and hence the solutions increase. For $y < -1.5$, the slopes are *negative*, and hence the solutions decrease. All solutions appear to diverge away from the equilibrium solution $y(t) = -1.5$.

5.



For $y > -1/2$, the slopes are *positive*, and hence the solutions increase. For $y < -1/2$, the slopes are *negative*, and hence the solutions decrease. All solutions diverge away from the equilibrium solution $y(t) = -1/2$.

6.



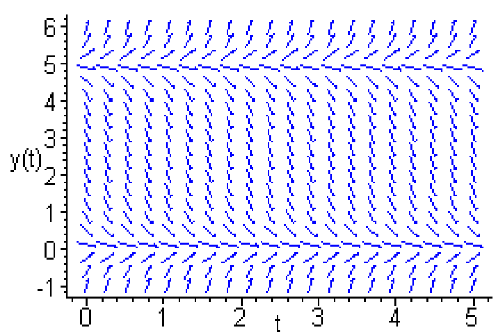
For $y > -2$, the slopes are *positive*, and hence the solutions increase. For $y < -2$, the slopes are *negative*, and hence the solutions decrease. All solutions diverge away from
the equilibrium solution $y(t) = -2$.

8. For *all* solutions to approach the equilibrium solution $y(t) = 2/3$, we must have $y' < 0$ for $y > 2/3$, and $y' > 0$ for $y < 2/3$. The required rates are satisfied by the differential equation $y' = 2 - 3y$.

9. For solutions *other* than $y(t) = 2$ to diverge from $y = 2$, $y(t)$ must be an *increasing* function for $y > 2$, and a *decreasing* function for $y < 2$. The simplest differential equation
whose solutions satisfy these criteria is $y' = y - 2$.

10. For solutions *other* than $y(t) = 1/3$ to diverge from $y = 1/3$, we must have $y' < 0$ for $y < 1/3$, and $y' > 0$ for $y > 1/3$. The required rates are satisfied by the differential equation $y' = 3y - 1$.

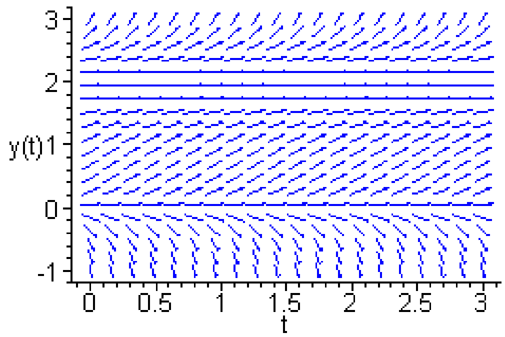
12.



Note that $y' = 0$ for $y = 0$ and $y = 5$. The two equilibrium solutions are $y(t) = 0$ and $y(t) = 5$. Based on the direction field, $y' > 0$ for $y > 5$; thus solutions with initial values *greater* than 5 diverge from the solution $y(t) = 5$. For $0 < y < 5$, the slopes are *negative*, and hence solutions with initial values *between* 0 and 5 all decrease toward the

solution $y(t) = 0$. For $y < 0$, the slopes are all *positive*; thus solutions with initial values
less than 0 approach the solution $y(t) = 0$.

14.

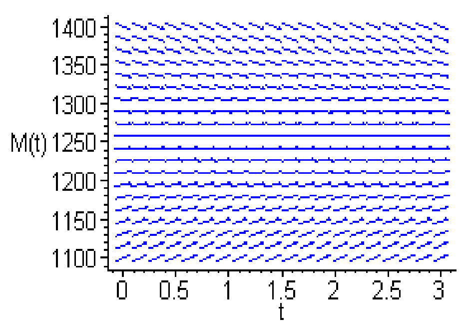


Observe that $y' = 0$ for $y = 0$ and $y = 2$. The two equilibrium solutions are $y(t) = 0$ and $y(t) = 2$. Based on the direction field, $y' > 0$ for $y > 2$; thus solutions with initial values *greater* than 2 diverge from $y(t) = 2$. For $0 < y < 2$, the slopes are also *positive*, and hence solutions with initial values *between* 0 and 2 all increase toward the solution
$y(t) = 2$. For $y < 0$, the slopes are all *negative*; thus solutions with initial values *less* than 0 diverge from the solution $y(t) = 0$.

16. (a) Let $M(t)$ be the total amount of the drug (*in milligrams*) in the patient's body at any
given time t (hrs). The drug is administered into the body at a *constant* rate of 500 mg/hr.
The rate at which the drug *leaves* the bloodstream is given by $0.4M(t)$. Hence the accumulation rate of the drug is described by the differential equation

$$\frac{dM}{dt} = 500 - 0.4\,M \quad (mg/hr)\,.$$

(b)



Based on the direction field, the amount of drug in the bloodstream approaches the equilibrium level of $1250\,mg$ $(within\,a\,few\,hours)$.

18. (a) Following the discussion in the text, the differential equation is

$$m\frac{dv}{dt} = mg - \gamma v^2$$

or equivalently,

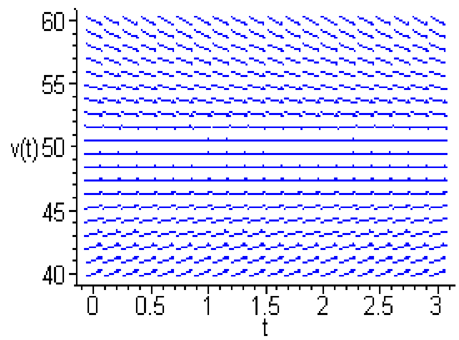
$$\frac{dv}{dt} = g - \frac{\gamma}{m}v^2 .$$

(b) After a long time, $\frac{dv}{dt} \approx 0$. Hence the object attains a *terminal velocity* given by

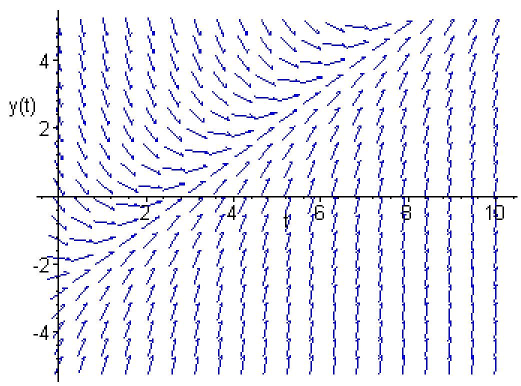
$$v_\infty = \sqrt{\frac{mg}{\gamma}} .$$

(c) Using the relation $\gamma v_\infty^2 = mg$, the required *drag coefficient* is $\gamma = 0.0408\, kg/sec$.

(d)

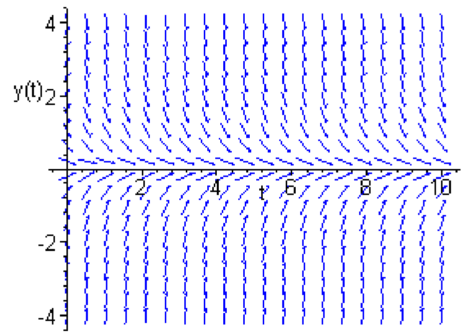


19.



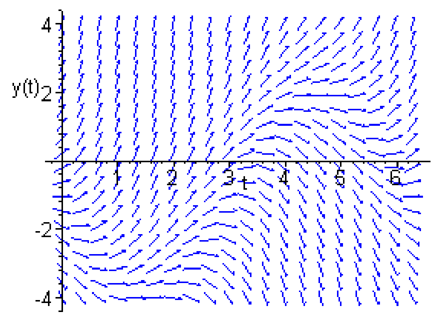
All solutions appear to approach a linear asymptote $(with\, slope\, equal\, to\, 1)$. It is easy to verify that $y(t) = t - 3$ is a solution.

20.



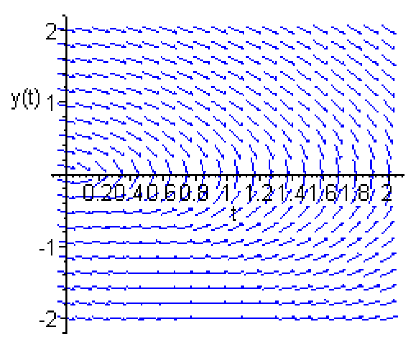
All solutions approach the equilibrium solution $y(t) = 0$.

23.



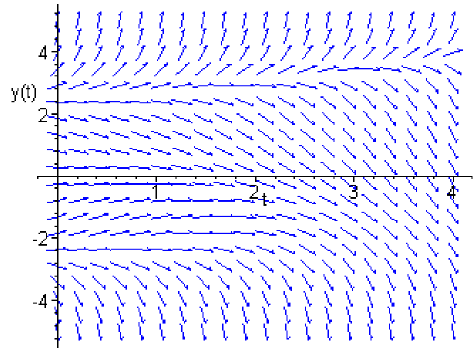
All solutions appear to *diverge* from the sinusoid $y(t) = -\frac{3}{\sqrt{2}} sin(t + \frac{\pi}{4}) - 1$,
which is also a solution corresponding to the initial value $y(0) = -5/2$.

25.

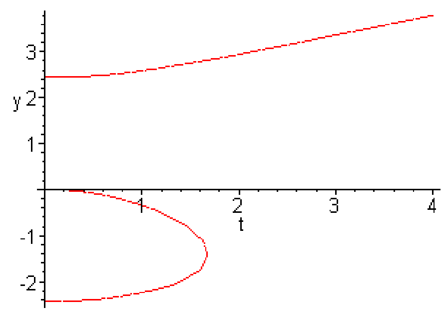


All solutions appear to converge to $y(t) = 0$. First, the rate of change is small. The slopes
eventually increase very rapidly in *magnitude*.

26.



The direction field is rather complicated. Nevertheless, the collection of points at which the slope field is *zero*, is given by the implicit equation $y^3 - 6y = 2t^2$. The graph of these points is shown below:



The *y-intercepts* of these curves are at $y = 0\,, \pm\sqrt{6}$. It follows that for solutions with initial values $y > \sqrt{6}$, all solutions increase without bound. For solutions with initial values in the range $y < -\sqrt{6}$ *and* $0 < y < \sqrt{6}$, the slopes remain *negative*, and hence
these solutions decrease without bound. Solutions with initial conditions in the range $-\sqrt{6} < y < 0$ initially increase. Once the solutions reach the critical value, given by the equation $y^3 - 6y = 2t^2$, the slopes become negative and *remain* negative. These solutions eventually decrease without bound.