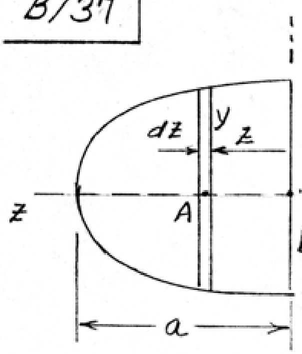
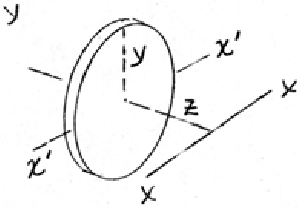
B/37





$$dI_{x'x'} = \frac{1}{4} dm\, y^2$$
$$= \frac{1}{4} \rho \pi y^2 dz\, y^2$$
$$= \frac{1}{4} \rho \pi y^4 dz$$

$$\frac{y^2}{b^2} + \frac{z^2}{a^2} = 1, \quad \text{so } y^2 = b^2\left(1 - \frac{z^2}{a^2}\right)$$

Thus $dI_{xx} = dI_{x'x'} + z^2 dm$

$$dI_{xx} = \frac{1}{4} \rho \pi y^4 dz + (\rho \pi y^2 dz) z^2$$
$$= \rho \pi \left(\frac{1}{4} b^4 \left[1 - \frac{z^2}{a^2}\right]^2 + b^2 z^2 \left[1 - \frac{z^2}{a^2}\right] \right) dz$$
$$I_{xx} = \int_0^a \rho \pi b^2 \left[\frac{b^2}{4} + \left(1 - \frac{b^2}{2a^2}\right) z^2 + \left(\frac{b^2}{4a^2} - 1\right) \frac{z^4}{a^2} \right] dz$$
$$= \rho \pi b^2 \left[\frac{ab^2}{4} + \frac{a^3}{3}\left(1 - \frac{b^2}{2a^2}\right) + \frac{a^3}{5}\left(\frac{b^2}{4a^2} - 1\right) \right] = \frac{2}{15} \rho \pi a b^2 (a^2 + b^2)$$

But $m = \int \rho\, dV = \rho \int \pi y^2 dz = \rho \pi \int_0^a b^2\left(1 - \frac{z^2}{a^2}\right) dz = \frac{2}{3} \rho \pi b^2 a$

so $\underline{I_{xx} = \frac{1}{5} m (a^2 + b^2)}$