

2/36 Particle 1 : $a = -kv$

$$-kv = \frac{dv}{dt}$$
$$-k \int_0^t dt = \int_{v_0}^v \frac{dv}{v} \Rightarrow \underline{v = v_0 e^{-kt}}$$

Then $\frac{ds}{dt} = v_0 e^{-kt}$

$$\int_{s_0=0}^s ds = \int_{t_0=0}^t v_0 e^{-kt} dt \Rightarrow \underline{s = \frac{v_0}{k} (1 - e^{-kt})}$$

Particle 2 : $a = -kt$

$$-kt = \frac{dv}{dt}$$
$$-k \int_0^t dt = \int_{v_0}^v dv \Rightarrow \underline{v = v_0 - \frac{1}{2}kt^2}$$

Then $\frac{ds}{dt} = v_0 - \frac{1}{2}kt^2$

$$\int_0^s ds = \int_0^t (v_0 - \frac{1}{2}kt^2) dt \Rightarrow \underline{s = v_0 t - \frac{1}{6}kt^3}$$

Particle 3 : $a = -ks$

$$-ks = v \frac{dv}{ds}$$

$$-k \int_0^s s \, ds = \int_{v_0}^v v \, dv \Rightarrow v = \pm \sqrt{v_0^2 - ks^2}$$

Then

$$\frac{ds}{dt} = \pm \sqrt{v_0^2 - ks^2}$$

$$\int_0^s \frac{ds}{\sqrt{v_0^2 - ks^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{k}} \sin^{-1}\left(\frac{\sqrt{k}}{v_0} s\right) = t \Rightarrow s = \frac{-v_0}{\sqrt{k}} \sin(\sqrt{k} t)$$

Note: Plus sign is chosen until first reversal ($v=0$), thereafter take minus sign, etc.

