B/43 From the figure in the solution for Prob. B/42 & the expression for dm, $\rho ab(1-\frac{x}{h})^2 dx$, the moment of inertia of dm about the z-axis is $dI_{zz} = dI_{z'z'} + x^2 dm$ by the transfer-of-axis theorem. Also, from the results of Prob. B/42 or Table D/4

$$dI_{z'z'} = \frac{1}{12} dm\, c^2 = \frac{1}{12} dm \left(1-\frac{x}{h}\right)^2 b^2$$

so $dI_{zz} = \left[\frac{1}{12} b^2 \left(1-\frac{x}{h}\right)^2 + x^2\right] dm = \rho ab\left[\frac{1}{12} b^2\left(1-\frac{x}{h}\right)^4 + x^2\left(1-\frac{x}{h}\right)^2\right] dx$

$$I_{zz} = \rho ab \int_0^h \left[\frac{1}{12} b^2 \left(1-\frac{x}{h}\right)^4 + x^2 \left(1-\frac{x}{h}\right)^2\right] dx$$

$$= \rho ab \left[\frac{b^2 h}{60} + \frac{h^3}{30}\right] = \frac{\rho abh}{30}\left(\frac{b^2}{2} + h^2\right)$$

& from solution to Prob. B/42, $m = \frac{1}{3}\rho abh$, so $\underline{I_{zz} = \frac{1}{10} m\left(\frac{b^2}{2} + h^2\right)}$