***B/47** Let ρ = mass per unit area

Panels: $\begin{cases} I_{xx} = 2\left\{\frac{1}{12}m(2r)^2 + m(2r)^2\right\} = \frac{104}{3}\rho r^4 \\ I_{yy} = 2\left\{\frac{1}{12}m(2r)^2\right\} = \frac{2}{3}mr^2 = \frac{8}{3}\rho r^4 \\ I_{zz} = 2\left\{\frac{1}{6}m(2r)^2 + m(2r)^2\right\} = \frac{28}{3}mr^2 = \frac{112}{3}\rho r^4 \end{cases}$

$(m = \rho(2r)^2$ = mass of each panel

Cylindrical shell (see Table D/4):

$$\begin{cases} I_{xx} = I_{yy} = \frac{1}{2}mr^2 + \frac{1}{12}mL^2 = \frac{m}{2}\left(r^2 + \frac{L^2}{6}\right) \\ \qquad\qquad = \pi r L\rho\left(r^2 + \frac{L^2}{6}\right) \\ I_{zz} = mr^2 = 2\pi r^3 L\rho \end{cases}$$

Complete model:

$$\begin{cases} I_{xx} = \frac{104}{3}\rho r^4 + \pi r L\rho\left(r^2 + \frac{L^2}{6}\right) \\ I_{yy} = \frac{8}{3}\rho r^4 + \pi r L\rho\left(r^2 + \frac{L^2}{6}\right) \\ I_{zz} = \frac{112}{3}\rho r^4 + 2\pi r^3 L\rho \end{cases}$$

Since $I_{yy} < I_{xx}$, I_{zz} must be less than I_{yy}:

$$\left(\frac{112}{3}\rho r^4 + 2\pi r^3 L\rho\right) < \left(\frac{8}{3}\rho r^4 + \pi r L\rho\left(r^2 + \frac{L^2}{6}\right)\right)$$

or $\frac{\pi}{6}\left(\frac{L}{r}\right)^3 - \pi\frac{L}{r} - \frac{104}{3} > 0$

Solve cubic for value of $\frac{L}{r}$ making left side zero to obtain $\frac{L}{r} = 4.54$

Inequality is satisfied if $\underline{L > 4.54r}$