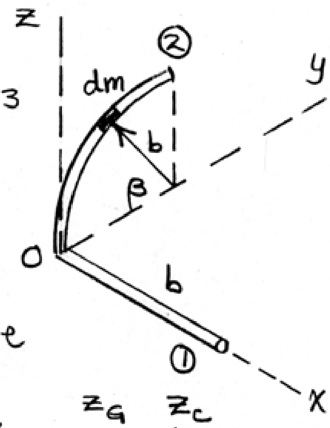
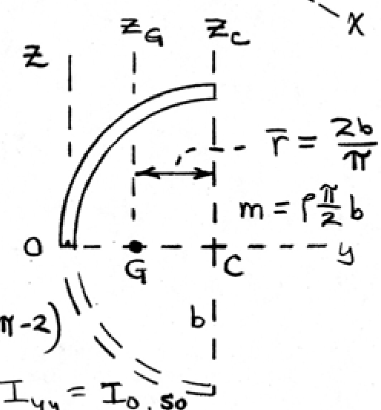
*B/72



Part ①: $I_{xx} = 0$, $I_{yy} = I_{zz} = \frac{1}{3}\rho b^3$

$I_{xy} = I_{xz} = I_{yz} = 0$

Part ②: I_{xx}, I_{yy}, & I_{zz} by symmetry are one-half those of semicircular ring.



$$I_O = I_G + m(b-\bar{r})^2$$

$$= I_C - m\bar{r}^2 + m(b-\bar{r})^2$$

$$= mb^2 + m(b^2 - 2b\bar{r})$$

$$= \rho\frac{\pi}{2}b^3\left[1 + 1 - \frac{4}{\pi}\right] = \rho b^3(\pi - 2)$$

$I_{yy} = \frac{1}{2}mb^2 = \frac{1}{4}\rho\pi b^3$, $I_{zz} + I_{yy} = I_O$, so

$$I_{zz} = \rho b^3\left(\frac{3\pi}{4} - 2\right)$$

$I_{xy} = 0$, $I_{xz} = 0$, $I_{yz} = \int yz\,dm$

$$I_{yz} = \int_0^{\pi/2}(b - b\cos\beta)(b\sin\beta)\rho b\,d\beta = \frac{1}{2}\rho b^3$$

For complete rod:

$I_{xx} = \rho b^3(\pi - 2) = 1.1416\,\rho b^3$, $I_{xy} = 0$

$I_{yy} = \rho b^3\left(\frac{\pi}{4} + \frac{1}{3}\right) = 1.1187\,\rho b^3$, $I_{xz} = 0$

$I_{zz} = \rho b^3\left(\frac{3\pi}{4} - \frac{5}{3}\right) = 0.6895\,\rho b^3$, $I_{yz} = \frac{1}{2}\rho b^3$

Substitute in Eq. B/11 & obtain for

$\frac{I}{\rho b^3} = I'$:

$$\begin{vmatrix} (1.1416 - I') & -0 & -0 \\ -0 & (1.1187 - I') & -0.5 \\ -0 & -0.5 & (0.6895 - I') \end{vmatrix} = 0$$

Expand: $I'^3 - 2.950 I'^2 + 2.586 I' - 0.5952 = 0$

Numerical solution (or by cubic formula)

$\underline{I_1 = 1.448\,\rho b^3}$, $\underline{I_2 = 0.360\,\rho b^3}$, $\underline{I_3 = 1.142\,\rho b^3}$

From Eq. B/12, the direction cosines for I_2-axis are

$(1.1416 - 0.360)\,l \quad -(0)m - (0)n = 0 \qquad (1)$

$-(0)l \quad + (1.1187 - 0.360)\,m - 0.5n = 0 \qquad (2)$

$-0(l) \quad - 0.5m + (0.6895 - 0.360)\,n = 0 \qquad (3)$

Solution: $\underline{l = 0}$, $\underline{m = 0.5503}$, $\underline{n = 0.8350}$