Dynamics Formula Overview

I. Particles

Kinematics

One dimensional motion

$$v = \frac{ds}{dt} = \dot{s} \tag{1}$$

$$a = \frac{dv}{dt} = \dot{v} = \ddot{s} \tag{2}$$

$$a\,ds = v\,dv \tag{3}$$

Planar motion

$$\underset{\sim}{v}(t) = \frac{d\underset{\sim}{r}(t)}{dt} \tag{4}$$

$$\underset{\sim}{a}(t) = \frac{d^2\underset{\sim}{r}(t)}{dt^2} = \frac{d\underset{\sim}{v}(t)}{dt} \tag{5}$$

Coordinate systems

Cartesian

$$\underset{\sim}{r}(t) = x(t)\hat{i} + y(t)\hat{j} \tag{6}$$

$$\underset{\sim}{v}(t) = \dot{x}(t)\hat{i} + \dot{y}(t)\hat{j} \tag{7}$$

$$\underset{\sim}{a}(t) = \ddot{x}(t)\hat{i} + \ddot{y}(t)\hat{j} \tag{8}$$

Polar

$$\underset{\sim}{r}(t) = r\underset{\sim}{e}_r \tag{9}$$

$$\underset{\sim}{v}(t) = \dot{r}\underset{\sim}{e}_r + r\dot{\theta}\underset{\sim}{e}_\theta \tag{10}$$

$$\underset{\sim}{a}(t) = (\ddot{r} - r\dot{\theta}^2)\underset{\sim}{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\underset{\sim}{e}_\theta \tag{11}$$

Path

$$\underset{\sim}{e}_t = \frac{d\underset{\sim}{r}}{ds} \tag{12}$$

$$\underset{\sim}{e}_n = \rho\frac{d\underset{\sim}{e}_t}{ds} \tag{13}$$

$$\rho = \left|\frac{d\underset{\sim}{e}_t}{dx}\right|^{-1} \tag{14}$$

$$\underset{\sim}{v} = v\underset{\sim}{e}_t \qquad v = \frac{ds}{dt} = \pm\|\underset{\sim}{v}\| \tag{15}$$

$$\underset{\sim}{a} = \frac{dv}{dt}\underset{\sim}{e}_t + \frac{v^2}{\rho}\underset{\sim}{e}_n \tag{16}$$

Kinetics

Newton's second law

$$\underset{\sim}{f}_P = m_P\underset{\sim}{a}_P \tag{17}$$

Work and energy

Work

$$dU = \underset{\sim}{f}\bullet d\underset{\sim}{r} \tag{18}$$

Energy

$$T = \frac{1}{2}mv^2 \tag{19}$$

$$V^e = \frac{1}{2}k(r - r_0)^2 \tag{20}$$

$$V^g = mgy \tag{21}$$

$$U_{1,2}^{non-cons} = \Delta T + \Delta V^g + \Delta V^e \tag{22}$$

Power

$$P = \underset{\sim}{f}\bullet\underset{\sim}{v} \tag{23}$$

Impulse-momentum

Linear impulse-momentum

$$\int_{t1}^{t2} \underset{\sim}{f} dt \quad (24)$$

$$\underset{\sim}{G} = m\underset{\sim}{v} \quad (25)$$

$$\sum \underset{\sim}{f} = \dot{\underset{\sim}{G}} \quad (26)$$

$$\int_{t1}^{t2} \sum \underset{\sim}{f} dt = \underset{\sim}{G}_2 - \underset{\sim}{G}_1 = \Delta \underset{\sim}{G} \quad (27)$$

Angular impulse-momentum

$$\int_{t1}^{t2} \underset{\sim}{M}_O dt = \int_{t1}^{t2} \underset{\sim}{r} \times \underset{\sim}{f} dt \quad (28)$$

$$\underset{\sim}{H}_O = \underset{\sim}{r} \times \underset{\sim}{G} = \underset{\sim}{r} \times (m\underset{\sim}{v}) \quad (29)$$

$$\sum \underset{\sim}{M}_O = \dot{\underset{\sim}{H}}_O \quad (30)$$

$$\int_{t1}^{t2} \sum \underset{\sim}{M}_O dt = (\underset{\sim}{H}_O)_2 - (\underset{\sim}{H}_O)_1 = \Delta(\underset{\sim}{H}_O) \quad (31)$$

One dimensional, central collision

$$m_1 v_1 + m_2 v_2 = m_1 v_1^{'} + m_2 v_2^{'} \quad (32)$$

$$e = \frac{v_1^{'} - v_2^{'}}{v_2 - v_1} \quad (33)$$

Perfect elastic: e=1

Perfect plastic: e=0

II. Systems of particles

Kinetics

General definitions

$$M = \sum_i m_i \quad (34)$$

$$\underset{\sim}{r}_G = \frac{\sum_i m_i \underset{\sim}{r}_i}{M} \quad (35)$$

Equation of motion

$$\sum_i \underset{\sim}{f}_i^{ext} = M\underset{\sim}{a}_G \quad (36)$$

Work and energy

Work

$$U_{1,2} = T_2 - T_1 \quad (37)$$

Including both external and internal forces

Energy

$$T = \sum T^i = \sum \frac{1}{2} m_i v_i^2 \quad (38)$$

$$T = \sum \frac{1}{2} m_i \left\| \dot{\rho}_i \right\|^2 + \frac{1}{2} M v_G^2 \quad (39)$$

$$U_{1,2}^{non-cons} = E_2 - E_1 = = (T_2 + V_2) - (T_1 + V_1) \quad (40)$$

V includes both external and internet forces

Impulse-momentum

Linear impulse-momentum

$$\underset{\sim}{G} = \sum_i \underset{\sim}{G}_i = \sum_i m_i \underset{\sim}{v}_i = M\underset{\sim}{v}_G \quad (41)$$

$$\underset{\sim}{F} = \sum_i \underset{\sim}{f}_i^{ext} \quad (42)$$

$$\underset{\sim}{F} = \dot{\underset{\sim}{G}} \quad (43)$$

$$\int_{t1}^{t2} \sum_i \underset{\sim}{f}_i^{ext} dt = \underset{\sim}{G}_2 - \underset{\sim}{G}_1 = \Delta \underset{\sim}{G} \quad (44)$$

Angular impulse-momentum

$$\underset{\sim}{M}_A = \sum_i \underset{\sim}{s}_i \times \underset{\sim}{f}_i^{ext} \quad (45)$$

$$\underset{\sim}{M}_A = \underset{\sim}{H}_A + \underset{\sim}{v}_A \times M \underset{\sim}{v}_G \quad (46)$$

Special cases

$$\underset{\sim}{M}_O = \dot{\underset{\sim}{H}}_O \quad (47)$$

About a fixed point O or the center of mass

$$\int_{t1}^{t2} \underset{\sim}{M}_G dt = (\underset{\sim}{H}_G)_2 - (\underset{\sim}{H}_G)_1 \quad (48)$$

III. Rigid bodies

Kinematics

General definitions

$$\omega = \frac{d\theta}{dt} = \dot{\theta} \quad (49)$$

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} = \ddot{\theta} \quad (50)$$

$$\alpha d\theta = \omega d\omega \quad (51)$$

$$\underset{\sim}{v} = \underset{\sim}{\omega} \times \underset{\sim}{r} \quad (52)$$

$$\underset{\sim}{a}_t = r\alpha \underset{\sim}{e}_t = \underset{\sim}{\alpha} \times \underset{\sim}{r} \quad (53)$$

$$\underset{\sim}{a}_n = r\omega^2 \underset{\sim}{e}_n = \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}) \quad (54)$$

$$\underset{\sim}{a} = \underset{\sim}{\alpha} \times \underset{\sim}{r} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}) \quad (55)$$

$$\underset{\sim}{a} = \underset{\sim}{\alpha} \times \underset{\sim}{r} - \omega^2 \underset{\sim}{r} \quad (56)$$

only valid for planar motion

$$\underset{\sim}{v}_Q = \underset{\sim}{v}_P + \underset{\sim}{\omega} \times \underset{\sim}{r}_{PQ} \quad (57)$$

$$\underset{\sim}{a}_Q = \underset{\sim}{a}_P + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{PQ} - \omega^2 \underset{\sim}{r}_{PQ} \quad (58)$$

only valid for planar motion

Kinetics

General definitions

$$M = \int_\Omega dm = \int_\Omega \rho_m dxdy \quad (59)$$

$$M = \rho A \quad (60)$$

for a homogeneous rigid body

$$r_G = \frac{\int_\Omega \underset{\sim}{r} dm}{M} \quad (61)$$

$$\underset{\sim}{r}_G = \frac{1}{A} \int_\Omega \underset{\sim}{r} dxdy \quad (62)$$

Impulse-momentum

Linear impulse-momentum

$$G = \int_\Omega \underset{\sim}{v} dm = M \underset{\sim}{v}_G \quad (63)$$

$$\underset{\sim}{F}^{ext} = \sum_{i=1}^{K} \underset{\sim}{f}_i = \dot{\underset{\sim}{G}} = M \underset{\sim}{a}_G \quad (64)$$

Mass-moment of Inertia

$$I_G = \int_\Omega \rho^2 dm \quad (65)$$

About the center of gravity

$$I_A = I_G + md^2 \quad (66)$$

About point A

Special cases

$$I_G = \frac{1}{12} m(h^2 + b^2) \quad (67)$$

homogeneous rectangle about center of mass

$$I_G = \frac{1}{12} ml^2 \quad (68)$$

slender bar about center of mass

$$I_A = \frac{1}{3} ml^2 \quad (69)$$

slender bar about its endpoint

$$I_G = \frac{1}{2} mr^2 \quad (70)$$

Homogenenous disk

Radius of gyration

$$\kappa_G = \sqrt{\frac{I_G}{M}} \quad (71)$$

Angular impulse-momentum

$$\underset{\sim}{H}_G = I_G \omega \hat{k} \quad (72)$$

$$\underset{\sim}{M}_G = \sum_{i=1}^{K} \underset{\sim}{\rho}_i \times \underset{\sim}{f}_i \quad (73)$$

$$\underset{\sim}{M}_G = \dot{\underset{\sim}{H}}_G = I_G \alpha \hat{k} \quad (74)$$

$$\underset{\sim}{H}_A = \underset{\sim}{H}_G + \underset{\sim}{r}_{AG} \times M\underset{\sim}{v}_G \quad (75)$$

$H_A = I_G\omega + Mv_G h$ (scalar!)

h is the perpendicular distance between the line of action of Vg and point A *(76)*

$$\underset{\sim}{M}_A = \dot{\underset{\sim}{H}}_G + \underset{\sim}{r}_{AG} \times M\underset{\sim}{a}_G \quad (77)$$

$M_A = I_G\alpha + Ma_G d$ (scalar!)

d is the perpendicular distance between point A and the center of mass *(78)*

$$\underset{\sim}{M}_B = I_B \alpha \hat{k} + \underset{\sim}{r}_{BG} \times M\underset{\sim}{a}_B \quad (79)$$

For a point **fixed** to the rigid body

Work and energy

Work

$$U_{1,2} = T_2 - T_1 \quad (80)$$

Energy

$$T = \int_\Omega \frac{1}{2}|\underset{\sim}{v}|^2\, dm = \frac{1}{2}|\underset{\sim}{v}|^2 \int_\Omega dm \quad (81)$$

pure translation of a rigid body

$$T = \frac{1}{2} M\, |\underset{\sim}{v}|^2 \quad (82)$$

pure translation of a homogeneous rigid body

$$T = \frac{1}{2} I_O \omega^2 \quad (83)$$

Pure rotation of a rigid body about the center of gravity

$$T = \frac{1}{2} M v_G^2 + \frac{1}{2} I_G \omega^2 \quad (84)$$

Total kinetic energy

$$T = \frac{1}{2} I_C \omega^2 \quad (85)$$

Total kinetic energy about instantanious center of rotation

$$U_{1,2}^{non-cons} = E_2 - E_1 = (T_2 + V_2) - (T_1 + V_1) \quad (86)$$

$$V^e = \frac{1}{2} k (r - r_0)^2 \quad (87)$$

$$V^g = mgy_G \quad (88)$$