Resit Examination
Introduction to Earth Observation (AE2–E02)
Faculty of Aerospace Engineering
Delft University of Technology

August 22, 2008, 14:00–17:00

VERSION with ANSWERS

Please read these instructions first:

This exam contains 10 questions totaling 130 points. The value of each question is given below, with the value of any subquestions listed in parentheses. Please use a new piece of paper for the start of each question in order to facilitate the distribution of the exam to the graders. Also be sure to write your name and 7-digit student ID clearly at the top right of each page, and number your pages (for example, page 3 of 12). Show all of your work, as incomplete or unsupported results will be penalized. **This is a closed-book exam**. You are not allowed to have any books, hand-outs, or notes on your table. Use of a pocket (non-programmable) calculator is allowed. An equation sheet, see p. 8–9, is added for your convenience.

Question 1: True/False, 10 pts. (2/2/2/2/2)

State whether the following statements are **True** or **False**. You must justify your answer with a short explanation (the right answer with a wrong or no explanation will be marked incorrect).

a) A change in albedo will result in a change in the observed brightness temperature of an object.
ANSWER: This is true. Albedo r is related to the emissivity ϵ by $\epsilon = 1 - r$. The brightness temperature T_b is related to the temperature T by $T_b = \epsilon T$. Therefore, a change in albedo results in a change in emissivity, which results in a different brightness temperature

b) In the expression of a horizontally polarized electromagnetic wave, $E_y = cE_0 \cos(\omega t - kz)$, the parameter ω represents the frequency in [Hz] and k is the wavelength in [m].
ANSWER: This is false. ω is the angular frequency in [rad/sec] and k is the wavenumber in [1/m].

c) Planck's law is only valid for a blackbody.
ANSWER: This is true. Planck's equation (see equation sheet) describes the electromagnetic/thermal radiation (*spectral radiance*, L_λ) of a black body of temperature T, at a specific frequency/wavelength.

d) The concepts of foreshortening and layover have a different meaning for optical than for radar imagery.
ANSWER: This is true. Radar measures distances, optical methods measure angles. Layover for radar means that a point that is further away from the nadir point of sensor as measured over the ground, will be closer to the sensor in slant range. Consequently, the 'top of the mountain' will be observed earlier than the foot. For an optical device, which is dependent on angles, the top of the mountain will be farther away. The same holds for foreshortening.

e) In a multi-spectral optical sensor, a diffraction grating is primarily used to focus the energy of the beam.
ANSWER: This is false. A multi-spectral sensor does not have a beam; it is a passive instrument. The diffraction grating is used for color (wavelength) separation.

Question 2: Ionosphere, 15 pts. (5/5/5)

The ionosphere is a dispersive medium.

a) Explain the characteristics of a dispersive medium.
ANSWER: In a dispersive medium, the propagation velocity of an electromagnetic wave is dependent of the frequency of the wave. This will affect the phase velocity and the group velocity.

The GPS system utilizes two frequencies, $\lambda_1 = 19$ cm (L1-frequency) and $\lambda_2 = 24$ cm (L2-frequency). This allows to separate the ionospheric signal delay $\frac{A}{f^2}$, which depends on the frequency as follows

$$\Delta t = \frac{\Delta s}{c_{\text{vac}}} + \frac{A}{f^2}$$

We observe that $\Delta t_{L1} - \Delta t_{L2} = 1 \cdot 10^{-8}$ s (corresponding to 3 m path length).

b) What does the parameter Δs represent?
ANSWER: Δs is the distance satellite–receiver. This follows (i) from the fact that this distance needs to be somewhere in the equations, or (ii) from dimension analysis: If $\Delta s/c_{\text{vac}}$ has the unit seconds, then the unit of Δs needs to be meters.

c) What is the value, the unit, and the sign of the constant A? What does the sign of A implicate?
ANSWER: $f_1 = c/\lambda_1 = 1.58$ GHz and $f_2 = c/\lambda_2 = 1.25$ GHz. Fill in equations:

$$\Delta t_{L1} - \Delta t_{L2} = 10^{-8}\,[\text{sec}] \tag{1}$$

$$\left(\frac{\Delta s}{c_{vac}} + \frac{A}{f_1^2}\right) - \left(\frac{\Delta s}{c_{vac}} + \frac{A}{f_2^2}\right) = \left(\frac{A}{f_1^2} - \frac{A}{f_2^2}\right) = A\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right) = 10^{-8} \tag{2}$$

$$A = \frac{10^{-8}}{\frac{1}{f_1^2} - \frac{1}{f_2^2}} = \frac{10^{-8}}{\frac{1}{1.58e9^2} - \frac{1}{1.25e9^2}} \approx -4e10\,[\text{Hz}^2\ \text{s}] \tag{3}$$

$$\approx -0.04\,[\text{MHz}^2\ \text{s}] \tag{4}$$

So the unit is $(\text{Hz})^2$s, or $(\text{MHz})^2$s. The sign is *negative*, which implicates that is is a *negative delay*, or rather a signal *advance*. (Better: it is a *phase* advance,whereas the *group* velocity is delayed.)

Question 3: EO Mission Design, 20 pts. (5/5/5/5)

Earth observation missions often use a sun-synchronous, repeat track orbit design. This allows repeated measurements over the same location to be made, while also giving the solar panels on the spacecraft the highest amount of exposure to the Sun.[1]

a) What is the primary physical process that allows a sun-synchronous orbit to exist?
ANSWER: The Earth's oblateness causes the line of nodes to precess, and for specific orbits, this precession can be timed to be the same as the Earth's rotation about the Sun.

b) What would the rate of the longitude of the ascending node, $\dot{\Omega}$, need to be in order to maintain this orbit?
ANSWER: 360/365.25 = .9856 deg/dy

c) Assume that a hypothetical laser altimetry satellite has been placed into a circular, sun-synchronous orbit at roughly 705 km in altitude, with a repeat period of 16 days, meaning that the satellite would fly over the same location on the Earth once every 16 days. What would the sampli ng density at the equator be for such a mission design? In other words, if you were to plot the ground track of the satellite, how far apart would the ascending tracks be along the equator?
ANSWER: From the equations below, the period is roughly 5932.5 s. The rest is simply recognizing that within 16 days you have n=(86400*16)/5932.5 = 233 orbits. The circumference of the Earth is 6378*2*pi = 40074 km, so the equatorial spacing of the ascending tracks is 40074/233 = 172 km. (lecture 3, slide 37)

[1] *Information on the equation sheet (p. 8–9) may or may not be of assistance to you for the following calculations.*

d) As discussed in the lectures and the textbook, altimetry crossover measurements are optimized when the ascending and descending tracks intersect at 90 degree angles. Using figure 1, taken from the text, what is the (approximate) latitude for which the laser altimetry satellite mentioned above obtains these optimal crossovers? What regions would be best observed by such a mission design?

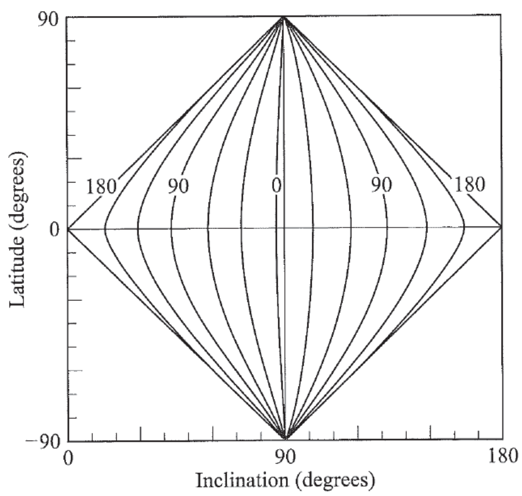

Figure 1: Crossing angle between ascending and descending sub-satellite tracks for satellites in low Earth orbits

ANSWER: Key here is to determine the inclination of the T/P mission from the information given in the previous sub-questions. For the mission described, i=98.8, which means that the optimal crossover point is rather high in latitude, around 75-80 deg lat. This means the ideal crossovers are obtained over the cryosphere.

Question 4: Radar, 15 pts. (5/5/5)

TerraSAR-X is an X-band (10 GHZ) SAR satellite at an altitude of 500 km with a radar antenna of 5 m long in along-track direction, and an off-nadir look angle of 30°. The pulse length is 10 μs and the bandwidth is 300 MHz.

a) Estimate the along-track resolution of a single radar pulse.

ANSWER: See Rees eqs,9.8 and 9.10 and section 9.6.1. With wavelength $\lambda = c/f = 3e8/10e9 = 0.03$ m, beam width $\beta = \lambda/L$ and radar range $R = 500km/\cos 30$: we find for the RAR along track resolution:

$$\Delta x = R \cdot \beta = \frac{H}{\cos\theta} \cdot \frac{\lambda}{L} = \frac{5e5}{\cos 30} \cdot \frac{0.03}{5} = \frac{5e5}{\frac{1}{2}\sqrt{3}} \cdot 0.006 = \frac{6e3}{\sqrt{3}} \approx 3464 \text{ [m]} \approx 3.5 \text{ [km]} \qquad (5)$$

b) Estimate the optimal along-track resolution for a focused SAR image.

ANSWER: (See Rees and lecture slides) SAR resolution is approximately half the antenna length: $L/2 \approx 2.5$ [m]

c) What is the influence of the satellite altitude on the along-track resolution of a focused SAR image?

ANSWER: (See Rees and lecture slides) The satellite altitude is evidently not relevant, as the SAR resolution is approximately half the antenna length: $L/2 \approx 2.5$ [m] (No height involved).

Question 5: Acoustics, 15 pts. (5/5/5)

a) Suppose that a bathymetric survey with a single beam echo-sounder system is carried out to study geological processes of the deep ocean sea floor. One of the objects of interest during the survey is a 0.5-km wide and 0.5-km deep fracture zone located in 6-km deep water. The situation at hand is depicted in Fig. 2. The single beam echo-sounder employed for the survey works at a frequency of 10 kHz. The source diameter of the single beam echo-sounder is 1.0 m. Is this sufficient for mapping the fracture zone features? Assume a sound speed of 1500 m/s in the water column.

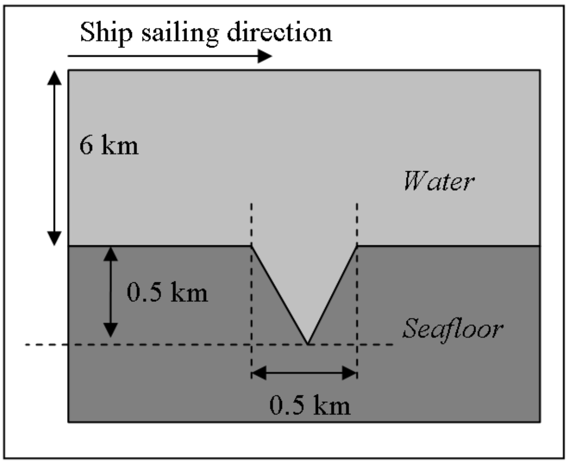

Figure 2: Oceanic cross section showing a sea-floor fracture zone in 6 km deep water.

ANSWER: The opening angle (degrees) for the single beam echo-sounder amounts to:

$$\beta = \frac{65\lambda}{d} = \frac{65 \times 1500}{1 \times 10000} = 9.8^\circ$$

with the wavelength and d the sonar source diameter. For a water depth of 6 km, the horizontal resolution becomes:

$$\frac{\beta\pi}{180} \times 6000 = 1021m$$

This resolution is not sufficient for mapping the 0.5-km wide fracture zone.

b) In addition to the single beam echo-sounder, used for the survey described above, also multi-beam echo-sounders are available for measuring the bathymetry. Mention at least one advantage and one disadvantage of using a multi beam echo-sounder instead of a single beam echo-sounder.

ANSWER: Advantages of using a multi beam system: better resolution, more measurements per ping along a swath perpendicular to the sailing direction (i.e. much better coverage) - Disadvantage of using a multi beam system: system is more expensive.

c) Explain why these acoustic systems can in principle also be used for classifying the seafloor surface sediments.

ANSWER: The interaction of sound with the seafloor is dependent on seafloor type. Therefore, the received signal (shape, strength) will be affected by the seafloor type. The received signals therefore contain information about seafloor type.

Question 6: Aliasing, 15 pts. (5/5/5)

Nearly all observations collected from Earth observing satellites are affected by errors associated with aliasing. In some cases, the influence of aliasing can completely change how the collected data is interpreted. For example, in the lectures, we gave examples of how aliasing could adversely impact satellite altimetry data, as well as the general case of a periodic signal that is sampled from a measurement device.

a) Explain in words AND with a diagram how the phenomenon of aliasing works.

ANSWER: At a minimum, there should be a drawing of a sinusoid that is undersampled showing an aliased signal. Text and descriptions should explain the diagram.

b) For the specific application of collecting satellite gravimetry data from GRACE (which generates monthly models of the Earth's gravity field), explain how the influence of aliasing would impact the resulting gravity model. In your answer, be sure to list the primary sources that would cause aliasing in GRACE.

ANSWER: The clue here is that GRACE samples global gravity at monthly timeframes. That means that any gravity changes that happen below periods of one month alias into the GRACE fields. The most obvious mass changes that happen at periods less than one month are changes in the tides/oceans and the atmosphere. Grading should be 3 pts if they recognize periods below one month get aliased, and one point for each valid source (up to two).

c) Assume that you have a function that is described by the following:

$$f_{\text{true}} = A \cdot \cos(\omega t + \phi),$$

where A, ω, ϕ, are all known constants, and t is in units of time. What would the maximum (constant) sampling rate need to be if the signal were to be perfectly recovered from discrete measurements in time?

ANSWER: The answer to this is given by the Nyquist sampling theorem, which states that you should sample a signal at a minimum of twice the frequency to recover the full waveform. In this case, it is simply $2 \times \omega$. (See lecture 10, slide 33)

Question 7: Data processing, 10 pts. (5/5)

As was demonstrated in the first assignment, the use of Gaussian smoothing can be a valuable data processing tool.

a) Describe at least two advantages and two disadvantages of using Gaussian smoothing on a data set (i.e., an image, gravity data, etc.) that is known to contain noise.

ANSWER: Pro's

- good for removing random, white noise
- easy to implement and can be applied uniformly (i.e., without prior knowledge of signal characteristics)
- parameters can be tuned to achieve varying levels of smoothing

Con's

- can also remove genuine high amplitude signal
- the averaging process can significantly attenuate signals, especially over regions of
- choice of smoothing parameters subjective

b) Assume that we wish to use a continuous probability density function with a "standard" normal distribution (i.e., a Gaussian function with μ=1 and σ=1) as the weighting kernel for our smoothing routine:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

what weight ($0 \leq W \leq 1$) would a point that is 300km from the origin be assigned to, assuming a 1-σ smoothing radius of 500km. Clearly state any assumptions you make and show all work. (Hint: you'll need to normalize the computed weights so that weights at the origin have a value of 1.0)

ANSWER: Setting $\sigma = 1$ and $\mu = 0$ simplifies the above equation. We also know that we want the 1-$sigma$ mark to be at 500km, so that x=300/500=.6. At x=0, the function gives .3989. At x=.6, you get .3332. Normalizing for a center weight of one, you get a final weight of W = .3332/.3989 = .8353.

Question 8: Black body, 10 pts. (5/5)

A black body of 20 °C with dimensions 1 m×1 m is situated in the direct sunlight.

a) When we integrate the total outgoing power [W] over all frequencies, how much will the black body radiate approximately?

ANSWER: The total power per unit area is also refered to as *radiant exittance*, M. We use Stefan's Law, (Rees, p.24. The equation and value for σ are given in equation sheet): $M = \sigma \cdot T^4$ [W/m^2], with $\sigma = 5.67e-8$ and $T = (273+20)^4$ we find $M = 417.9$ W/m^2 ≈ 420 W/m^2. For an object of 1 m^2 the total outgoing power is called the *radiant flux*, $\Phi = 420$ W (=[J/s])

b) What is the corresponding emissivity of the black body?

ANSWER: A black body is a perfect absorber and therefore a perfect emittor, i.e., $\epsilon = 1$. The temperature is not relevant.

Question 9: Solid angle, 15 pts. (5/2/3/2/3)

The concept of the solid angle is used in many radiative equations.

a) Explain why this concept is needed.

ANSWER: Rees, p.19-21, lecture 10 sheets To express the radiant intensity, I, we need to know how much radiant flux (in [W] or [J/s]) is coming from a particular area in the sky. This means that we need an expression for a part of a sphere, as seen from the inside. As this sphere has an infinite radius, square meters are not desirable. Therefore we need an angular parameter, similar to a cone. Note however, that the area projected at the inside of the sphere does not have to be a circle. We also need it for the radiance L, which is the radiant intensity per unit projected source

b) What is the unit of the solid angle?

ANSWER: Rees, p.19-21, lecture 10 sheets The unit of the solid angle is the steradian (in Dutch *steradiaal*)

c) Suppose we have a flat, horizontal terrain. If we consider the visible open hemisphere in terms of its solid angle, how much of these units does it cover?

ANSWER: Rees, p.19-21, lecture 10 sheets The visible open hemisphere covers a solid angle of $\Omega = 2\pi$ sr. 'sr' is the correct abbriviation of the unit steradian. Note for graders: the solid angle is defined as a surface element of a sphere, divided by the sphere's radius R squared, i.e., solid angle $\Omega = A/R^2$. As a full sphere has surface $A = 4\pi R^2$, a semi-sphere or hemisphere has $2\pi R^2$, and the corresponding solid angle, Ω, is $\frac{2\pi R^2}{R^2} = 2\pi$.

d) What is the approximate solid angle covered by the Sun when viewed from Earth?

ANSWER: Rees, p.19-21, lecture 10 sheets See equation sheet. Distance Earth-Sun: $R = 1.5e11$ m, Radius Sun: $r = 7e8$ km. Area of Sun is thus $\pi \cdot r^2 = \pi \cdot 49e16$. Solid angle is area of 'cap' divided by distance, or $A/R^2 = \pi \cdot 49e16/2.25e22 = 7 \times 10^{-5}$ sr = 0.00007 nsr.

e) What is the approximate solid angle covered by the Moon when viewed from Earth, and what does this mean for a solar eclipse?

ANSWER: Rees, p.19-21, lecture 10 sheets See equation sheet. Distance Earth-Moon: $R = 384403$ km, Radius moon is half the diameter: $r = 1738$ km. Area of moon is thus $\pi \cdot r^2 = \pi \cdot 1738^2$. Solid angle is area of 'cap' divided by distance, or $A/R^2 = \pi \cdot 1738^2/384403^2 = 0.00006$ sr. This value is comparable to the solid angle for the Sun, so this implies that a full Eclipse will be exactly covering the Sun's disk.

Question 10: Time, 15 pts. (15) Describe with words or pictures the difference between Universal Time (UT1), International Atomic Time (TAI), and Coordinated Universal Time (UTC).

ANSWER: (See lecture 4, slide 44-45)

Equation sheet AE2-E02, Introduction to Earth Observation

Physical constants		
c	Speed of light *in vacuo*	2.9979×10^{8} m s^{-1} (defined as 299 792 458 m s^{-1})
h	Planck constant	6.6261×10^{-34} J s
e	Charge on the proton	1.6022×10^{-19} C
m_e	Mass of the electron	9.1094×10^{-31} kg
u	Atomic mass unit	1.6605×10^{-27} kg
m_0	Permeability of free space	1.2566×10^{-6} H m^{-1} (defined as $4\pi \times 10^{-7}$ H m^{-1})
ε_0	Permittivity of free space	8.8542×10^{-12} F m^{-1}
Z_0	Impedance of fee space	$3.7673 \times 10^{2}\,\Omega$
G	Gravitational constant	6.6726×10^{-11} N m^{2} kg^{-2}
R	Gass constant	8.3145 J K^{-1} mol^{-1}
N_A	Avogadro number	6.0221×10^{23} mol^{-1}
k	Boltzmann constant	1.3807×10^{-23} J K^{-1}
σ	Stefan-Boltzmann constant	5.6705×10^{-8} W m^{-2} K^{-4}
A	Wien's displacement constant	2.8978×10^{-3} K m

Units		
AU	Astronomical unit	1.496×10^{11} m

Properties of the Sun and Earth	
Sun's radius	6.96×10^{8} m
Sun's mass	1.99×10^{30} kg
Total radiated solar power	3.85×10^{26} W
Sun's black-body temperature	5770 K
Earth's equatorial radius	6378135 m
Earth's polar radius	6356775 m
Semi-major axis of Earth's orbit around the Sun	1.496×10^{11} m
Earth's mass $M_\oplus$	5.976×10^{24} kg
Standard gravitational parameter $\mu = GM_\oplus$	3.986×10^{14} m^{3} s^{-2}
Mean global albedo	0.35
Moon's radius	1.738×10^{6} m
Average Earth-Moon distance	3.844×10^{8} m

Main equations	
Maxwell	$$\begin{aligned}\nabla\cdot\mathbf{E} &= 0\\ \nabla\cdot\mathbf{B} &= 0\\ \nabla\times\mathbf{E} &= -\dot{\mathbf{B}}\\ \nabla\times\mathbf{B} &= \varepsilon_0\,\mu_0\,\dot{\mathbf{E}}\end{aligned}$$
Planck	$$L_\lambda = \frac{2hc^2}{\lambda^5\big(e^{hc/\lambda kT}-1\big)}.$$
Rayleigh-Jeans	$$L_f \approx \frac{2kTf^2}{c^2} = \frac{2kT}{\lambda^2}$$
Stefan	$$M = \sigma T^4$$
Wien	$$\lambda_{\mathsf{max}} = A/T$$
Kirchhoff	$$L_\lambda = \varepsilon(\lambda) L_{\lambda,\mathsf{bb}}$$
Fourier	$$f(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} a(\omega)\exp(i\omega t)d\omega$$
Inverse Fourier	$$a(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)\exp(-i\omega t)dt$$

$$\begin{aligned}P &= \frac{2\pi}{n}\\ n &= \sqrt{\frac{\mu}{a^3}}\\ \frac{d\Omega}{dt} &= -2.06474\times10^{14}\frac{\cos i}{a^{3.5}(1-e^2)^2}\\ \bar{F} = m\bar{a} &= \frac{\mu m}{r^2}\frac{\bar{r}}{r}\end{aligned}$$