Examination
Introduction to Earth Observation (AE2–E02)
Faculty of Aerospace Engineering
Delft University of Technology

July 3, 2008, 14:00–17:00

VERSION with ANSWERS

Please read these instructions first:

This exam contains 10 questions totaling 180 points. The value of each question is given below, with the value of any subquestions listed in parentheses. Please use a new piece of paper for the start of each question in order to facilitate the distribution of the exam to the graders. Also be sure to write your name and 7-digit student ID clearly at the top right of each page, and number your pages (for example, page 3 of 12). Show all of your work, as incomplete or unsupported results will be penalized. **This is a closed-book exam**. You are not allowed to have any books, hand-outs, or notes on your table. Use of a pocket (non-programmable) calculator is allowed.

Question 1: True/False, 10 pts. (2/2/2/2/2)

(10 min.) State whether the following statements are **True** or **False**. You must justify your answer with a short explanation (the right answer with a wrong explanation will be marked incorrect, and vice versa).

a) A geostationary orbit can be geosynchronous, but not all geosynchronous orbits are geostationary.
ANSWER: TRUE, you can have geosynchronous orbits that are not equatorial (lecture 3, slide 43, and AE1-801 lecture notes)

b) A repeat orbit is often defined in terms of the number of orbits within the repeat time frame. Is it possible to have a low Earth orbit (LEO) repeat track of 200/8, i.e. 200 orbits every 8 days? Assume a circular orbit with an angular velocity defined by $w_0 = (\frac{\mu}{a^3})^{1/2} = 1.107\text{e-}3$ rad/s.
ANSWER: FALSE, a LEO orbit lasts roughly 90 minutes, so there are 16 orbits per day...16*8 is 128, which is the maximum number of orbits in an 8 day period, so 200 is not possible. The angular velocity value (based on a 500km orbit) was only provided to allow the orbital period to be computed for those who did not know off hand that a LEO is 90 min (lecture 3, slides 22, 29-33)

c) An orbit can be both in a sun-synchronous orbit and in a repeat orbit at the same time.
ANSWER: TRUE, this is often the case for altimetry missions (lecture 3, slide 36)

d) Until the early 1920's, a clock did not exist that was more accurate than that of the Earth's daily rotation about its axis.
ANSWER: TRUE, it was only after the 1920's that man-made clocks became more accurate than the Earth's rotation (especially the atomic clocks developed later in the 1950s) (lecture 4, slide 42))

e) The geoid is a mathematically perfect ellipsoid that is sized to coincide with the mean sea level.
ANSWER: FALSE, the geoid does coincide with mean sea level, but it is not an ellipsoid (lecture 5, slide 41-45).

Question 2: Platforms, 10 pts.

Describe at least two advantages and two disadvantages of using an airborne platform (i.e., as compared to a spaceborne platform) to collect observations.

ANSWER: (See lecture 2, slide 17)

Pro's

- flexible in operation and payload
- wide range of altitudes and speeds
- high spatial resolution (compared with satellites)

Con's

- short mission duration no time-continuity of data stream
- smaller spatial coverage unsuitable for large areas
- platform dynamics (atmosphere, vibrations)

Question 3: Time, 15 pts. Describe with words or pictures the difference between Universal Time (UT1), International Atomic Time (TAI), and Coordinated Universal Time (UTC). **ANSWER:** (See lecture 4, slide 44-45)

Question 4: The Global Positioning System (GPS), 15 pts. (5/5/5)

We are able to determine our absolute position using the Global Positioning System (GPS) through signals transmitted by the various orbiting GPS satellites. These signals must propagate through the ionosphere (and the rest of the atmosphere), whose high electron content can often cause distortions. This, in turn, can lead to errors in the estimated position at the ground level. The designers of the GPS satellite network were aware of this problem in advance, but used the fact that, when not in a vacuum, different frequencies travel at different velocities to their advantage to help eliminate these errors.

a) Explain in words or pictures how the GPS network is able to significantly reduce these ionospheric errors.
ANSWER: The main point to recognize here is that GPS broadcasts on two frequencies, L1 and L2, and that these can be used to cancel out most of the ionospheric effects.

b) The measured travel time delay (Δt_{iono}) due to the ionosphere for a GPS signal of a given frequency, f, is described as:

$$\Delta t_{iono} \approx \frac{\Delta s}{c_{vac}} + \frac{A}{f^2}$$

Using this equation, describe how one could solve for the offset in geometric distance (Δs) caused by the ionosphere. Assume that the value for the speed of light in a vacuum (c_{vac}) is also well known.
ANSWER: (See lecture 3, slide 11)
A is unknown, so use the given equation and the fact that GPS broadcasts on two frequencies to solve for A. Then solve for Δs.

c) In addition to atmospheric errors, a number of other error sources were discussed in the lectures that can impact the accuracy of the position estimates from GPS. List at least three of them.
ANSWER: (See lecture 3, slide 16)
(a) Clock errors
(b) Atmospheric effects
(c) Imprecise knowledge of GPS orbits
(d) Relativistic effects

(e) Encryption
(f) Dilution of precision (i.e., geometry)
(g) Multipath

Question 5: Gravity, 30 pts. (10/10/10)

A new satellite will be launched later this year by ESA called the Gravity field and steady-state Ocean Circulation Explorer (GOCE) mission. The goal of the GOCE mission will be to measure the Earth's static (i.e., non-variable) gravity field using the technique of satellite gravity gradiometry (SGG). This mission will complement the currently flying GRACE mission, which relies on a technique based on satellite-to-satellite tracking (SST) to infer gravitational changes.

a) Explain in words and/or pictures the basic working principle behind the SGG approach used by GOCE and the SST method used by GRACE, and highlight any differences between the two measurement techniques.

ANSWER: (See lecture 6, slides 10-33 for all) The main points to recognize here are that GOCE measures gravity gradients directly using the three pairs of accelerometers, while GRACE infers the gravity from changes in the inter-satellite range measurements. A drawing of the six accelerometer pairs for GOCE was expected, as well as a diagram showing the two GRACE satellites. Additional comments or descriptions were also expected if the concepts were not obvious from the diagrams. The absolute positioning with GPS (needed for both missions) should also have been mentioned.

b) Both GRACE and GOCE make use of accelerometers to accurately measure external forces acting on the satellite(s). For GOCE, the accelerometer pairs are separated by roughly 0.5 meter, and each accelerometer has an accuracy (i.e. σ), of 1.5e-12 m/s^2. If the GOCE satellite is launched into an orbit with an altitude of 250km, what is the expected overall accuracy of the the on-board gradiometer, expressed in units of Eötvös (1 Eötvös = 1E = 1e-9 Gal/cm = 1e-9 $1/s^2$, and 1 Gal = .01 m/s^2)?

ANSWER: In short, if you have accelerometers accurate to 1.5e-12, and there is a 0.5m difference between the vertical accelerometers, then the accuracy of the gradiometer is simply 1.5e-12/.5 = 3 mE.

c) The accelerometers on GOCE are extremely sensitive because they must detect small variations in the gravity field, caused by mass elements located at or below the Earth's surface, from a distance of 250km away. The required sensitivity of the accelerometers would decrease if the gradiometer of GOCE could take measurements directly at the Earth's surface. Assuming this could be done, what level of sensitivity would the accelerometers need to have in order for the gradiometer to maintain the same level of accuracy as that described in part b? Express your answer in units of m/s^2. Assume the radius of the Earth to be 6378.136 km. In addition, the following information may be useful for your calculations.

$$F = ma = \frac{\mu m}{r^2}\bar{r}$$

$$\mu = GM = 398600.4418 km^3/s^2$$

ANSWER: Here, you need to use the given inverse square law and some information from part b. Using this, you can easily derive an equation that relates the change in field intensity from the satellite orbit to the Earth's surface, i.e., $\frac{a_e}{a_{sat}} = \frac{r_{sat}^2}{r_e^2} = 1.08$. This means that you will need an accelerometer that is accurate to 1.5*1.08e-12 = 1.62e-12 m/s^2.

Wavelength	λ	0.05	m
Gain	G	3000000	
Antenna surface	A_{ant}	600	m^2
Distance Earth-Venus	R	$4 \cdot 10^{10}$	m
Radius Venus	r_v	$6050 \cdot 10^3$	m
Radar cross section Venus	σ	10^{13}	m^2
Boltzmann constant	k	$1.3807 \cdot 10^{-23}$	JK^{-1}
Radar bandwidth	B	$5 \cdot 10^6$	Hz
Thermal noise temperature	T_{sys}	1000	K
Pulse length	τ	$50 \cdot 10^{-6}$	s

Table 1: Radar properties

Question 6: RADAR, 15 pts. (5/5/5)

If we try to detect the planet Venus using an Earth-based radar system with properties as described in table 1 with an SNR of 10 dB,

a) How much transmit power [Watts] should that radar system have?

b) What would be the range resolution [m] if the system transmits a short 'rectangular' pulse of 50 μs, without frequency modulation? Explain your answer.

c) What would be the range resolution [m] if the system transmits a chirp, with changing frequencies over a bandwidth of 5 MHz? Explain your answer.

ANSWER:

a) *Goal: understand and apply the* **radar equation** *given the table as help, and understand how to work with dB's. The question can be answered by approximation.* (see Lecture 16, slides 40–49, and Rees Chap. 9.3)

The question can be rephrased as 'give the transmit power P_t for SNR=$\frac{P_r}{P_n} = 10$.' (the linear SN-ratio is equal to the value in dB's: $10 \cdot {}^{10}\log 10 = 10$ dB). Derive P_r using the radar equation.

$$P_r = \frac{P_t}{4\pi R^2} \cdot G \cdot \sigma \cdot \frac{A_{\text{ant}}}{4\pi R^2} \quad \text{[W]} \tag{1}$$

The noise part is:

$$P_n = k \cdot B \cdot T_{\text{sys}} \quad \text{[W]} \tag{2}$$

Thus the equation for the SNR is

$$\text{SNR} = \frac{P_r}{P_n} \tag{3}$$

It is requested that the SNR should be 10 (at least), so therefore we solve eqs. (1)–(3) to derive P_t.

b) *Goal: understand the way resolution can be improved in* **range**. *The example was discussed explicitly in class (two 'trees' and a rectangular pulse bouncing on both of them). In this answer they should show that they understand this by stating (or figuring out on the spot) the equation* (See Lecture 18, slides 36–37. rees Chap. 8.3.2, eq. (8.13))

For a rectangular pulse of length τ seconds, traveling with the speed of light c, the range resolution Δ_r is

$$\Delta_r = \frac{1}{2}\tau c \text{ [m]} \tag{4}$$

(The factor $0,5$ is due to the 2-way travel of the pulse, to Venus and back to earth)

c) *Goals: understand that (1) the* **chirp** *is the way to improve range resolution, (2) that a chirp means varying* **frequencies** *over a certain* **bandwidth***, and (3) that the final resolution is inversely proportional to the bandwidth. In class, it was emphasized that* **bandwidth** *is an extremely important parameter in signal processing and therefore also for earth observation. The equation was mentioned in class, but could also be derived on the spot form dimension analysis.*

(See Lecture 18, slides 35–39. Rees p.211)

The range resolution of a chirp signal is

$$\Delta_r = \frac{c}{2B} = \frac{3 \cdot 10^8}{2 \cdot 5 \cdot 10^6} = \frac{300}{10} = 30\,[\mathrm{m}] \tag{5}$$

Question 7: Albedo, 20 pts. (5/5/5/5)

If the albedo of the planet Venus is 0.65, and the true surface temperature of Venus is 750 K,

a) What would be the brightness temperature observed by a microwave sensor?

b) Explain the physical relationship(s) you used.

c) Would the brightness temperature change if we would observe Venus at other, e.g., optical, wavelengths?

d) Describe the role of a blackbody in this calculation.

ANSWER: Goal: understanding of the concepts **albedo**, **emissivity**, **brightness temperature**, **blackbody**.

a) (Lecture 16 and Rees, p.48) The emissivity ε is related to the albedo r by $\varepsilon = 1 - r$, thus $\varepsilon = 0.35$. The brightness temperature $T_b = \varepsilon T = 0.35 \cdot 750 = 257.5$ K. (using the Rayleigh Jeans approximation, because the question refers to microwave observations)

b) **Kirchhof's law** is applied which relates the radiance of a black body to the radiance of a true body, using the emissivity ε. The **Rayleigh-Jeans** approximation is applied, since we are using a microwave sensor, that is, relatively long wavelengths. The Rayleigh-Jeans approximation suggests a linear relation between the true temperature and the brightness temperature: $T_b = \varepsilon T$ (Rees, p. 27)

c) **Yes**, the *brightness temperature* is a function of the *emissivity*, which is dependent on the *wavelength*. (Rees, p. 25/26, eq. 2.38)

d) The brightness temperature is only equivalent to the real, physical temperature if the object is a perfect emitter ($\varepsilon = 1$), and therefore a perfect absorber. Such an object is called a blackbody.

Question 8: Maxwell's Equations, 30 pts. (15/15) Given a Cartesian reference system $\{x, y, z\}$ and the Maxwell equations

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\dot{\mathbf{B}} \\ \nabla \times \mathbf{B} &= \varepsilon_0\, \mu_0\, \dot{\mathbf{E}} \end{aligned}$$

a) Prove that a horizontally polarized electromagnetic wave, propagating in the z-direction, with the electric field vector at position z and time t described by

$$E_y = c\, E_0\, \cos(\omega t - kz)$$

satisfies the third Maxwell equation.

b) Derive the expression for the velocity of light as a function of the electric permittivity and the magnetic permeability.

ANSWER: Goal: understanding of Maxwell's equations and their relevance

a)
- The general wave equation is $E_x = E_0 \cos(\omega t - kz - \phi_x)$, where $k = 2\pi/\lambda$. Here, however, the equation is given for E_y in stead of E_x.
- horizontally polarized, therefore $E_x = 0$
- magnetic field vector is orthogonal to electric field vector, with a minus-sign, and scaled by $\frac{1}{c}$, therefore

$$B_x = -\frac{cE_0}{c}\cos(\omega t - kz)$$
$$B_y = 0$$

  (Note that the minus sign follows from the right-hand rule, but it also can be a posteriori detected to make Maxwell-3 fit)
- The direction of propagation field vectors are zero by definition: $E_z = B_z = 0$
- Moreover, as it is horizontally, i.e., plane, polarized, the rotation ϕ_x has no meaning and can be set to zero.

Thus we have:

$$\begin{aligned} E_y &= c\,E_0 \cos(\omega t - kz) \\ E_x &= E_z = 0 \\ B_y &= B_z = 0 \\ B_x &= -E_0 \cos(\omega t - kz) \end{aligned} \tag{6}$$

and derive using derivatives, and using $\omega = c \cdot k$.

b) *Goal: understand what electric permittivity ε_0 and the magnetic permeability μ_0 are and use it to derive c. See Rees, Chap. 2, page 9, and Derivation in Lecture 11.*
Use fourth Maxwell equation with $\omega = c \cdot k$ to derive c, which is the theoretical derivation of the speed of light.

Question 9: Plank's Law, 20 pts. (10/10)

Planck's law, describing the spectral radiance L_λ of a blackbody as a function of various parameters is given by

$$L_\lambda = \frac{2hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}.$$

a) Show that when dealing with visible and near-infrared wavelengths, the thermal emission of the Earth can be ignored with respect to reflected solar radiation.

b) Show how significant thermal emission is for radiowaves, say 1 GHz and 1 THz.

h	Planck constant	6.6261×10^{-34} J s
c	Speed of light *in vacuo*	2.9979×10^{8} m s^{-1}
k	Boltzmann constant	1.3807×10^{-23} J K^{-1}

ANSWER: Goals: (1) understanding of Planck's law, what it describes and what parameters are involved, (2) Understand that reflected sunlight is in fact radiation from the Sun, with a specific spectral radiance dependent on the Sun's temperature, (3) know the wavelengths of visible light.

(Rees, p. 22-23. and question 5, p.33)

a) Consider the thermal emission of the Earth and the reflected solar radiation on Earth. It is stated that the thermal emission can be ignored with respect to the reflected solar radiation. In other words, state the ratio between both, and show that this ratio is very very small. The question should be answered for visible wavelengths and for near-infrared wavelengths.

Given $c = f \cdot \lambda$, and estimate temperature of the earth, e.g. $T \approx 300$ K. For reflected solar radiation use temperature of the sun $T \approx 6000$ K. Approximate wavelengths of the visual spectrum: 0.4–0.7 μm $\approx 0.5 \times 10^{-6}$m. Infrared wavelengths (should have a bit longer wavelength than visible light), say: 1 μm $\approx 1 \times 10^{-6}$m.

To determine the influence of thermal radiation with respect to reflected solar radiation, the ratio between the two can be computed:

$$\frac{L_{\lambda,T=300}}{L_{\lambda,T=6000}} = \frac{\frac{2hc^2}{\lambda^5\left(e^{hc/\lambda k T_{300}}-1\right)}}{\frac{2hc^2}{\lambda^5\left(e^{hc/\lambda k T_{6000}}-1\right)}} = \frac{e^{hc/\lambda k T_{6000}} - 1}{e^{hc/\lambda k T_{300}} - 1}$$

Using an average value for the visual wavelengths, say 0.5μm, we find

$$\frac{L_{\lambda,T=300}}{L_{\lambda,T=6000}} \approx 10^{-50} \tag{7}$$

Synthesis: From this result it shows that the thermal radiation of the earth $L_{\lambda,T=300}$ is about 50 orders of magnitude smaller than the reflected radiation of the sun $L_{\lambda,T=6000}$. Using visible ($\approx 0,5\mu$m) or infrared ($\approx 1\mu$m) does not really matter considering the equations above.

b) Where the previous question relates to the visible/infrared wavelenghts, this one now refers to radiowaves, at two wavelengths.

For radio waves, $f = 1$ GHz, the equations are

$$\frac{L_{\lambda,T=300}}{L_{\lambda,T=6000}} \approx \frac{10^{-5}}{2 \cdot 10^{-4}} \approx 0,05$$

For f=1 THz the equations are

$$\frac{L_{\lambda,T=300}}{L_{\lambda,T=6000}} \approx \frac{10^{-2}}{0,22} \approx 0,05$$

Synthesis: From this result it shows that the thermal radiation of the earth $L_{\lambda,T=300}$ is about 2 orders of magnitude smaller than the reflected radiation of the sun $L_{\lambda,T=6000}$, when considering radiowaves of 1 GHz and 1 THz. This ratio is much smaller than the ratio for visible ($\approx 0,5\mu$m) or infrared ($\approx 1\mu$m) wavelengths, showing that the influence of the earth's thermal radiation will be much more significant for the radiowaves.

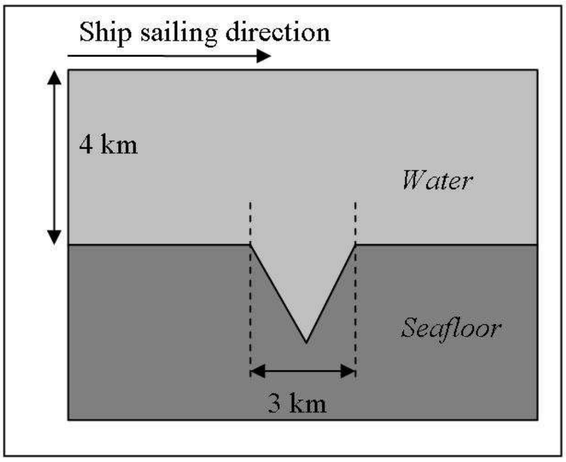
Question 10: Acoustics, 15 pts. (5/5/5)

a) What are the two main reasons that sound is being used for studying the underwater environment?

b) A bathymetric survey is carried out to study geological processes of the deep ocean sea floor. Use is made of a single beam echo-sounder. One of the survey goals is to investigate a 3-km wide fracture zone in 4-km deep water (as depicted in the figure below). What are the two main design parameters of the single beam echo-sounder system? Motivate your answer.

c) The single beam echo-sounder employed for the survey works at a frequency of 12 kHz. We consider again the situation described in part b, depicted in the figure below. The source diameter of the single beam echo-sounder is 1.6 m. Is this sufficient for mapping the fracture zone features? Assume a sound speed of 1500 m/s in the water column.

ANSWER:



a)
- In water, sound is absorbed to a much lesser extent than other wave types (depends on frequency).
- Due to the sound speed profile channeling occurs, thereby keeping the (geometrical) propagation loss limited (cylindrical spreading instead of spherical spreading).

b)
- **Frequency**. Due to the large water depth, the absorption coefficient should be low; otherwise the signal is attenuated too much when arriving at the receiver and therefore it can not be detected. A low absorption coefficient is obtained when a low frequency is selected. The frequency cannot be too low, because of the increasing ambient noise level and the lower directivity of the source. Simulations and experience show the optimum frequency to be around 12 kHz (instead of several 100 kHz for shallow-water systems).
- **Horizontal resolution.** The horizontal resolution should be small enough for imaging the features of interest. The horizontal resolution is determined by the water depth, the frequency and the sonar size.

c) The opening angle β (degrees) for the single beam echo-sounder amounts to:

$$\beta = \frac{65\lambda}{d} = \frac{65 \times 1500}{1.6 \times 12000} = 5.1^\circ \tag{8}$$

with λ the wavelength and d the sonar source diameter. For a water depth of 4 km, the horizontal resolution becomes:

$$\frac{\beta\pi}{180} \times 4000 = 355\,\mathrm{m}. \tag{9}$$

This should be sufficient to map the 3-km wide fracture zone as the horizontal is one order of magnitude smaller than the width of the fracture zone.