FLIGHT DYNAMICS AE3-302

EXERCISES

ANSWERS

Delft University of Technology

Faculty of Aerospace Engineering

Control and Simulation Division

1. (a)

Relative density	$\mu_c = 69.34$
Elevator effectiveness	$C_{m_\delta} = -2.0 \text{ rad}^{-1}$
Neutral point stick free	$x_{n_{free}} = 0.5\,\bar{c}$
Manoeuvre point stick free	$x_{m_{free}} = 0.6\,\bar{c}$

   (b) $0.38 < \frac{x_{cg}}{\bar{c}} < 0.50$

   (c) $0.318 < \frac{x_{cg}}{\bar{c}} < 0.466$

2. Use the expression for the stick displacement per g,

$$\frac{d\delta_e}{dn} = \frac{-1}{C_{m_\delta}}\left(\frac{W}{\frac{1}{2}\rho V^2 S}\left\{\left(\frac{x_{cg} - x_{n_{fix}}}{\bar{c}}\right) + \left(\frac{C_{m_q}}{2\mu_c}\right)\right\}\right)$$

   and fill in the three measurements one after the other.

   (a) $C_{m_\delta} = -1.5 \text{ rad}^{-1}$

   (b) $C_{m_q} = -22,6$

   (c) $x_{n_{fix}} = 0.364$

   (d) i. h = 0 m: $x_{m_{fix}} = 0.50\bar{c}$

   ii. h = 4000 m: $x_{m_{fix}} = 0.45\bar{c}$

3. (a) $F_e = K \cdot \delta_e = \frac{-K}{C_{m_\delta}}\left\{C_{m_0} + \frac{W}{\frac{1}{2}\rho V^2 S}\left(\frac{x_{cg} - x_{n_{fix}}}{\bar{c}}\right)\right\}$

   (b) $\frac{x_{n_{free}}}{\bar{c}} = C_{m_0} \cdot \left(\frac{\frac{1}{2}\rho V^2 S}{W}\right) + \frac{x_{cg}}{\bar{c}}$

   Control force stability if $x_{cg} < x_{n_{fix}}$.

   (c)

$$i_h = \frac{C_{m_{ac}}}{C_{N_{h_\alpha}}\left(\frac{V_h}{V}\right)^2\left(\frac{S_h}{S}\right)\left(\frac{l_h}{\bar{c}}\right)} - \alpha_0 + \frac{\frac{W}{\frac{1}{2}\rho V^2 S} \cdot \frac{x_{cg} - x_{n_{fix}}}{\bar{c}}}{C_{N_{h_\alpha}}\left(\frac{V_h}{V}\right)^2\left(\frac{S_h}{S}\right)\left(\frac{l_h}{\bar{c}}\right)}$$

4. See the lecture notes.

5. See the lecture notes.

6. (a) $\left(\delta_{t_e} - \delta_{t_{e_0}}\right) = 0.241$

   $\left(\dfrac{dF_e}{dV}\right)_{F_e=0} = 4.165.$

   (b) $F_e = -39.15$ N

   The pilot has to exert a *pulling* force on the control column.

   (c) $\left(\delta_{t_e} - \delta_{t_{e_0}}\right) = 0.287$

   The trim tab should be rotated *backwards*.

   (d) $\left(\dfrac{dF_e}{dV}\right)_{F_e=0} = 4.95$, so control force stability has been increased.

7. (a) See the lecture notes.

   (b)

$$\frac{d\delta_\phi}{d\left(\frac{rb}{2V}\right)} = \frac{4\mu_b}{C_L} \qquad \frac{d\delta_a}{d\left(\frac{rb}{2V}\right)} = \frac{-C_{l_r}}{C_{l_{\delta_a}}} \qquad \frac{d\delta_r}{d\left(\frac{rb}{2V}\right)} = \frac{-C_{n_r}}{C_{n_{\delta_r}}}$$

$$\phi > 0 \qquad \delta_a > 0 \qquad \text{and} \qquad \delta_r < 0 \tag{1}$$

   (c) $K < 0$

   (d) The adverse yawing is increased, see the lecture notes.

8. See the lecture notes.

   (b) $\lambda_{1,2} = -0.065 \pm 0.084i$

   (d)

Period	$P = 2.64$ s
Damping ratio	$\zeta = 0.61$
Time to damp to half amplitude	$T_{\frac{1}{2}} = 0.38$ s
Eigenfrequency	$\omega_n = 2.39$ rad/s

9. (a) See the lecture notes.

   (b) $\delta_r = 0$:

$$\frac{d\delta_a}{d\left(\frac{rb}{2V}\right)} < 0 \qquad \frac{d\delta_r}{d\left(\frac{rb}{2V}\right)} \qquad \frac{d\phi}{d\left(\frac{rb}{2V}\right)} \qquad \frac{d\beta}{d\left(\frac{rb}{2V}\right)}$$

   $\delta_a = 0$:

$$\frac{d\delta_r}{d\left(\frac{rb}{2V}\right)} > 0 \qquad \frac{d\phi}{d\left(\frac{rb}{2V}\right)} > 0 \qquad \frac{d\beta}{d\left(\frac{rb}{2V}\right)} > 0$$

(c) $\delta_r = 0$:

$$\frac{d\delta_a}{d\left(\frac{rb}{2V}\right)} > 0 \qquad \frac{d\phi}{d\left(\frac{rb}{2V}\right)} > 0 \qquad \frac{d\beta}{d\left(\frac{rb}{2V}\right)} > 0$$

$\delta_a = 0$:

$$\frac{d\delta_r}{d\left(\frac{rb}{2V}\right)} < 0 \qquad \frac{d\phi}{d\left(\frac{rb}{2V}\right)} > 0 \qquad \frac{d\beta}{d\left(\frac{rb}{2V}\right)} > 0$$

10. See the lecture notes.

11. (a)
    - The forces along the Y-axis:

$$W \sin\phi + Y = mVr$$
$$C_L \cdot \phi + C_{Y_\beta} \cdot \beta + \left(C_{Y_r} - 4\mu_b\right) \frac{rb}{2V} = 0$$

    - The moment equation about the X-axis:

$$L + W \cos\phi \cdot \Delta y = 0$$
$$C_{l_\beta} \cdot \beta + C_{l_r} \cdot \frac{rb}{2V} + C_L \cdot \frac{\Delta y}{b} = 0$$

    - The moment equation about the Z-axis:

$$N = 0$$
$$C_{n_\beta} \cdot \beta + C_{n_r} \cdot \frac{rb}{2V} = 0$$

(b) $\dfrac{d\left(\frac{\Delta y}{b}\right)}{d\left(\frac{rb}{2V}\right)} = \dfrac{1}{C_L} \cdot \left(\dfrac{C_{l_\beta} C_{n_r} - C_{l_r} C_{n_\beta}}{C_{n_\beta}}\right)$
$C_{n_\beta} < 0$ and $C_{n_r} > 0$, so $\beta > 0$.

(c) $\dfrac{d\left(\frac{\Delta y}{b}\right)}{d\left(\frac{rb}{2V}\right)} > 0.$
See the lecture notes.