FLIGHT DYNAMICS AE3-302

EXERCISES

Delft University of Technology

Faculty of Aerospace Engineering

Control and Simulation Division

W = 150000 N	g = 9.81 m s^{-2}	$\left(\frac{V_h}{V}\right)^2 = 1$
S = 60 m^2	S_h = 15 m^2	S_e = 3 m^2
$\bar{c}$ = 3.0 m	l_h = 12 m	$\bar{c}_e$ = 0.4 m
$\frac{d\delta_e}{ds_e}$ = 2.25 rad m^{-1}	C_{m_α} = -1.5 rad^{-1}	C_{m_q} = -14 rad^{-1}
x_{cg} = 0.25 $\bar{c}$	C_{N_α} = 6 rad^{-1}	$C_{N_{h_\delta}}$ = 2.0 rad^{-1}
ρ = 1.225 kg m^{-3}	C_{h_α} = 0	C_{h_δ} = -0.4 rad^{-1}

Table 1: Aircraft parameters

1. For a conventional aircraft the following aircraft parameters were determined, see Table 1.

   (a) Calculate the relative density μ_c, the elevator effectiveness C_{m_δ}, and both the location of the neutral point, stick free, and the location of the neutral point, stick fixed, $x_{n_{free}}$ and $x_{m_{free}}$, respectively.

   (b) It is required, that

   - the aircraft possesses control force stability as well as control position stability.
   - the stick force per g, $\frac{dF_e}{dn}$, has the required sign.
   - $100 \ \frac{N}{g} < \left|\frac{dF_e}{dn}\right| < 300 \ \frac{N}{g}$.

   Calculate the range of locations of the center of gravity for which the previous requirements all hold.

   (c) For a flight level at which ρ = 0.450 kg/m^3, determine both $\frac{x_{m_{free}}}{\bar{c}}$ and the range of locations of the center of gravity for which the requirements of Question (b) hold.

	$\frac{x_{cg}}{\bar{c}}$ [-]	h [m]	ρ [kg m^{-3}]	$\frac{1}{2}\rho V^2$ [N m^{-2}]	$\frac{d\delta_e}{dn}$ [rad g^{-1}]
1	0.20	0	1.225	6000	-0.20
2	0.35	0	1.225	6000	-0.10
3	0.20	4000	0.819	6000	-0.17

Table 2: Aircraft parameters

2. For a conventional aircraft the following aircraft parameters hold,

W = 1500000 N S = 250 m^2 $\bar{c} = 6.0$ m

g = 9.81 m/sec^2

During pull-up manoeuvres both at different flight altitudes h and at different locations of the center of gravity x_{cg}, the stick displacement per g, $\frac{d\delta_e}{dn}$, has been determined, see Table 2. (Assume that only C_{m_α} depends on the location of the center of gravity x_{cg}. All other aerodynamic derivatives are *not* dependent on the flight conditions as well as the aircraft configuration)

(a) Determine the elevator effectiveness C_{m_δ}.

(b) Determine the stability derivative C_{m_q}.

(c) Calculate the position of the neutral point stick fixed, $x_{n_{fix}}$.

(d) Calculate the position of the manoeuvre point stick fixed, $x_{m_{fix}}$, for both h = 0 m and h = 4000 m.

3. Consider a conventional aircraft which has a hydraulic elevator control mechanism in such a way, that the aerodynamic hinge moment has no direct effect on the elevator angle δ_e, and the pilot does not feel its influence. The control mechanism exerts control forces on the control stick, which are proportional to the elevator deflection s_e. The aircraft can be trimmed by adjusting the horizontal stabilizer setting i_h.

   (a) Derive an expression for the control force as a function of the airspeed V, assuming that the trim setting is fixed. Draw a sketch of the elevator control force curve for two different settings of the horizontal stabilizer.

   (b) Determine the location of the neutral point, stick free $x_{n_{free}}$. Under which circumstances does the aircraft possess control force stability?

   (c) Derive an expression for the setting of the horizontal stabilizer as a function of the airspeed V, assuming that the control force is trimmed to zero. Draw a sketch of the curve for two different locations of the center of gravity.

4. (a) What is the required sign of the control force stability?

   (b) Why is it necessary to set both an upper limit and a lower limit for the control force stability?

   (c) Suppose that the control force stability does not have the required sign. If you consider a steady straight flight during which the control force $F_e = 0$, what will happen to the aircraft behaviour when atmospheric turbulence causes a positive change of the angle of attack α?

   (d) Suppose that the control *position* stability has the required sign in contrast with the control force stability, which has *not* the required sign. What will happen to the elevator angle δ_e when atmospheric turbulence causes a positive change of the angle of attack α, assuming a stick free condition and a steady straight flight condition?

   (e) Considering the flight conditions of the previous question, what is the position of both the neutral point, stick free $x_{n_{free}}$ and the aircraft center of gravity x_{cg} relative to the neutral point, stick fixed $x_{n_{free}}$?

   (f) If C_{h_δ} has the required (negative) sign, what is the sign of C_{h_α} in Question (c)?

   (g) What happens to the aircraft behaviour after a positive change of the angle of attack, if the sign of C_{h_α} would have been different?

5. (a) Why is it useful not only to consider static stability stick *fixed*, but also static stability stick *free*? Why should the sign of C_{h_δ} be negative? Motivate your answer.

   (b) Derive an expression for the contribution of the horizontal tailplane to the static stability, stick free, $C_{m_{\alpha_{free}}}$.
   Which term of this expression determines the difference between the static stability, stick fixed and the static stability, stick free?

   (c) Indicate how the sign of C_{h_α} determines whether or not the aircraft is static stick free more stable than static stick fixed, and how the sign indicates whether the two static conditions are equally stable.
   Show that an aircraft which is static stable, stick free also possesses control force stability.

   (d) Consider an aircraft for which it holds that the location of the center of gravity x_{cg} is located between the neutral point stick free $x_{n_{free}}$ and the neutral point stick fixed $x_{n_{fix}}$ ($C_{m_\alpha} < 0$). At a certain trim tab angle $\delta_{t_{e_0}}$ the control force F_e does not change with the airspeed V.
   Should the pilot exert a pulling force or a pushing force on the control stick?

   (e) Again, consider the situation of the previous question. Now the trim tab is adjusted in such a way that at a certain airspeed V_{trim} the control force F_e is equal to zero.
   In which direction is the trim tab rotated with respect to $\delta_{t_{e_0}}$? And does the aircraft possess control force stability? Motivate your answers by drawing a sketch of the elevator control force curve as a function of V.

W = 150000 N	ρ = 0.6 kg m^{-3}	V = 100 m s^{-1}
S = 70 m^2	S_e = 3 m^2	$\left(\frac{V_h}{V}\right)^2 = 1$
$\bar{c}$ = 2.5 m	$\bar{c}_e$ = 0.4 m	$\frac{d\delta_e}{ds_e}$ = 2.0 rad m^{-1}
C_{m_δ} = -2.0 rad^{-1}	C_{h_δ} = -0.3 rad^{-1}	$C_{h_{\delta_t}}$ = -0.12 rad
$\frac{x_{cg}}{\bar{c}} = 0.35$	$\frac{x_{n_{los}}}{\bar{c}} = 0.62$	

Table 3: Aircraft parameters

6. Consider a conventional aircraft during a trimmed horizontal flight ($F_e = 0$), and for which the aircraft parameters in Table 3 hold:

   (a) Calculate both $\left(\delta_{t_e} - \delta_{t_{e_0}}\right)$ and the control force stability $\left(\frac{dF_e}{dV}\right)_{F_e=0}$.

   (b) At a certain altitude a weight of 20000 N is being released out of the aircraft, causing a relocation of the center of gravity at $x_{cg_{new}} = 0.25\ \bar{c}$.
   Calculate the magnitude as well as the sign of the control force, which is necessary to keep flying at the original airspeed V. Assume that the trim tab is not being adjusted. Should the pilot exert a pulling force of a pushing force at the control column?

   (c) Calculate the change in trim tab angle if the control force is being reduced to zero, and assuming that the airspeed does not change. Should the trim tab be rotated forward or backward?

   (d) Does the aircraft possess more control force stability or less control force stability after the weight release?

7. (a) Starting from the asymmetric equations of motion, derive the equations of motion which hold for steady horizontal turns ($\beta \neq 0$). Point out which parameter is related to the component of the aircraft's weight.

   (b) Assuming that for steady turns the parameters C_{Y_r}, $C_{Y_{\delta_r}}$, $C_{Y_{\delta_a}}$, $C_{\ell_{\delta_r}}$ and $C_{n_{\delta_a}}$ can be neglected, derive the following expressions, for turns with sideslip angle $\beta = 0$,

$$\frac{d\delta_\phi}{d\left(\frac{rb}{2V}\right)} \qquad \frac{d\delta_a}{d\left(\frac{rb}{2V}\right)} \qquad \frac{d\delta_r}{d\left(\frac{rb}{2V}\right)}$$

   What is the sign of ϕ, δ_a and δ_r for a right turn?

   (c) Consider a mechanical coupling between the ailerons and the rudder in such a manner that co-ordinated turns ($\beta = 0$) are possible. Only the ailerons are used to turn the aircraft. The coupling between the ailerons and the rudder is written in mathematical form,

$$\delta_r = K \cdot \delta_a$$

   Use the results of the previous question to find an expression for K.
   What is the sign of K?

   (d) Now consider an aircraft without a mechanical coupling between the ailerons and the rudder. If 'adverse yawing' occurs during the initiation of a turn with only use of the ailerons, then explain accurately whether the earlier mentioned mechanical coupling would increase or decrease the adverse yawing, or whether there would be no difference.

V = 45 m s^{-1}	$\bar{c}$ = 1.5875 m	K_Y = 1.083	μ_c = 58.66
C_{X_0} = 0	C_{Z_0} = -0.902		
C_{X_u} = -0.211	C_{Z_u} = -1.72	C_{m_u} = 0.041	
C_{X_α} = 0.372	C_{Z_α} = -5.82	C_{m_α} = -1.55	
C_{X_q} = -0.532	$C_{Z_q} + C_{Z_{\dot{\alpha}}}$ = -3.82	$C_{m_q} + C_{m_{\dot{\alpha}}}$ = -18.0	

Table 4: Aircraft parameters

8. Of the former laboratory aircraft, a De Havilland DHC-2 'Beaver', the following parameters were derived from flighttest measurements during non-steady symmetric flight, see Table 4.

   (a) Which of the flight parameters vary most during the short period mode, and during the phugoid?

   (b) Write down the symmetrical equations of motion in the components $\hat{u}$, γ, θ and $\frac{q\bar{c}}{V}$ (γ is the flight path angle).
   One of the symmetrical characteristic aircraft modes can accurately be approximated by assuming that V = constant and γ = 0. Which mode is being mentioned here?
   Derive the simplified equations of motion for this characteristic mode and calculate the roots of its characteristic equation.

   (c) Draw the characteristic modes' roots in the complex plain and show how the damping ratio ζ can be described by a line in this complex plain. Show in your figure where the roots of the other characteristic mode are positioned.

   (d) Calculate the period P, the damping ratio ζ, the time to damp to half amplitude $T_{\frac{1}{2}}$ and the eigenfrequency ω_n of the characteristic mode mentioned in question (b).

9. (a) Consider the asymmetrical equations of motion, and derive the equations of motion for steady horizontal turns by neglecting all terms, which contrain time derivatives, by neglecting C_{Y_r}, $C_{Y_{\delta_r}}$, $C_{Y_{\delta_a}}$, $C_{\ell_{\delta_r}}$, and $C_{n_{\delta_a}}$, and by assuming that $\frac{pb}{2V} = 0$.

   (b) Now consider turns during which only the ailerons are being used ($\delta_r = 0$) as well as turns during which only the rudder is being used ($\delta_a = 0$).
   If all derivatives have the required sign and the aircraft possesses spiral stability, determine the sign of the following expressions,

$$\frac{d\delta_a}{d\left(\frac{rb}{2V}\right)} \qquad \frac{d\delta_r}{d\left(\frac{rb}{2V}\right)} \qquad \frac{d\phi}{d\left(\frac{rb}{2V}\right)} \qquad \text{and} \qquad \frac{d\beta}{d\left(\frac{rb}{2V}\right)}$$

   (c) Again answer the previous question, now assuming that the aircraft does *not* possess spiral stability.

10. (a) Derive briefly how the contribution of the horizontal tailplane to the stability derivatives $C_{Z_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$ is established. Express these contributions in terms of the characteristic geometrical and aerodynamical derivatives of the tailplane.

    (b) Derive also briefly how the contribution of the horizontal tailplane to C_{Z_q} and C_{m_q} is established.

    (c) What is the required sign and the order of magnitude of the derivatives $C_{Z_{\dot{\alpha}}}$, $C_{m_{\dot{\alpha}}}$, C_{Z_q} and C_{m_q} for conventional aircraft with a horizontal tailplane?

    (d) Show that the contribution of a straight wing to the derivative C_{Z_q} and C_{m_q} is rather small relative to the contribution of the horizontal tailplane.

    (e) Show that in the equations of motion the derivatives $C_{Z_{\dot{\alpha}}}$ and C_{Z_q} can often be neglected.

11. A so-called hang glider has neither ailerons nor a rudder. Lateral flight control is possible by changing the location of the center of gravity along the lateral axis.

    (a) Derive the equations of motion for steady horizontal turns.

    (b) Derive an expression for $\dfrac{d\left(\frac{\Delta y}{b}\right)}{d\left(\frac{rb}{2V}\right)}$, in which Δy is the change of location of the center of gravity in lateral direction.
    Assuming that $\dfrac{rb}{2V} > 0$, what is the sign of the sideslip angle β?

    (c) What is the required sign of $\dfrac{d\left(\frac{\Delta y}{b}\right)}{d\left(\frac{rb}{2V}\right)}$ and why? Which condition can be derived from this and which requirement should be met?

    (d) In general, hang gliders have either a very small vertical tailplane, or they do not have one at all. Considering the requirement you found in the previous question, explain why this is so.
    Show in a lateral stability diagram in which area the hang gliders should be situated.