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# FLIGHT DYNAMICS AE3302

## EXAMINATION

April 8th, 2010

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**Delft University of Technology**  
**Faculty of Aerospace Engineering**  
**Control and Simulation Division**

This exam contains 6 questions.

You may use the formulae on the given formula sheets.

Some answers need to be filled in on the Results Sheet.

Hand in your derivations and your Results Sheet!

### PLEASE NOTE

Always write down the correct units for each computed parameter value. Be mindful for any required conversion before making any computations. **Always** write down the derivations of your answers. Use the Results Sheet when required.

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**Question 1 (10 %)**

Which of the following statements are true and which are false:

- (a) Newtons laws of motion are only valid when expressed in an inertial reference frame.
- (b) When an aircraft is dynamically unstable it must be statically unstable.
- (c)  $C_{m_q}$  is the most important parameter for static longitudinal stability.
- (d) The aerodynamic center is the point on the mean aerodynamic chord about which the longitudinal aerodynamic moment is zero.
- (e) Positive deflections of elevator, ailerons and rudder result in negative contributions to pitch, roll and yaw moments.
- (f) The contribution of the fuselage to  $C_{n_\beta}$  is destabilizing.
- (g) Most pilots will find an aircraft unpleasant to fly if the elevator control force curve has a negative slope:  $(\frac{dF_e}{dV})_{F_e=0} < 0$
- (h) The linearized equations of motion describe exactly the true motions of an aircraft.
- (i) The stability derivative  $C_{m_{\dot{\alpha}}}$  is used to account for the time delay of the wing induced downwash to hit the horizontal tail.
- (j) Both wing-sweep angle and dihedral affect the stability derivative  $C_{l_\beta}$ .

**Question 2 (10 %)**

Transformation matrices are used often in the field of flight dynamics. The most commonly used transformation in flight dynamics is the 3-2-1 rotation. The 3-2-1 rotation transforms the vehicle carried earth reference frame  $F_O$  to the body-fixed reference frame  $F_b$ . Derive the transformation matrix  $T_{bO}$  for the 3-2-1 rotation such that  $F_b = T_{bO}F_O$ . You do not have to perform the matrix multiplications; write  $T_{bO}$  as a product of 3 matrices.

**Question 3 (20 %):** Static stability of an aircraft with a canard

The force and moment equilibrium for an aircraft with a canard configuration differs significantly from that of an aircraft with a conventional horizontal tail plane.

- (a) In Figure 5 of the Results Sheet, draw the non-dimensional forces and moments necessary for longitudinal equilibrium for an aircraft with a canard configuration and the given center of gravity. Make sure that the forces and moments have the correct direction! Assume that for the canard,  $C_{T_h}$  and  $C_{m_{ac_h}}$  are negligible. For the main wing, assume that the contribution of  $C_{T_w}$  to the aerodynamic moment is negligible.
- (b) Formulate the equilibrium equation for the pitching moment coefficient  $C_m$ . Note that for a canard configuration it is safe to assume that  $\left(\frac{V_h}{V}\right)^2 = 1$ , while the factor  $\frac{S_h}{S}$  must not be neglected.
- (c) Derive from the moment equation the expression for the static stability  $C_{m_\alpha}$ .
- (d) Derive an expression for the x-coordinate of the neutral point stick-fixed  $x_{n_{fix}}$ .
- (e) The situation in Figure 5 is not very efficient nor statically stable. Suggest a configuration change in Figure 5 resulting in a more efficient, and stable design. Mark the changed forces, moments, lengths and points with an asterisk (\*).



**Figure 1:** Northrop B-2 Spirit

**Question 4 (20 %)**

A flying wing is an aircraft without a tailplane, see Figure 1. As a result, the characteristic modes of the flying wing differ significantly from those of a conventional aircraft. In this question the dynamics of the short period mode of a flying wing will be examined.

- From the full linearized longitudinal equations of motion, derive a simplified form describing the short period motion. Assume that  $\hat{u} = 0$  and that  $\theta = 0$ . Additionally, the stability derivatives  $C_{m_{\dot{\alpha}}}$ ,  $C_{Z_{\dot{\alpha}}}$ ,  $C_{m_q}$  and  $C_{Z_q}$  are assumed to be zero.
- Derive the characteristic equation of the simplified short period motion and determine the non-dimensional eigenvalues. Does the flying wing have a stable short period motion? Use the numerical data from Table 1.
- In order to improve the flying qualities of the flying wing, a simple pitch-rate feedback controller is integrated with the flight control system. This feedback controller has the following definition:  $\delta_e = k_q \frac{q}{V}$ . Derive the new characteristic equation for the simplified short period motion from part (b).
- For what range of the feedback gain  $k_q$  is the short period both stable and periodic? Use the numerical data from Table 1.

$V = 180[m s^{-1}]$	$\mu_c = 205$	$K_Y^2 = 0.98$	$C_{X_0} = 0$	$C_{X_u} = -0.22$	$C_{X_\alpha} = 0.465$
$C_{Z_0} = -1.14$	$C_{Z_u} = -2.72$	$C_{Z_\alpha} = 0.5$	$C_{Z_q} = 0$	$C_{Z_{\dot{\alpha}}} = 0$	$C_{Z_{\delta_e}} = -0.44$
$C_{m_\alpha} = -0.45$	$C_{m_u} = 0$	$C_{m_{\dot{\alpha}}} = 0$	$C_{m_{\delta_e}} = -1.28$	$C_{m_q} = 0$	

**Table 1:** Aircraft data for question 4

**Question 5 (20 %) : Flighth test data analysis**

During the flight test phase of a new aircraft, the characteristic modes of an aircraft are investigated. In this case the pilots excite the three **lateral** characteristic modes. The results from the flight tests are shown in Figure 2, Figure 3 and Figure 4

- (a) Based on the results from Figure 2, Figure 3 and Figure 4, calculate the nondimensional eigenvalues  $\lambda_{b_1}, \lambda_{b_2}, \lambda_{b_3}, \lambda_{b_4}$  corresponding with the characteristic motions. Use the numerical data from Table 2. Plot your results on the Results Sheet.

**Hint:** Remember that for the lateral case, nondimensional time is given by  $s_b = \frac{b}{V} \cdot t$ . Additionally, the period of the oscillation is given by  $P = \frac{2\pi}{Im(\lambda_b)} \frac{b}{V}$ .

- (b) Based on the results you obtained in part (a), calculate the undamped ( $\omega_0$ ) and damped ( $\omega_d$ ) natural frequencies, as well as the damping ratio ( $\zeta$ ) corresponding with each eigenvalue. Fill in your results on the Result Summary Sheet.

**Hint:** The damping ratio ( $\zeta$ ) is given by the equation:

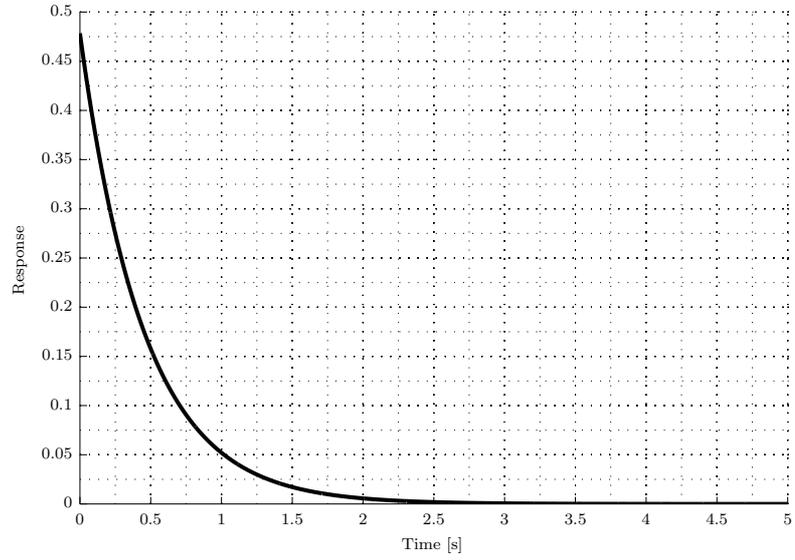
$$\zeta = \frac{-Re(\lambda_b)}{\sqrt{(Re^2(\lambda_b) + Im^2(\lambda_b))}}$$

- (c) Given the eigenvalues calculated in part (a), qualitatively describe the behavior of the aircraft over time after the pilot gives a positive pulse input on the ailerons. In your answer include the sideslip angle  $\beta$  and the roll angle  $\phi$ .
- (d) The lateral stability derivatives  $C_{l_r}$  and  $C_{n_r}$  are important for the lateral stability of an aircraft. Explain why we have for the signs of these derivatives:
- $C_{l_r} > 0$  and
  - $C_{n_r} < 0$
- (e) The design team is not happy with the flying qualities of the aircraft, and after some research finds that the parameter  $E$  has an incorrect sign:  $E < 0$  with  $E$  given by:

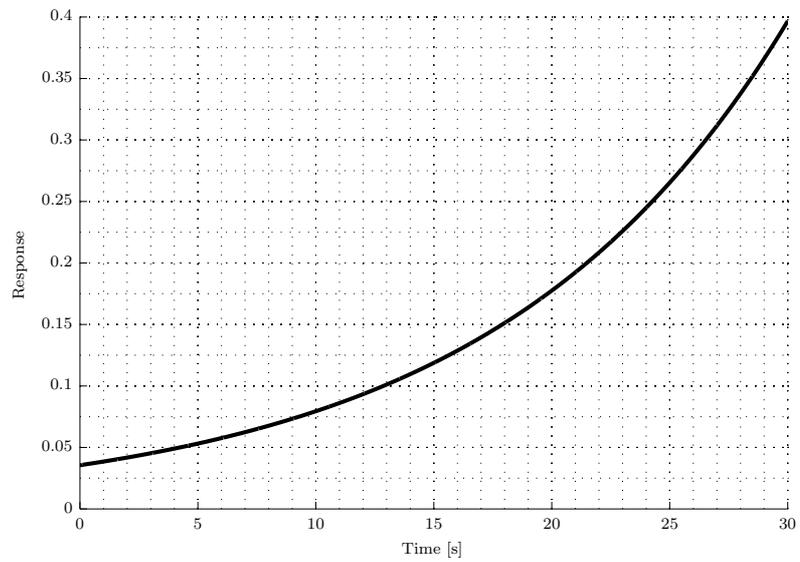
$$E = C_L (C_{l_\beta} C_{n_r} - C_{n_\beta} C_{l_r}) \quad (1)$$

Name 2 structural changes that can be made to the aircraft to improve the flying qualities by changing  $E$  such that  $E > 0$ ? Which of the parameters in Equation 1 are affected by the structural changes?

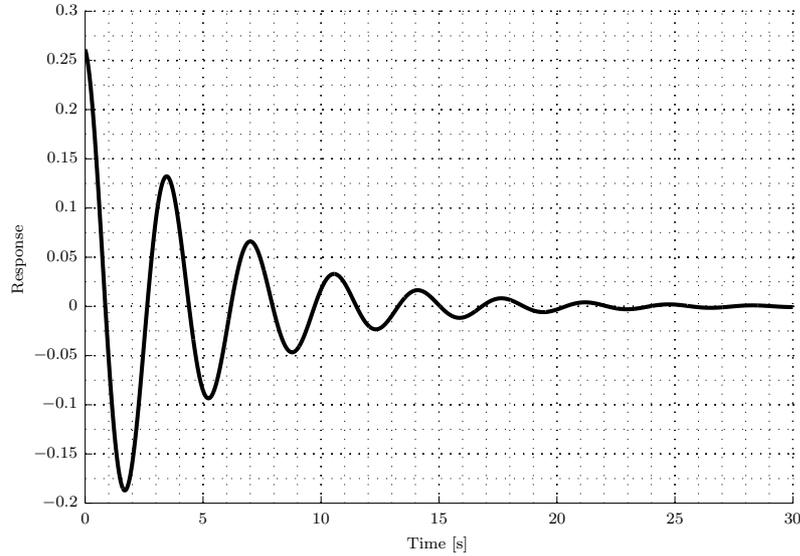
- (f) **Bonus Question:** While the natural frequencies and damping ratios can be calculated analytically, they can also be derived geometrically from the plot of the eigenvalues in the complex plane. On the Results Sheet, clearly show for 1 eigenvalue how the undamped ( $\omega_0$ ) and damped ( $\omega_d$ ) natural frequencies and the damping ratio  $\zeta$  can be derived geometrically.



**Figure 2:** First lateral eigenmode



**Figure 3:** Second lateral eigenmode



**Figure 4:** Third lateral eigenmode

$V = 125[m s^{-1}]$	$b = 13.36[m]$	$\mu_b = 15.5$	$K_X^2 = 0.012$	$K_{XZ} = 0.002$
$K_Z^2 = 0.037$	$C_L = 1.136$	$C_{Y_\beta} = -0.9896$	$C_{Y_p} = -0.087$	$C_{Y_r} = 0.43$
$C_{Y_\beta} = 0$	$C_{l_\beta} = -0.0772$	$C_{l_p} = -0.3444$	$C_{l_r} = 0.28$	$C_{n_\beta} = 0.1638$
$C_{n_p} = -0.0108$	$C_{n_r} = -0.1930$	$C_{n_\beta} = -0.15$		

**Table 2:** Aircraft data for question 5 and 6

**Question 6 (20 %)**

The linearized equations of motion can be significantly simplified without losing too much of their accuracy for predicting the characteristic modes. In this Question, the linearized asymmetric equations of motion will be simplified for the Dutch roll motion.

- Starting with the full, linearized asymmetric equations of motion, derive the simplified set of equations describing the Dutch roll motion assuming that the roll angle  $\phi$  and roll rate  $p$  can be discarded. Also assume that  $C_{Y_\beta}$  can be neglected and that  $C_{Y_r}$  is insignificant relative to  $4\mu_b$ . Note that in this case  $C_{n_\beta}$  will **not** be neglected.
- Using the numerical data in Table 2, calculate the eigenvalues corresponding with the simplified set of equations of motion from part (a).
- Calculate the range of values of  $C_{n_\beta}$  for which the Dutch roll mode becomes non-periodic.
- Is the range of values for  $C_{n_\beta}$  you found in part (c) beneficial for **static** lateral stability? Explain your answer.

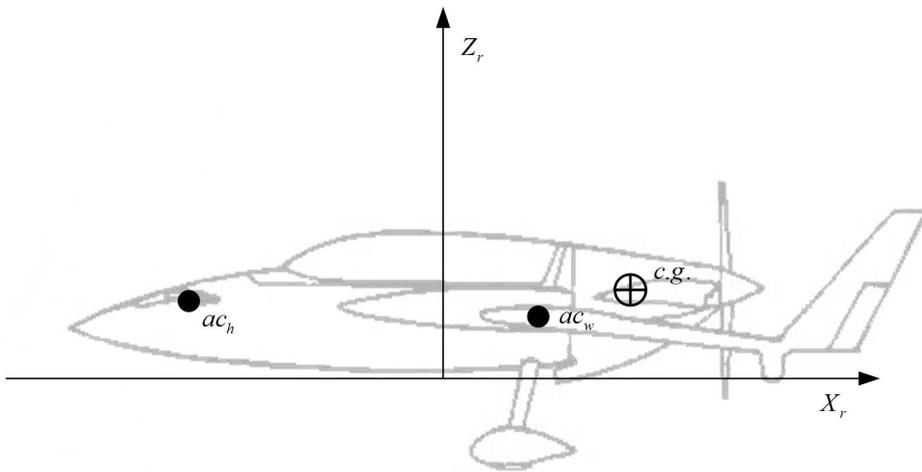


**Results Sheet AE3302 - April 8th, 2010**

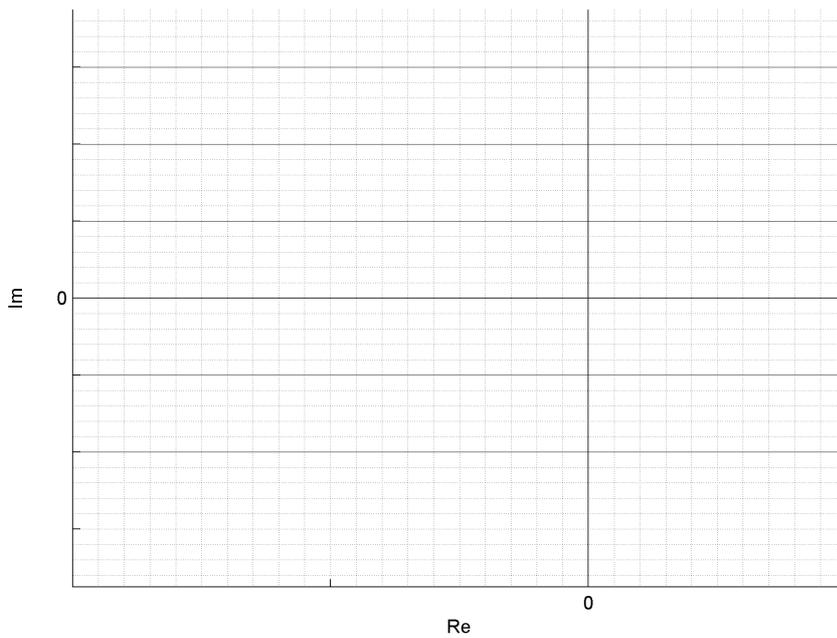
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**Figure 5:** Answer to Question 3a and 3e



**Figure 6:** Answer to Question 5a and 5f