Aerodynamic Forces & Moments

The aerodynamic forces and moments on the body are due to only 2 basic sources:

1. Pressure distribution over the body surface
2. Shear stress distribution over the body surface

The figure below illustrates the pressure & shear distribution on an aerodynamic surface.

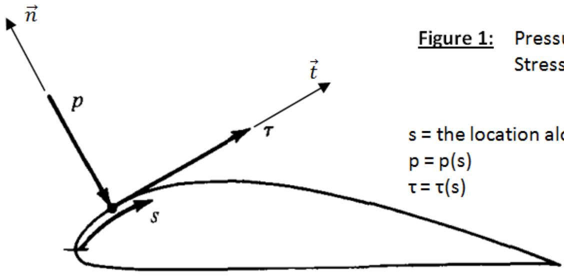

Figure 1: Pressure and Shear Stress on a surface

s = the location along the surface
p = p(s)
τ = τ(s)

Shear stress is due to the tugging action on the surface, which is caused by friction between the body & the air.

The net effects of p & τ distributions integrated over the complete body surface have a resultant aerodynamic force $\vec{R}$ & moment $\vec{M}$ over the body as shown in figure 2.

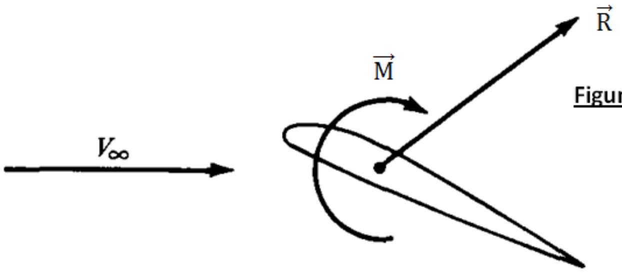

Figure 2: Resultant Aerodynamic Force & Moment on the Body

V_∞ = the free stream velocity

1. Analytical Expressions

There are 3 axes-systems:

- Ground
- Body
- Aerodynamic

Therefore the resultant aerodynamic force $\vec{R}$ could be split into components in the body or aerodynamic axes systems, as shown in figure 3.

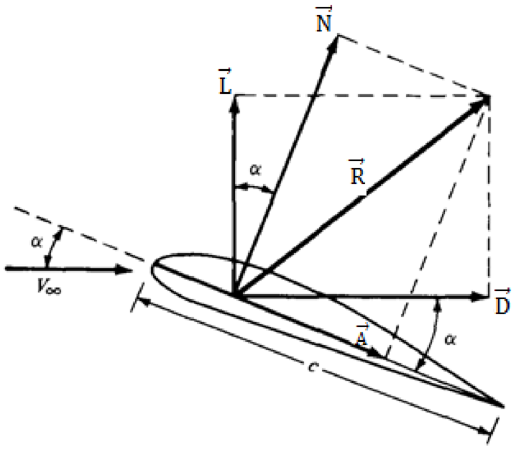

Figure 3: Resultant Aerodynamic Force

Where:
$\vec{N}$ & $\vec{A}$ are defined by body axis system
$\vec{L}$ & $\vec{D}$ are defined by aerodynamic axis system

$\vec{N}$ = normal force
$\vec{A}$ = axial force
$\vec{L}$ = lift force
$\vec{D}$ = drag force

The geometrical relations between these 2 sets of components are:

$$L = N\cos(\alpha) - A\sin(\alpha)$$
$$D = N\sin(\alpha) + A\cos(\alpha)$$

2. Calculations of Forces & Moments

Consider the 2-dimensional body defined by figure 4

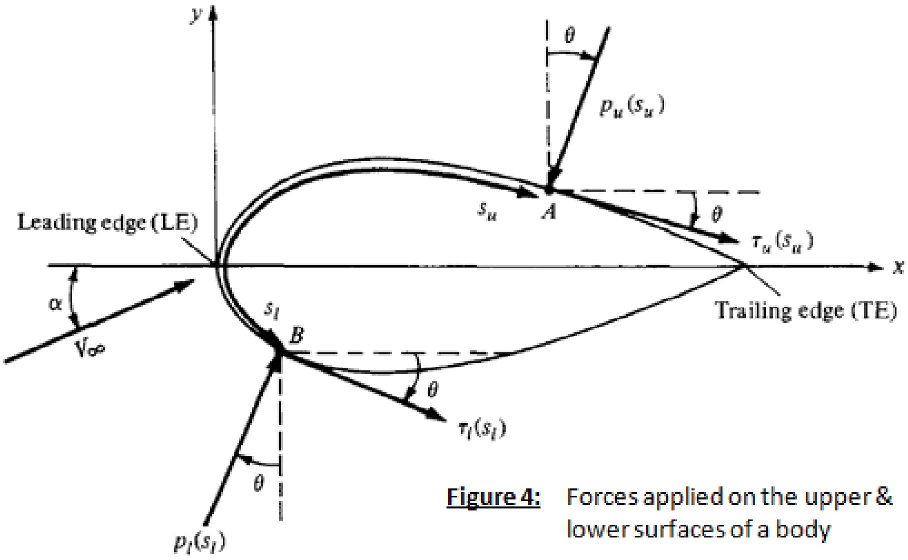

Figure 4: Forces applied on the upper & lower surfaces of a body

Now consider the 2D shape in figure 4 as a cross-section of an infinitely long cylinder of uniform section (***a unit span***) such a cylinder is shown in figure 5. Consider an elemental surface area (ds) of this cylinder, where ds = (ds) x 1

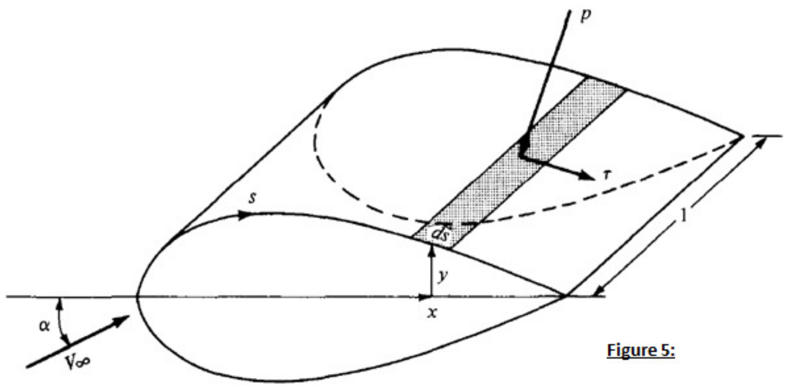

Figure 5:

The elemental normal force dN and axial force dA acting on the elemental surface ds on the upper body surface are:

$$(1) \Rightarrow \boxed{dN_u = -p_u ds_u \cos(\theta) - \tau_u ds_u \sin(\theta)}$$
$$(2) \Rightarrow \boxed{dA_u = -p_u ds_u \sin(\theta) + \tau_u ds_u \cos(\theta)}$$

In the same way on the lower surface we have:

$$(3) \Rightarrow \boxed{dN_l = p_l ds_l \cos(\theta) - \tau_l ds_l \sin(\theta)}$$
$$(4) \Rightarrow \boxed{dA_l = p_l ds_l \sin(\theta) + \tau_l ds_l \cos(\theta)}$$

To determine the normal force N & the axial force N, equations (1) to (4) must be integrated from the leading edge (LE) to the trailing edge (TE)

$$N = \int (dN + dA) \quad \text{per unit area}$$

Where:

$$dN = dN_u + dN_l$$
$$dA = dA_u + dA_l$$

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Therefore:

$$(5) \Rightarrow \boxed{N = -\int_{LE}^{TE} (p_u \cos(\theta) + \tau_u \sin(\theta))ds_u + \int_{LE}^{TE} (p_l \cos(\theta) + \tau_l \sin(\theta))ds_l}$$

&

$$(6) \Rightarrow \boxed{A = \int_{LE}^{TE} (-p_u \sin(\theta) + \tau_u \cos(\theta))ds_u + \int_{LE}^{TE} (p_l \sin(\theta) + \tau_l \cos(\theta))ds_l}$$

Because of the relations:

$$L = N\cos(\alpha) - A\sin(\alpha)$$
$$D = N\sin(\alpha) + A\cos(\alpha)$$

The lift & drag can be calculated using (5) & (6)

Aerodynamic Moment Exerted on the Body

It depends on the point about which moments are taken.

Sign convention for aerodynamic moments.

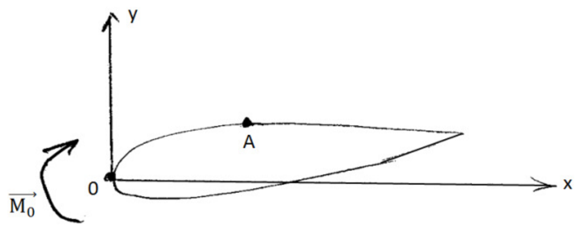
Consider the aerodynamic moment calculated about the leading edge (LE). The moment per unit span about the leading edge due to p & τ (figures 4 & 5) on the elemental area (ds) on the upper & lower surfaces are:

$$(7) \Rightarrow \boxed{dM_u = [p_u \cos(\theta) + \tau_u \sin(\theta)]x.ds_u + [-p_u \sin(\theta) + \tau_u \cos(\theta)]y.ds_u}$$

&

$$(8) \Rightarrow \boxed{dM_l = [-p_l \cos(\theta) + \tau_l \sin(\theta)]x.ds_l + [p_l \sin(\theta) + \tau_l \cos(\theta)]y.ds_l}$$

Note:



$$\overrightarrow{M_0} = \int \left(d\vec{N} + d\vec{A}\right) \Lambda\, \overrightarrow{0A}$$

$$(9) \Rightarrow \boxed{M_{LE} = \int_{LE}^{TE} \left[dM_{u,LE} + dM_{l,LE}\right]}$$

To define the dynamic pressure that arises when the fluid is in motion:

$$q_\infty = \frac{1}{2}\rho_\infty {v_\infty}^2 \quad \text{(freestream conditions)}$$

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Lift Coefficient: $C_L = \frac{l}{q_\infty s}$

Drag Coefficient: $C_D = \frac{D}{q_\infty s}$

Normal Coefficient: $C_N = \frac{N}{q_\infty s}$

Axial Coefficient: $C_A = \frac{A}{q_\infty s}$

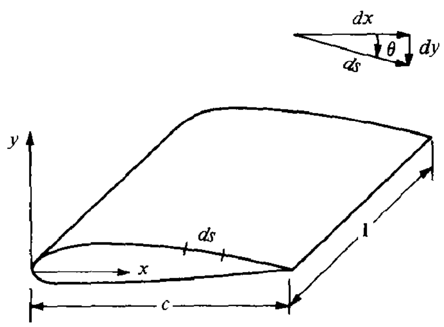
Moment Coefficient: $C_M = \frac{M}{q_\infty s l}$

s = a reference area
l = a reference length

Let's consider 2 additional non-dimensional coefficients:

Pressure Coefficient: $C_p = \frac{P-P_\infty}{q_\infty}$

Skin Friction Coefficient: $C_F = \frac{\tau}{q_\infty}$



Using the above geometry we can write:

$$dx = ds.\cos(\theta) \quad \& \quad dy = -ds.\sin(\theta)$$
$$\& \quad S = c(l)$$

$$\boxed{C_N = \frac{1}{c}\left[\int_0^c (C_{p,l} - C_{p,u})dx + \int_{LE}^{TE} (C_{F,u} - C_{F,l})dy\right]}$$

$$\boxed{C_A = \frac{1}{c}\left[\int_{LE}^{TE} (C_{p,u} - C_{p,l})dy + \int_0^c (C_{F,u} - C_{F,l})dx\right]}$$

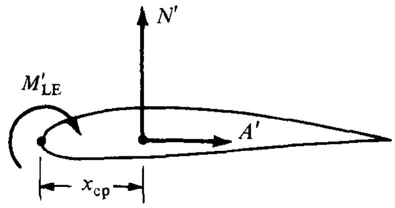
$$\boxed{C_{M,LE} = \frac{1}{c^2}\left[\begin{array}{l} \int_0^c (C_{p,u} - C_{p,l})x.dx - \int_{LE}^{TE} (C_{F,u} - C_{F,l})x.dy \\ + \int_{LE}^{TE} (C_{p,u} - C_{p,l})y.dy + \int_0^c (C_{F,u} - C_{F,l})y.dx \end{array}\right]}$$

3. Centre of Pressure

The centre of pressure is the location where the resultant of a distributed load effectively acts on the body. If the moments were taken about the centre of pressure, the integrated effect of the distributed loads would be zero.

⇨ An alternate definition for the centre of pressure is the point on the body at which the aerodynamic moment is zero.

$$\boxed{x_{cp} = \frac{-M_{LE}}{N}}$$



If α is small then:

$$\sin(\alpha) = 0 \quad \& \quad \cos(\alpha) = 1$$

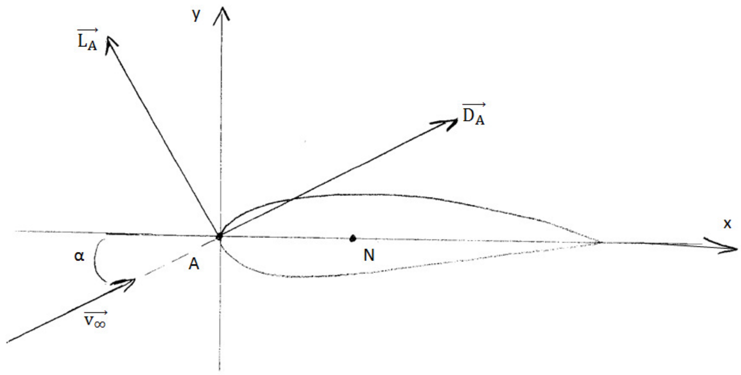
In this case, L = N, therefore:

$$x_{cp} = \frac{-M_{LE}}{L}$$

So as N & L decrease, x_{cp} increases
As the force approaches 0, $x_{cp} \to \infty$

Note: the centre of pressure is not always a convenient concept in aerodynamics

4. Change of Centre for an Airfoil



$$L_A = q_\infty . l . C_{L,A}$$
$$D_A = q_\infty . l . C_{D,A}$$

$$M_N = M_A + \overrightarrow{R_A} \wedge \overrightarrow{AN}$$

With: $\overrightarrow{R_A} = \overrightarrow{L_A} + \overrightarrow{D_A}$

$$M_N = M_A - L_A x_N \cos(\alpha) - D_A x_N \sin(\alpha)$$
$$\therefore M_N = M_A - x_N[L_A \cos(\alpha) + D_A \sin(\alpha)]$$

Dividing by $q_\infty l^2$

$$\frac{M_N}{q_\infty l^2} = \frac{M_A}{q_\infty l^2} - \frac{x_N}{l}\left[\frac{L_A}{q_\infty l}\cos(\alpha) + \frac{D_A}{q_\infty l}\sin(\alpha)\right]$$

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$$\boxed{C_{M,N} = C_{M,A} - \frac{x_N}{l}\left[C_{L,A}\cos(\alpha) + C_{D,A}\sin(\alpha)\right]}$$

When $\alpha \to 0$:

$$\sin(\alpha) = 0 \quad \& \quad \cos(\alpha) = 1$$

Therefore:

$$\boxed{C_{M,N} = C_{M,A} - \frac{x_N}{l}C_{L,A}}$$

$C_{M,0} = C_{M,A} \quad \text{when} \quad C_{L,A} = 0$

Therefore:

$$\boxed{C_{M,A} = C_{M,0} + \frac{\partial C_{M,A}}{\partial C_{L,A}} \times C_{L,A}}$$

$$\boxed{C_{M,N} = C_{M,0} + C_{L,A}\left(\frac{\partial C_{M,A}}{\partial C_{L,A}} - \frac{x_N}{l}\right)}$$

Consider a point F along the chord where

$$C_{M,F} = C_{M,0} \qquad \forall\, C_{L,A}$$

$$\therefore \frac{\partial C_{M,A}}{\partial C_{L,A}} - \frac{x_F}{l} = 0$$

$$\therefore \frac{x_F}{l} = \frac{\partial C_{M,A}}{\partial C_{L,A}}$$

F is called the aerodynamic centre and does not depend on the incidence.