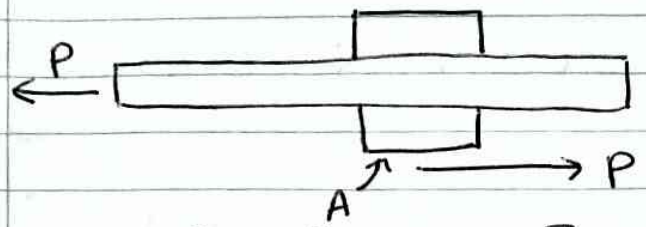
Introduction to Aerospace Engineering II Exam (01/30/2015)

Part A:

Material	Aluminium 2024-T3	Medium carbon steel
σ_{ult} [MPa]	483	800
σ_y [MPa]	345	640
τ_{ult} [MPa]	283	470

A1 FBD:



$P = 60\,kN = 60 \cdot 10^3\,N$

$\tau_{ult} = 470\,MPa = 470 \cdot 10^6\,Pa$

$\tau = \frac{F}{A} \rightarrow A = \frac{F}{\tau_{ult}}$

$$A = \frac{60 \cdot 10^3}{470 \cdot 10^6} = \frac{3}{23500} = \frac{\pi}{4} d^2$$

Thus,

$$d = \sqrt{\frac{3 \cdot 4}{23500\pi}} = 0.0127\,m = 12.7\,mm$$

A2 The size of the bold limits the effective cross-sectional area.

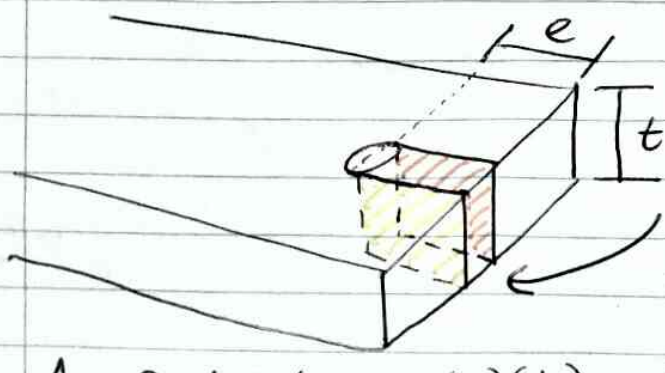
$$\sigma_y = \frac{F}{A} = \frac{P}{A} \rightarrow A_{ef.} = \frac{P}{\sigma_y} = \frac{60 \cdot 10^3}{345 \cdot 10^6} = \frac{1}{5750}\,m^2$$

$A_{ef} = A_T - A_{bolt} = W_T t - W_{bolt} t$ $\quad W_T = 60\,mm,\ t = 4mm$

$$\frac{1}{5750} = 60 \cdot 10^{-3} \cdot 4 \cdot 10^{-3} - W_{bolt} \cdot 4 \cdot 10^{-3}$$

$$W_{bolt} = d = 0.0165\,m = 16.5\,mm$$

A3



$\tau = \frac{F}{A}$

tear-out occurs, thus two areas are required.

$A = 2 \cdot W \cdot t = 2 \cdot (e)(4) = 8e\,mm^2$ (e is in mm)

$$\tau = \frac{F}{8e \cdot 10^{-6}} \rightarrow e = \frac{F}{\tau \cdot 8 \cdot 10^{-6}} = \frac{60 \cdot 10^3}{283 \cdot 10^6 \cdot 8 \cdot 10^{-6}} = 26.5\,mm$$

A4 $A = W \cdot t = \frac{F}{\sigma_y} \rightarrow t = \frac{F}{\sigma_y \cdot W}$

$t = \frac{60 \cdot 10^3}{345 \cdot 10^6 \cdot 60 \cdot 10^{-3}} = 2.9 \cdot 10^{-3}\ m = 2.9\ mm$

A5 See answers on BrightSpace.

A6 $E_{carbon} = 220\ GPa$ with $\sigma_{uts} = 760\ MPa$

$E_{resin} = 5\ GPa$ with $\sigma_{uts} = 40 MPa$

$\varepsilon = \frac{\sigma}{E} = \frac{760 \cdot 10^6}{220 \cdot 10^9} = 0.0034545 \approx 0.35\ \%$

A7 $f = 0.57$

$E = 0.57 \cdot 220 + 0.43 \cdot 5 = 127.55 \approx 128\ GPa$

A8 5GPa, since the resin is isotropic but the fibres work in a perpendicular direction.

A9 $0.57 \cdot 1700 + 0.43 \cdot 1200 = 1485\ kg/m^3$

Part B:

$m = 62500\ kg$

$a = 1.9 \cdot g$ (landing)

$g = 9.81\ m/s^2$

B1 $F = ma$

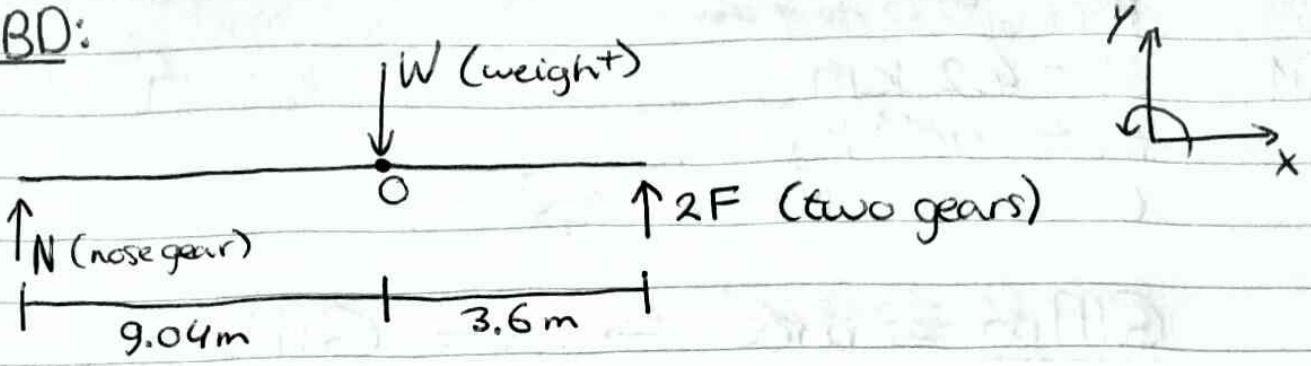
$F_{total} = 62500 \cdot 1.9 \cdot 9.81 = 1164937\ N$

$F_{1\ gear} = \frac{F_{total}}{2} = 583\ N$

B2 $M = 430 \cdot 10^3 \cdot \underline{(3.795 - 1.8)} = 8.58 \cdot 10^5\ Nm$

See figure 2 on the exam.

B3 FBD:



$+\uparrow \Sigma F_y = 0;$

$N + 2F - W = 0 \rightarrow N = W - 2F$

$\circlearrowleft \Sigma M_o = 0;$

$2F \cdot 3.6 - (W - 2F) \cdot 9.04 = 0$

$7.2F - 9.04W + 18.08F = 0$

$25.28F = 9.04 \cdot 62500 \cdot 9.81$

$$F = \frac{9.04 \cdot 62500 \cdot 9.81}{25.28} = 219250.3956\ N$$

$N = W - 2F = 62500 \cdot 9.81 - 2 \cdot 219250.3956 = 174625\ N$

$\approx 175\ kN$

B4 The spar webs are loaded in shear.

B5 The wing rib at location B should be forged and subsequently machined.

B6 $\sqrt[3]{E}/\rho$

B7 B & D

B8 $q = \tau t = \left[\frac{N}{m^2}\right] \cdot [m] = \left[\frac{N}{m}\right]$

B9 $k = \frac{F}{\delta} = \left[\frac{N}{m}\right]$

Part C:

See answers on BrightSpace

D1

Part D:

$d = 4.2\ km$

$m = 1 \cdot 10^{13}\ kg$

$G = 6.673 \cdot 10^{-11}\ m^3/kgs^2$

$$\frac{GMm}{r^2} = ma \rightarrow a = \frac{GM}{r^2}$$

$$a = \frac{6.673 \cdot 10^{-11} \cdot 1 \cdot 10^{13}}{\left(\frac{4.2 \cdot 10^3}{2}\right)^2} = 1.51 \cdot 10^{-4}\ m/s^2 = 1.51 \cdot 10^{-1}\ mm/s^2 = 0.151\ mm/s^2$$

D2

$m_b = 12000\ kg$ (begin/initial mass)

$m_e = 4000\ kg$ (burn-out/end mass)

$t_b = 60s$

$T = 250\ kN$

$$\dot{m} = \frac{m_{propellant}}{t_b} = \frac{12000 - 4000}{60} = 129.41\ kg/s$$

$$T = \dot{m} V_e \longrightarrow V_e = \frac{T}{\dot{m}} = \frac{250 \cdot 10^3}{129.41} = 1931.818\ m/s$$

Tsiolkovsky equation:

$$V_{end} = V_e \cdot \ln\left(\frac{m_b}{m_e}\right) = 1931.060 \cdot \ln\left(\frac{12000}{4000}\right) = 2246.9959\ m/s \approx 2.25\ km/s$$

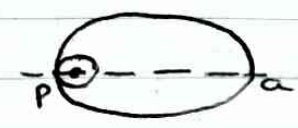
D3

Mars has a smaller mass than ~~the~~ Earth.

D4

$h_a = 5000\ km$

$h_p = 0\ km$



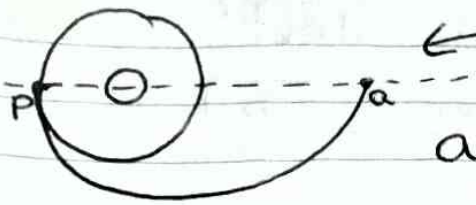
$2a = 2 \cdot Re + h_p + h_a = 2 \cdot 6378.136 + 0 + 5000 = 18556.272\ km$

$a = 9278.136\ km$

$$V_p^2 = \mu\left(\frac{2}{r_p} - \frac{1}{a}\right) = 398600.441 \cdot \left(\frac{2}{6378.136} - \frac{1}{9278.136}\right) = 82.02837\ km^2/s^2$$

$V_p = 9.057\ km/s$

D5



← Hohmann transfer

$$a = \frac{2R_e + 200 + 35800}{2} = 24378.136\ km/s$$

$$V_{200} = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{398600.441}{200+6378.136}} = 7.784\ km/s \quad \text{(circular orbit)}$$

$$V_p = \sqrt{\mu\left(\frac{2}{r_p} - \frac{1}{a}\right)} \quad \text{(derived from vis viva)}$$

$$= \sqrt{398600.441\left(\frac{2}{200+6378.136} - \frac{1}{24378.136}\right)} = 10.24\ km/s$$

(perigee velocity)

So the first part of the ΔV budget is:

$$\Delta V_p = |10.24 - 7.78| = 2.45\ km/s$$

$$V_{35800} = \sqrt{\frac{\mu}{r_2}} = 3.074\ km/s \quad \text{(circular orbit)}$$

$$V_a = \sqrt{\mu\left(\frac{2}{r_a} - \frac{1}{a}\right)} = 1.5968\ km/s$$

$$\Delta V_a = |3.074 - 1.5968| = 1.477\ km/s$$

$$\Delta V = \Delta V_p + \Delta V_a = 2.45 + 1.477 = 3.927 \approx 3.93\ km/s$$

D6 $m_{dry} = 1000\,kg$ } $m_{wet} = 5000\,kg$
$m_{prop} = 4000\,kg$
$I_{sp} = 300\,s$
$\dot{m} = 50\,kg/s$

$$w = I_{sp} \cdot g_0 \longrightarrow T = \dot{m} w = \dot{m} \cdot I_{sp} \cdot g_0 = 14709.75\,N$$

Formula sheet gives:

$$ma = T - mg - C_D \cdot \tfrac{1}{2}\rho v^2 S \quad (=0)$$

$$a = \frac{T}{m_{dry}} - g = \frac{14709.75}{1000} - 9.81 = 137.3\ m/s^2$$

$$a = \frac{T}{m_{dry}} = 147.1\ m/s^2$$

D7

$d = 380\,000$ km
$b = 1000$ bit/s
$f = 2.6$ GHz
$P_{trans} = 20$ W

Formula sheet:

$$D = \frac{6000\,d}{f}\sqrt{\frac{b}{P}}$$

$$D = \frac{6000 \cdot 380000}{2.6 \cdot 10^9} \cdot \sqrt{\frac{1000}{20}} = 6.2 \text{ m}$$

$$P = \frac{b \cdot (6000\,d)^2}{(Df)^2} = \frac{1000 \cdot (6000 \cdot 380000)^2}{(2 \cdot 2.6 \cdot 10^9)^2} = 192.2 \text{ W}$$

D8

$DC = 0.01\,A^2$
$OC = 15 + \frac{25}{A}$
$\Big\}$ $T = DC + OC = 0.01A^2 + \frac{25}{A} + 15$

$$\frac{dT}{dA} = 0.02A - \frac{25}{A^2} = 0$$

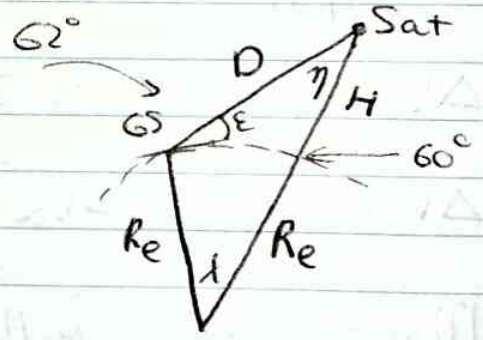
$$0.02\,A^3 - 25 = 0$$

$$A^3 = 1250 \rightarrow A = 10.77\%$$

$$T = 18.48$$

D9

$h = 800$ km
$i = 60°$
$GS = 62°$
$\lambda = 2°$ (see figure)



$Re + H = 7178.136$ km

cosine rule:

$$D^2 = (Re+H)^2 + Re^2 - 2 \cdot (Re+H)(Re)\cos(\lambda)$$
$$= 695779.688 \text{ km}^2$$
$$D = 834.134 \text{ km}$$

sine rule:

$$\frac{D}{\sin(\lambda)} = \frac{Re+H}{\sin(90° + \varepsilon)}$$

$$23901.347 = \frac{7178.136}{\sin(90° + \varepsilon)}$$

$$90° + \varepsilon = 17.477° \rightarrow \varepsilon = -72.52°$$

Sign correction: $\varepsilon = 72.52°$

D_{10} will increase with time