Summary AE1102: Materials & Structures

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Based on Reader Materials & Structures (v. 21/12/2010) – R. Alderliesten

Chapter 1: Material Physics & Properties

Similitude principle … says that when comparing material properties, dimensional aspects should be left out of the equation.

Different properties are based on that principle. ***Strain*** (extension normalized by length):

$$\epsilon = \frac{\Delta L}{L},$$

and ***stress*** (load normalized by area):

$$\sigma = \frac{P}{A}.$$

When the actual A is used, σ is the *true stress*. When the initial A is used, σ is the *engineering stress* (which is used most often).

Stress and strain are related to each other by ***normal stress***:

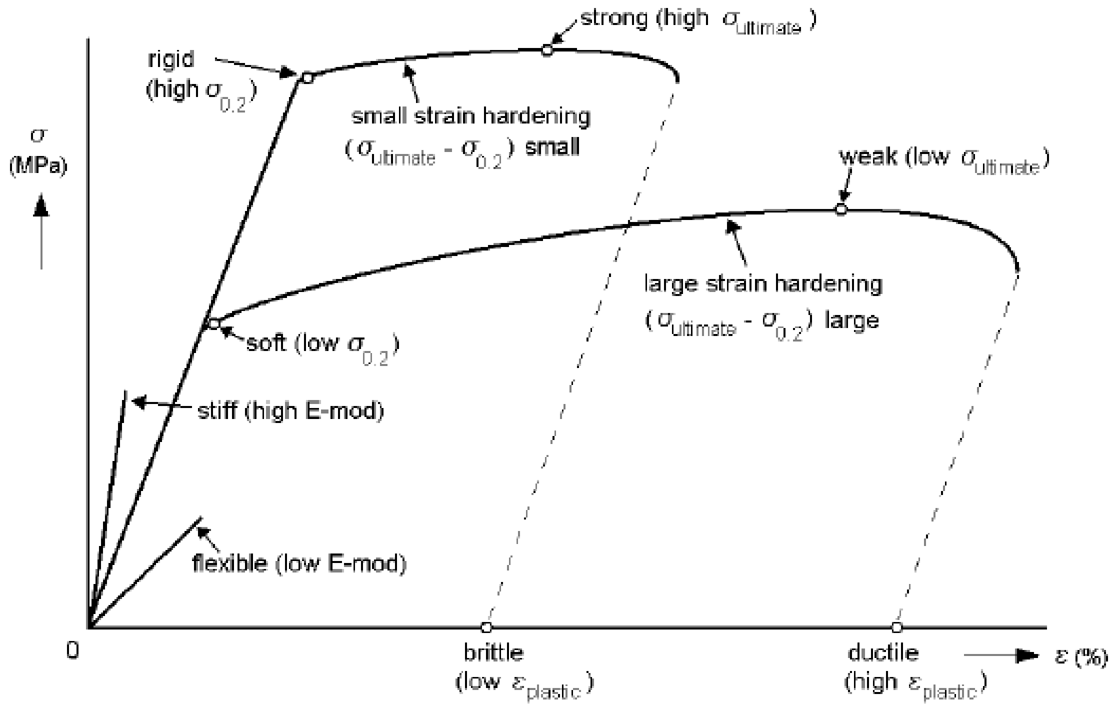
$$\sigma = E \cdot \epsilon,$$

in which the *Youngs's modulus* (or *modulus of elasticity*), a material constant (the higher, the stiffer the material), serves as a sort of 'spring constant'.

Yield point Point in the stress-strain curve, where the deformation transforms from *elastic* to *plastic*.

Yield stress Stress at yield point, + 0,2% (denoted by $\sigma_{0,2}$).

Perfect plastic Hypothetical case in which the material becomes fully plastic after yielding.



The relation between *lateral* and *transverse strain* is given by:

$$\epsilon_t = v \cdot \epsilon_l = v \cdot \frac{\sigma_l}{E},$$

where v is the Poisson's ratio (also a material constant).

Just as ordinary stress, ***shear stress*** (which occurs if the material is in strain or in torsion) is given by a force divided by an area:

$$\tau = \frac{F}{A}$$

Shear stress (τ) and ***strain*** (γ) are related by:

$$\tau = G \cdot \gamma,$$

where G is equivalent to E in earlier formulas, and represents the *shear modulus of elasticity*.

When loads are applied in two directions, stresses can be superimposed. That gives ***Hooke's Law*** (for a sheet in bi-axial stress):

$$\epsilon_x = \frac{\sigma_x}{E} - v\frac{\sigma_y}{E};\ \epsilon_y = -v\frac{\sigma_x}{E} + \frac{\sigma_y}{E}$$

If a sheet is loaded while being clamped ($\sigma_x = 0$), the *apparent Youngs's modulus* has to be used:

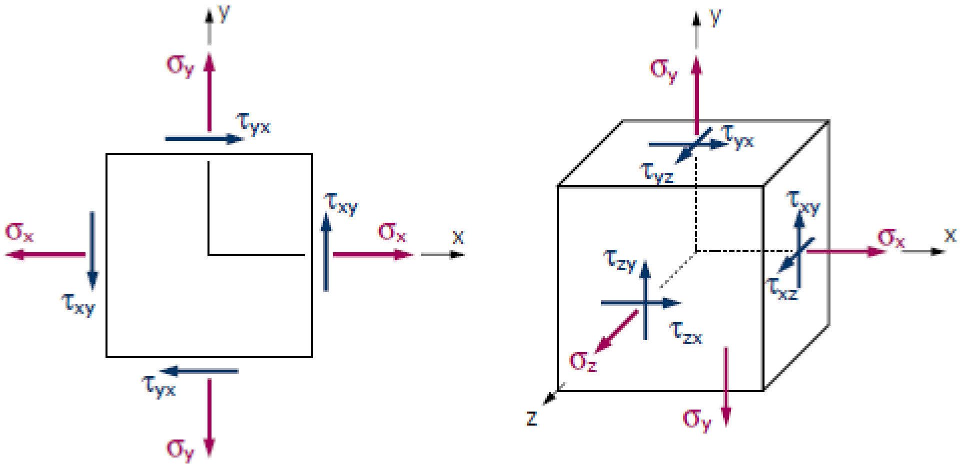
$$E^* = \frac{1}{1-v^2}E$$

Hooke's Law and the equations for *shear stress* and *strain* can be combined into matrix formulation:

$$\begin{bmatrix}\epsilon_x \\ \epsilon_y \\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}\frac{1}{E_x} & -\frac{v_{yx}}{E_y} & 0 \\ -\frac{v_{xy}}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}}\end{bmatrix}\begin{bmatrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{bmatrix}$$

For isotropic deformation, the subscripts in the main matrix (3 x 3) may be neglected. For anisotropic deformation, that is not the case. The illustrations on page 12 (PDF: 16) show the difference.

Note on subscripts: For the shear stress τ_{xy} and shear strain γ_{xy} the first subscript indicates the axis perpendicular to the face that the shear stress and strain are acting on, while the second subscript indicates the positive direction of the shear stress and strain, see figure below.



When performing calculations on composites, keep in mind that the strength and stiffness in the composite (in line with the fibres) is given by the strength and stiffness of the fibres. However, perpendicular to the fibres, the strength and stiffness of the composite is given by the strength and stiffness of the matrix.

Toughness … represents the resistance of the material against fracture, formation of damage or impact.

Toughness is given by the area under the stress-strain curve. Mind the difference between *toughness* and *fracture toughness* (which represents the resistance of a material against fracture in the presence of a crack)!

Chapter 2: Environment & Durability

Obviously, temperature has an effect on material properties. In general, the effect of increasing temperature is that most mechanical and fatigue properties of engineering materials decrease. ***Creep*** is also a phenomenon related to higher temperatures, where materials under constant stress steadily (and slowly) deforms.

The thermal expansion coefficient α (a material property) indicates how much the change in volume of a certain material is, per change in temperature:

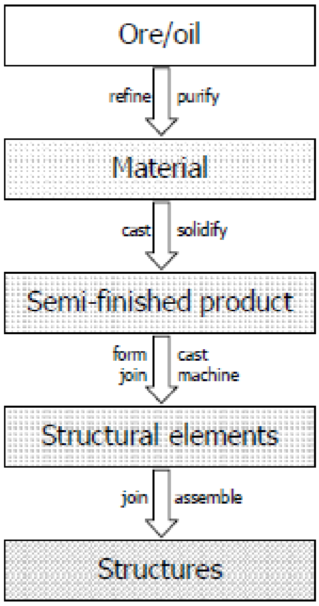
$$\alpha = \frac{1}{V}\frac{dV}{dT}.$$

In general:

Low temperature	Properties of metallic materials increase. Higher resistance against (both elastic and plastic) deformation. Some low carbon steel alloys become more brittle (T2-ships).
Humidity	Metallic materials are more effected by corrosion. Composites face ingress of moisture into the matrix (which is then weakened). In FMLs, only outer layers are affected. (Moisture ingress is stopped by metal layers).
Moisture/salt	Metallic materials corrode more quickly.
Outer space	Radiation (UV) / free radicals / vacuum start playing a (negative) role.

Chapter 3: Material Types

Before materials can be used, they have to be 'harvested' and processed:



There are four categories of materials that are suited for application in aerospace structures:

- Metal alloys (aluminium, titanium, …)

Table 3.1 Typical mechanical properties some metals

Metal	Alloy	E	G	σ_y	σ_{ult}	ε_{ult}	ν	ρ
		[GPa]	[GPa]	[MPa]	[MPa]	[%]	[-]	[g/cm^3]
Steel	AISI 301	193	71	965	1275	40	0.3	8.00
	AISI 4340	205	80	470	745	22	0.29	7.85
	D6AC	210	84	1724	1931	7	0.32	7.87
Aluminium	AA 2024-T3	72	27	345	483	18	0.33	2.78
	AA 7475-T761	70	27	448	517	12	0.33	2.81
Titanium	Ti6Al-4V (5)	114	44	880	950	14	0.34	4.43
Magnesium	AZ31B-H24	45	17	221	290	15	0.35	1.78

  - Used in structures and components that require high strength in both tension and compression.
    - Main fuselage
    - Wing
  - Titanium is used for high temperature applications (Blackbird, Concorde, …).

- Polymers
  - Exhibits lower stiffness and strength (compared to metals).
  - Many applications, such as tires, seals, coatings, liners, plastics.
- Ceramics
  - Very brittle, not suited for structural parts.
  - Used as heat protection (Space Shuttle), and in glass (for windows, fibres, …).
- Composites (see: below)

Composite materials	Engineering materials in which <u>two or more distinct</u> and structurally complementary substances with different physical or chemical properties re combined to produce structural or functional properties <u>not present in any individual component</u>. (Above definition includes all combinations of materials. However, these days, a composite is composed of carbon fibres and polymer.)

Fibre reinforced polymers have <u>high specific properties</u> and a <u>considerable stiffness-weight ratio</u>, but (most often) behave <u>plastic until failure</u>, thus <u>limiting the toughness</u>. A very important advantage, however, is the fact that composites (due to their directionality) can be perfectly tailored for their intended use. The downside is, that to create a *quasi-isotropic* sheet, you need multiple layers, adding weight to the structure.

Besides ***quasi-isotropic*** laminates (that try to behave isotropic, equal in all directions), there are ***cross-ply*** laminates (a number of layers with fibres orientated at, alternating, 0 and 90 degrees) and ***uni-directional*** laminates (fibres in one direction).

Properties (density, E-modulus) of a composite can be roughly estimated by the ***rule of mixtures***:

$$M_{FRP} = M_F + M_M$$
$$\rho_{FRP} V_{FRP} = \rho_F V_F + \rho_M V_M$$
$$\rho_{FRP} = \rho_F v_F + \rho_M V_M$$

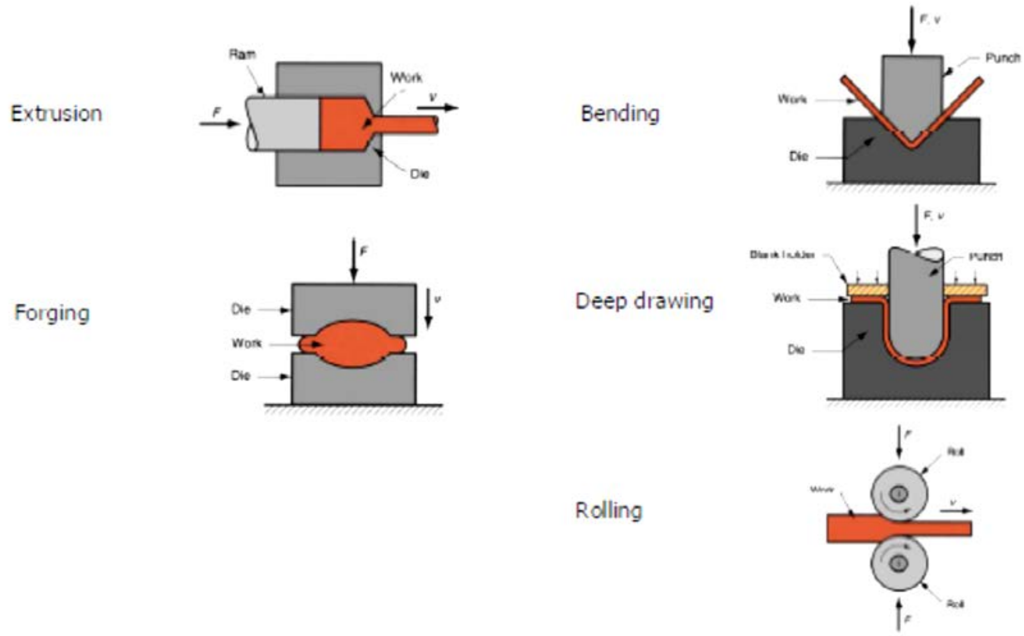
in which v indicate the *volume fraction of that particular 'ingredient'* ($0 \leq v \leq 1$).

Besides material properties, it is important to have materials that have good *workshop properties*, i.e. are not impossible to manufacture / form into the required shapes.

Chapter 4: Manufacturing

Each material requires different manufacturing techniques, that each have their advantages and disadvantages:

- Metals
  - Casting
    - Metal is molten, poured into a mould, and left to cool.
  - Machining
    - Component is cut from a larger piece of material by drilling/cutting.
    - Can be performed with great accuracy, since process works at room temperature.
  - Forming
    - Of bulk material
      - Extrusion
      - Forging
    - Of sheet material
      - Bending
      - Deep drawing
      - Roll bending



Since forming is based on plastic deformation of material, ***spring back*** has to be taken into account. The amount of spring back (dependent on the level of stress imposed by the machine) needs to be considered, as well as the possible tolerances.

- Composites
  - Composites can be manufactured by placement of fibres in dry or wet direction, or pre-impregnated fibres can be used.
    Production techniques include:
    - Filament winding (around a *mandrel*, works with dry and wet fibres))
    - Pultrusion (fibres are pulled through a resin, which is heated/cured afterwards)

- Lay-up (both automated or manual, and for dry, wet and pre-impregnated fibres)
- Resin transfer moulding (RTM) (dry fibres are placed in a mould, through which resin is drawn by a pressure difference)
    - Vacuum infusion (similar to RTM, but mould is closed and is made vacuum)
    - Vacuum assisted RTM (VARTM) (midway between RTM and VI)

Pressure	P_1 (outside)	P_2 (inside)	ΔP
Resin Transfer Moulding	> 1 bar	1 bar	> 1 bar
Vacuum Assisted Resin Transfer Moulding	> 1 bar	< 1 bar	> 1 bar
Vacuum infusion	1 bar	< 1 bar	~ 1 bar

Resins can be both made from thermoplastics as well as from thermosets. The first has the advantages of quick manufacturing and the fact that products can be welded and recycled. On the downside, high temperatures and pressures are required, and storage life is limited.

Aspect	Thermoset	Thermoplastic
Material	Liquid components A and B	Single solid matrix
Melting step	No	Yes
Impregnating fibres	Yes	Yes
Chemical reaction	Yes	No
Material after cooling	Solid matrix	Solid matrix

Chapter 5: Aircraft & Spacecraft Structures

To distinct elements that are considered to be part of the structure of an aircraft/spacecraft from other elements, these elements are often referred to as ***airframe***:

- The aircraft or spacecraft without installed equipment and furnishing.
- The skin and framework (skeleton) that provide aerodynamic shapes.
- The load bearing parts that take up forces in flight.
- The parts that together protect the contents from the environment.

Primary structure	Structural elements that in case of damage or failure could lead to failure of the entire craft.
Secondary structure	Structural elements that are not part of the primary structure.

Through the years, different structural concepts have been explored:

- Truss structures

  Load transfer primarily via tension/compression. Rigidity is obtained by diagonal elements (wires, rod or sheet material). That last set-up resulted in the development of a beam: a web plate (the former sheet) with two girders, placed perpendicular on both ends (in place of the rods used in the truss)
- Shell structures

  Based on the most 'modern' truss structure, the one with sheet material as diagonal element. Stiffeners are attached to increase performance, where hat-shaped stiffeners provide more stability than L- or Z-shaped ones. The combination is a shell structure.
- Sandwich structures

  Very similar to a beam, but with *core material* acting as web plate (taking up shear loads). The core material (most often a honeycomb structure) can be made from aluminium, polymer, or many other materials.

  Primary advantage is high bending stiffness, which is why sandwiches are often applied as floor concept. Disadvantages concern the fact that the material cannot be simply bolted to other parts of the structure. Furthermore, moisture absorption is very harmful to this structure.
- Integrally stiffened structures

  Similar to a stiffened shell structure, but with the difference that stiffeners aren't added afterwards (by bolting/riveting), but are interconnected with the shell (by machining the complete element at once).

  Advantages include possible weight savings and relative low cost of production. Disadvantages should be sought in the fact that complex structures cannot be machined. In addition, the integral structure is less damage tolerant when compared to a built-up structure: there is no natural barrier for cracks.

Fuselage structures:

- Skin
- Frames
- Stringers (‘stiffeners’)
- Bulkheads
- Splices/joints

Wing structures:

- Skin
- Ribs (from TE to LE)

  Introduce aerodynamic / fuel loads as well as local loads (landing gear, engines, control surfaces) to the wing structure and provide stability against panel crushing and buckling.
- Spars (from root to tip)

  Carry wing bending, which is why spars are thicker at the root than at the wing tips. Two spars can counteract torsional moments on the wing, but a torsion box (as it has a closed surface) do better at this job. Besides, the torsion box proves a lighter solution and creates the ability to engineer torsional and bending stiffness separately.

Selection of material and manufacturing method used for ribs and spars depend on:

- Loads
- Design philosophy
- Available equipment / experience
- Costs

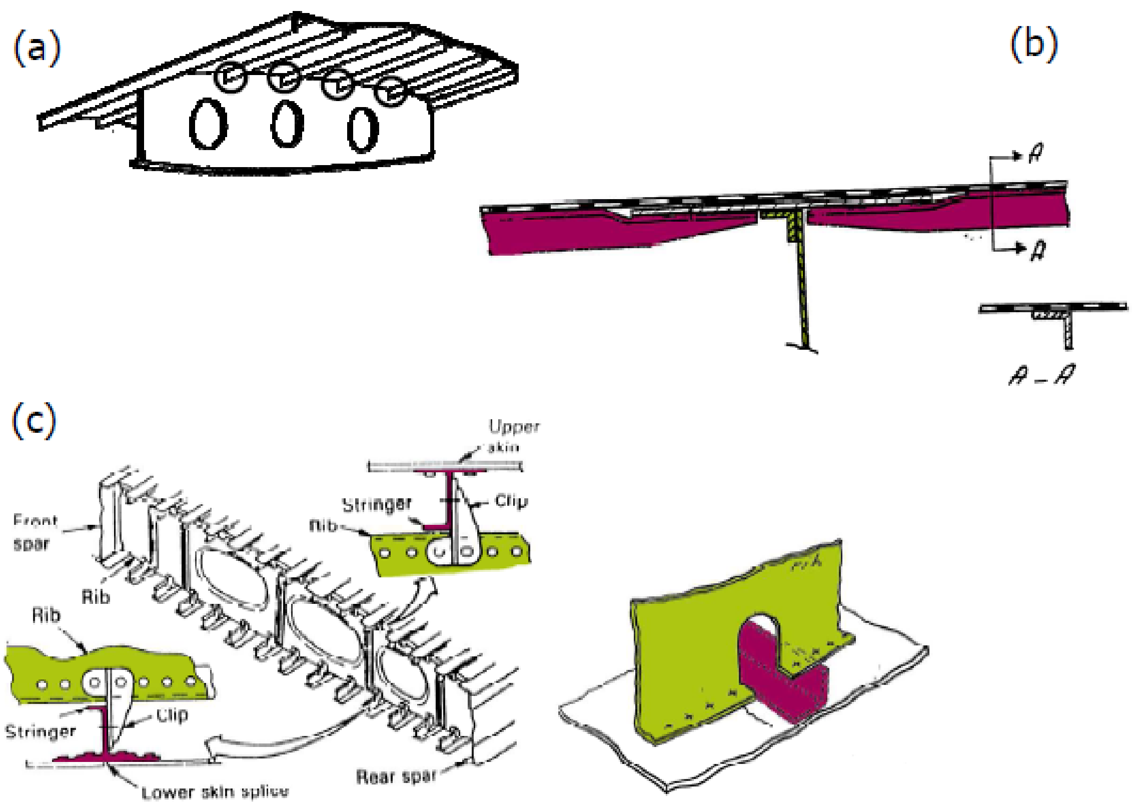
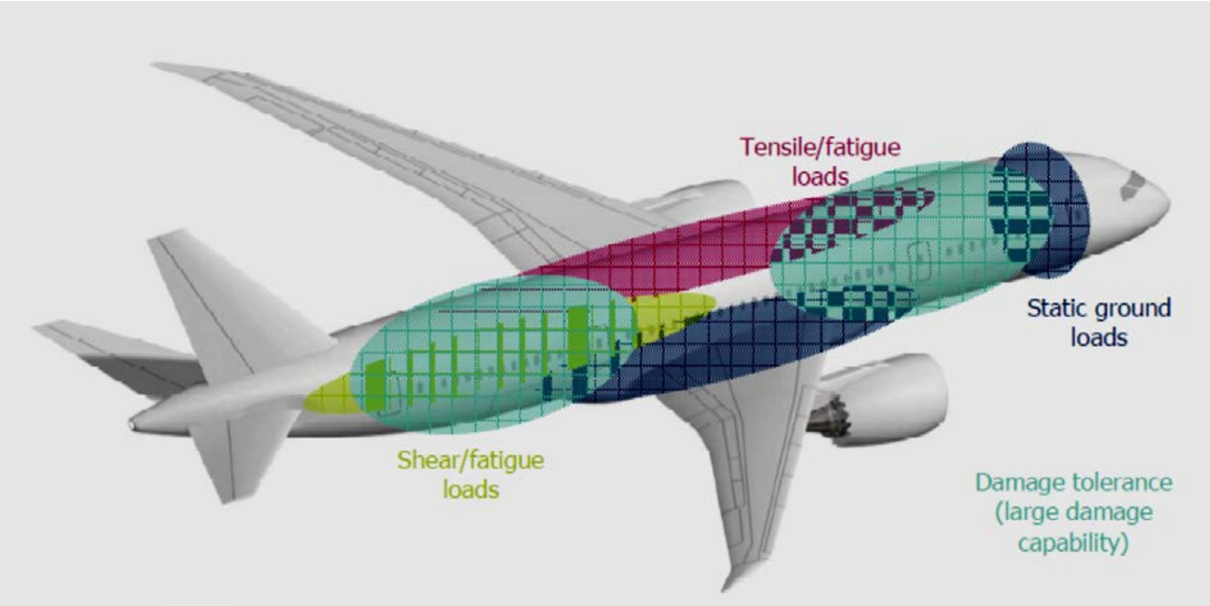


Figure 5.32 Illustration possible rib stringer intersections; both not interrupted (a), stringer interrupted (b), rib interrupted (c)

Chapter 6: Aircraft & Spacecraft Loads

An aircraft or spacecraft is loaded by a combination of distributed (weight on landing gears) and concentrated loads (lift), and faces a certain amount of load cases (such as manoeuvres, pressurization, …).

Load path	A physical trajectory that links the location of applied force and forces elsewhere that provide equilibrium with the applied force.



As the picture above shows, some aircraft loads act in specific areas. Static ground loads will be significant near the landing gears, tension and fatigue dominant load cases in the upper fuselage, compression loads in the lower fuselage and shear and fatigue loads will be faced by the aft side fuselage (loads are introduced by the empennage (= tail section)).

Spacecraft face loads which are similar to an aircraft's, but also some others:

- Pre-launch
  - Gravity
  - Vibration and acoustic test
- During launch
  - Quasi-static
  - Sine vibration
  - Acoustic noise and random vibration
  - Shock loads
- In orbit
  - Shock loads
  - Structurally transmitted loads
  - Internal pressure
  - Thermal stress

Steady state loads are mostly directed in axial direction (thrust) or lateral direction (wind gusts).

Chapter 7: Translating Loads to Stresses

The loads an aircraft is facing in its lifetime translate to all kinds of stresses:

Stresses in a pressure vessel:

$$\sigma_{circ} = \frac{\Delta pR}{t}$$

$$\sigma_{long} = \frac{\Delta pR}{2t}$$

Since $\sigma_{circ} > \sigma_{long}$ (factor 2), welds should be in circumferential direction.

$$\sigma_{sphere} = \sigma_1 = \sigma_2 = \frac{\Delta pR}{2t}$$

Strain in a pressure vessel (combining stress formula's and Hooke's Law):

$$\epsilon_{circ} = \frac{\sigma_{circ}}{E}\left(1 - \frac{v}{2}\right)$$

$$\epsilon_{long} = \frac{\sigma_{long}}{E}\left(\frac{1}{2} - v\right)$$

$$\epsilon_{sphere} = \frac{\Delta pR}{2t}\frac{1}{E}(1 - v) = \frac{\sigma_{sphere}}{E}(1 - v)$$

When attaching pressure bulkheads to the tubular fuselage, pay attention to the thicknesses: the circumferential strain should be equal to the spherical strain ($\epsilon_{circ} = \epsilon_{sphere}$, results in $t_{sphere} = 0{,}41 \cdot t_{cylinder}$).

To reduce stress, a pressure at 2400/3000m is maintained as cabin pressure.

If a thin walled shell structure is loaded by a torsional moment (wind blowing against the vertical tail surface) denoted by M_T, then the ***shear flow*** in that shell is solely determined by the enclosed area A. That gives a formula for *shear flow* q:

$$q = \frac{M_T}{2A}$$

This shows that a torsional moment is resisted by shear stresses in the walls. That explains why cut-outs (windows, doors, …) need reinforcing.

For wing bending (mentioned earlier), the shear function of the spar webs proves essential. Those structural parts maintain an equilibrium between web plates and frames (/girders/*spar caps*), in which normal forces are apparent. Thus, the <u>webs transfer transverse forces into shear flows</u>, while the <u>caps transfer shear-flows into normal forces</u>.

An explanation (+ graphics!) is given in Section 7.5 (*Case Study: Bending of a Wing Spar*), pp. 121-127. A couple of general(ized) formula's follow from that case study:

- Shear flow
  - Transverse shear force D at n^{th} shear web:

    $$D_n = F_1 + F_2 + \cdots + F_n = \sum_1^n F_n$$

  - Shear flow q in each web (with h being the height of the web):

    $$q_n = \frac{D_n}{h}$$

- Shear stress
  - Shear stress in the webs:

    $$\tau_x = \frac{q_x}{t_x} = \frac{D_x}{ht_x}$$

- Normal forces
  - Normal forces in upper and lower caps at the location of the vertical stiffener (with l being the length of the web):

$$N_{m+1} = \frac{1}{h}\sum_{n=1}^{m} D_n \cdot l_n$$

  - Normal forces in-between the stringers (between webs 1 and 2):

$$N_{U_x} = \frac{D_2}{h}(l_2 - x) + \frac{D_1}{h} l_1$$

- Normal stress
  - Normal stress in the spar caps (with hA_x being the moment of resistance W):

$$\sigma_x = \pm\frac{N_x}{A_x} = \pm\frac{M_x}{hA_x}$$

- Bending moment
  - Bending moment at $n+1$th shear web:

$$M_{n+1} = \sum_{n}^{m} D_n \cdot l_n$$

  - Bending moment at location x:

$$M_x = N_{U_x} \cdot h = D_2(l_2 - x) + D_1 l_1$$

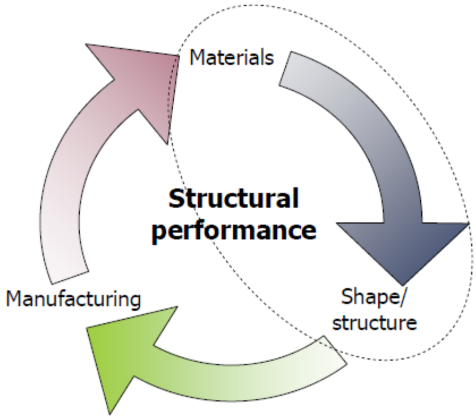
Valid for each location on the spar: $\frac{dM_x}{dx} = -D_x$

  - Normal forces in the spar caps:

$$N = \frac{M}{h}$$

Chapter 8: Considering Strength & Stiffness

The ***structural performance*** is a function of material properties, geometrical features and dimensional aspects and manufacturing aspects. Keep in mind that the introduction of a new material might need a change in geometry as well.



The ***strength-to-weight ratio*** is often considered to select the best material for a structure (used in the aerospace field).

Specific tensile strength — The ratio between ultimate strength σ_{ult} and density ρ.

Breaking length — The length to which a bar (hanging from the ceiling) can be extended without failing. (Graphic example on p. 132.)

$$L_{ult} = \frac{\sigma_{ult}}{\rho g} \text{ (most often in [km])}$$

Material	σ_{ult} [MPa]	σ_{ult}/ρ [10^6 Nmm²/kg]	ρg [N/dm³]	L_{ult} [km]
Steel AISI 301	1275	159	78.4	16.2
Steel D6AC	1931	248	77.2	25.0
Aluminium 2024-T3	483	174	27.3	17.7
Aluminium 7475-T761	517	184	27.6	18.7
Magnesium AZ31-H24	290	163	17.5	16.6
Titanium Ti-6Al-4V (5)	950	214	43.5	21.8
Quasi-isotropic CFRP	500	327	15.0	33.3

Next to the strength-to-weight ratio, one should also consider the specific buckling strength:

$$\sigma_{cr} = \frac{Et^2}{Lb},$$ with L being length, b being width and t being thickness of the sheet.

Then, the buckling load is given by

$$F_{cr} = \sigma_{cr} \cdot b \cdot t$$

and the required thickness is calculated by

$$t = \sqrt[3]{\frac{F_{cr}L}{E}}.$$

In other words, to compare the specific properties for the case of compression buckling, one has to search for the highest value of $\sqrt[3]{E}/\rho$.

Material	σ_{ult} [MPa]	E [GPa]	$\sqrt[3]{E}/\rho$ [10^6 $mm^{7/3}N^{2/3}$]
Steel AISI 301	1275	193	0.72
Steel D6AC	1931	210	0.76
Aluminium 2024-T3	483	72	1.50
Aluminium 7475-T761	517	70	1.47
Magnesium AZ31-H24	290	45	2.00
Titanium Ti-6Al-4V (5)	950	114	1.09
Quasi-isotropic CFRP	500	60	2.56

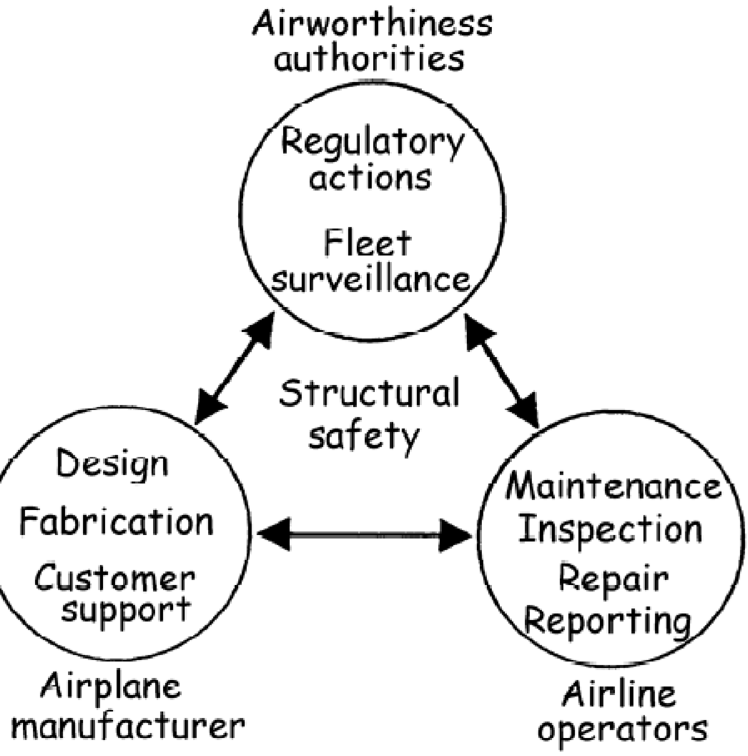
As stated earlier, the geometry of a structure also plays an important role. In addition, the performance of different materials and/or geometries can change for different load cases (as previously stated).

The table below shows what gives the minimum weight for given stiffness or strength in a couple of load cases structures may encounter.

	Minimum weight for given	
Loading mode	Stiffness	Strength
Compression / buckling	$\sqrt[3]{E}/\rho$	
Bending	$\sqrt[3]{E}/\rho$	$\sqrt{\sigma}/\rho$
Torsion (thin walled)	E/ρ	σ/ρ
Pressurization (thin walled)	$E/(1-v)\rho$	

Chpater 9: Design & Certification

Obviously, safety is very important. Three major bodies have a crucial responsibility to achieve safe flying.



For aeronautical structures, a couple of requirements have to be met to ensure a certain level of safety. These requirements can be categorized as follows:

- Strength
- Loads

  T most important and most difficult aspect.
- Life time

  Accurate logs are crucial to evaluate whether loading is more/less severe than the structure was designed for.

There are three important ***structural design philosophies***:

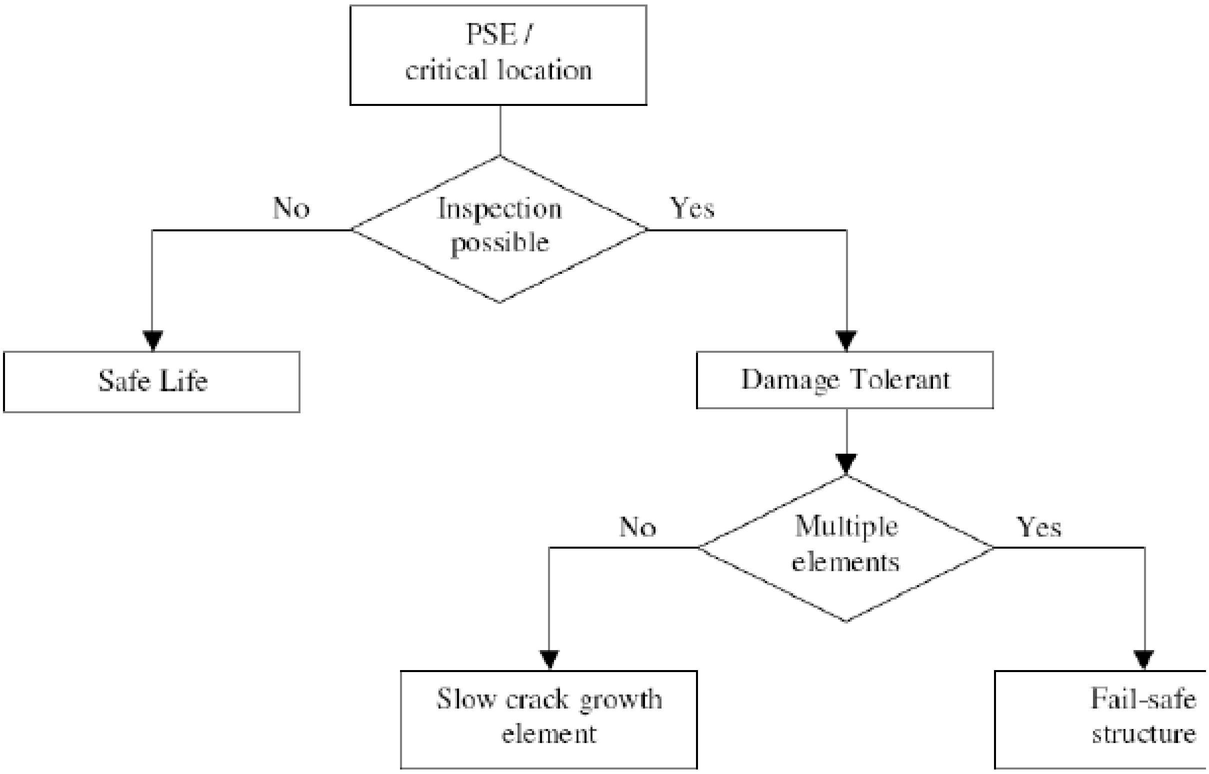
1. Safe life (safety by retirement)

   Safe-life of a structure is the number of flights, landings or flight hours, during which there is a low probability that the strength will degrade below its design strength.
2. Fail safe (safety by design)

   Fail-safe is the attribute of the structure that permits it to retain required residential strength for a period of un-repaired use after failure or partial failure of a structural element.
3. Damage tolerance (safety by inspection)

   The ability of the structure to sustain anticipated loads in the presence of fatigue, corrosion or accidental damage until such damage is detected through inspections or malfunctions and is repaired.

Whereas in the first design philosophy, components had to be replaced after a certain lifetime (indifferent of the integrity of the component), the second design philosophy stated that components should be kept in service until the damage was observed. The third philosophy combines the earlier two. The graph beneath shows it clearly.



MSD (Multiple Site Damage)	The occurrence of multiple cracks in adjacent components or elements.
Durability	The ability of the structure to sustain degradation from sources as fatigue, corrosion, accidental damage and environmental deterioration to the extent that they can be controlled by economically acceptable maintenance and inspection programs.

Chapter 10: Space Structures

There are some specific things to pay attention to when designing / analysing spacecraft.

There are two categories of spacecraft structures, that differ by the way the transfer loads from the thrust (at the lower end) through the structure of the launch vehicle:

- Strutted structures
  Based on trusses.
- Central cylindrical shell structures (like EnviSat)
  Based on a cylinder, that bears the thrust-load and to which other parts are attached.

Typical launch vehicle structures include:

- Payload fairing
- Stage structures
- Thrust structures
- Adaptors (transfers load from launch vehicle to payload)

Just as for aircraft, the mass should be minimized, while strength and stiffness are maximized. In addition, it is necessary to assure that the lowest natural frequency present in the structure is well above the specified minimum frequency, which can be achieved by *structural sizing*. This is done to avoid resonance between launch vehicle and the spacecraft carried within.

An equation for the two frequencies is derived (pp. 162-164) by expressing two 'spring constants', one in axial (x-) and one in lateral (y-) direction:

$$k_x = \frac{EA}{L}; k_y = \frac{3EI}{L^3}$$

in which EA represents the stiffness, and EI represents the bending stiffness of the 'spring'.\
The natural frequency is defined as:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

combining those formulas and that requirement given in the previous paragraph, this results in:

$$\frac{EA}{L} \geq (2\pi f_n)^2 m \text{ (axial direction)}$$

$$\frac{3EI}{L^3} \geq (2\pi f_n)^2 m \text{ (lateral direction)}$$

Once the requirements concerning natural frequencies are met, the structure can be designed for quasi-static loads: loads related to the acceleration of the vehicle, defined as

$$F = mg_x$$

This results in the following equations for axial and lateral stress:

$$\sigma_x = \frac{F}{A}; \sigma_y = \frac{(mg_y L)y}{I}$$

(In which I is the second M. of Inertia, y the distance from the neutral line to the outer surface of the beam.)

The maximum stress should be lower than the allowable stress, calculated by superimposing:

$$\sigma_{tot} = \sigma_x + \sigma_y \leq \sigma_{allowable}$$

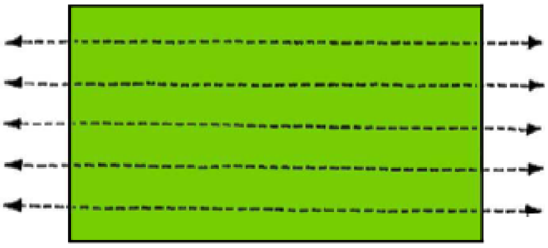
The buckling load applied (Euler buckling load) depends on the bending stiffness, EI:

$$F_{Euler} = \frac{\pi^2 EI}{4L^2}$$

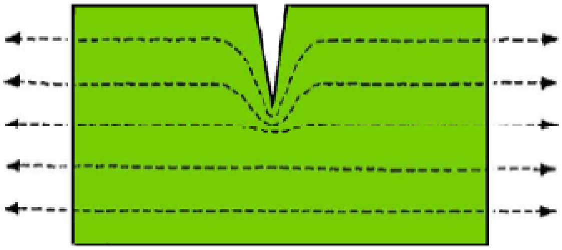
which should be larger or equal to $g_x \cdot M$.

Chapter 11: Fatigue & Durability

Although the formula for stress given in chapter 1 is correct, it does not cover all cases. When the geometry of the shape is damaged, a disturbance in the ***stress field*** is formed.



Homogeneous stress field



Stress field with disturbance

Because the actual area is reduced, the stress increases by a factor A/A^*. This gives:

$$\sigma_{nom} = \frac{F}{A^*} = \frac{A}{A^*}\sigma_{gross}$$

However, a component will fail <u>before</u> $\sigma_{nom} = \sigma_{ult}$, which is due to the fact that the stress is concentrated around a cut or crack (as the picture shows). This is indicated with a ***stress concentration factor***, given by:

$$K_t = \frac{\sigma_{peak}}{\sigma_{nom}}$$

Saint Venant's principle … states that the effect of a local disturbance remains limited to its direct neighbourhood.

Because of this principle, a stress analysis can be divided into two parts:

- Global stress analysis (complete structure)
- Local stress analysis (disturbances)

Repairs can extend the lifespan of a component, but attention has to be paid: when a structure is reinforced, the geometry changes, which might cause altered load cases. That way, a repair might 'attract' more load than that part originally had to face. Although the repair itself might survive, the edges of the repair might not.

Fatigue Damage phenomenon induced by <u>a large number</u> of load cycles <u>below the ultimate strength</u> of materials / structures causing permanent deterioration of materials / structures resulting in a reduction in load bearing capability.

<u>All engineering materials</u>, in one way or another, <u>suffer from fatigue</u>.

Nominal load / limit load Load case that is considered to occur only once in a lifetime, and at which failure <u>may not occur</u>.

S-N curves or ***Wohler curves*** can represent the influence / effects of fatigue. The upper asymptote is related to the maximum stress reaching the ultimate strength. The lower asymptote shows the ***fatigue limit***. It is defined as <u>the stress amplitude below which fatigue failure does not occur</u>. And, in general, is reduced by a factor K_t for damaged structures.

One thing to keep in mind is that the Wohler curves are based on constant amplitude loading. Variable amplitude loading can be estimated (very inaccurately) by the ***Miner rule***, which gives the damage fraction for the i^{th} load cycle:

$$D_i = \frac{n_i}{N_i}$$

where N_i is the fatigue life, from the S-N curve.

The component fails when the summation of all D_i equals one.

Each material has different fatigue characteristics:

- In metallic materials, cyclic tensile loading may initiate and/or propagate cracks.
- In composites, delaminations and transverse shear cracks may occur.

In addition to the *stress concentration factor*, there is a ***stress intensity factor*** (which should not be confused!). It assess the residual strength in a damaged piece of material and is given by:

$$K = S\sqrt{\pi a}$$

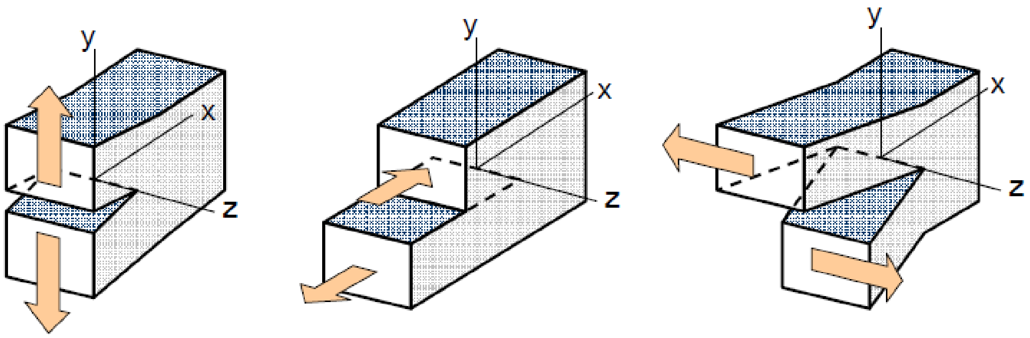


Figure 11.17 Three different opening modes of a crack; tensile mode I, shear mode II, and transverse shear mode III

The ***fracture toughness***, a material property, indicates the sensitivity to cracks:

$$K_{Ic} = S_{crit}\sqrt{\pi a_{crit}}$$

The subscript I refers to the crack opening mode (see: figure 11.17 above).

Formula's

Stress, strain, shear, strength, loads, …	
$\epsilon = \frac{\Delta L}{L}$ $\sigma = \frac{P}{A}$ $\sigma = E \cdot \epsilon$ $\epsilon_t = v \cdot \epsilon_l$ $\tau = \frac{F}{A}$ $\tau_{xy} = G \cdot \gamma_{xy}$ $G = \frac{E}{2(1+v)}$	ϵ: strain (ϵ_l: lateral / ϵ_t: transverse) σ: stress τ: shear strain γ: shear stress E: Youngs's modulus E^*: apparent Youngs's modulus G: shear modulus of elasticity v: Poisson's ratio
$\epsilon_x = \frac{\sigma_x}{E} - v\frac{\sigma_y}{E}$; $\epsilon_y = -v\frac{\sigma_x}{E} + \frac{\sigma_y}{E}$ $E^* = \frac{1}{1-v^2}E$	Hooke's Law, for bi-axial loading
$L_{ult} = \frac{\sigma_{ult}}{\rho g}$ $\sigma_{cr} = \frac{Et^2}{Lb}$ $F_{cr} = \sigma_{cr} bt$ $t = \sqrt[3]{\frac{F_{cr}L}{E}}$	L_{ult}: length at failure strength σ_{ult}: ultimate stress σ_{cr}: critical buckling strength F_{cr}: critical buckling load b: width (breedte) L: length t: thickness
$\sigma_{circ} = \frac{\Delta pR}{t}$ $\sigma_{long} = \frac{\Delta pR}{2t}$ $\epsilon_{circ} = \frac{\sigma_{circ}}{E}\left(1 - \frac{v}{2}\right)$ $\epsilon_{long} = \frac{\sigma_{circ}}{E}\left(\frac{1}{2} - v\right)$ $\epsilon_{sphere} = \frac{\Delta pR}{2t}\frac{1}{E}(1-v)$	R: radius t: thickness From Hooke's Law v: Poisson's ratio
$q = \frac{M_T}{2A}$	q: shear flow M_T: torsional moment A: enclosed area (cabin/torsion box/…)
$K_t = \frac{\sigma_{peak}}{\sigma_{nom}}$ $K = S\sqrt{\pi a}$	K_t: stress concentration K: stress intensity
Matrix formulation (combining Hooke's law and formula for shear strain/stress)	
$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{v_{yx}}{E_y} & 0 \\ -\frac{v_{xy}}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$	For anisotropic materials. For isotropic materials, neglect subscript in main (3 x 3) matrix.

Rule of mixtures (INACCURATE!!)	
$M_{FRP} = M_F + M_M$ $\rho_{FRP}V_{FRP} = \rho_F V_F + \rho_M V_M$ $\rho_{FRP} = \rho_F v_F + \rho_M V_M$	M: mass V: volume ρ: density v: volume fraction (V_F/V_{FRP} or V_M/V_{FRP})
Bending of a wing spar	
$D_n = F_1 + F_2 + \cdots + F_n = \sum_1^n F_n$	shear force at nth spar
$q_n = \frac{D_n}{h}$	shear flow
$N_{m+1} = \frac{1}{h}\sum_{n=1}^{m} D_n l_n$	normal forces at location of vert. stiffener
$N_{U_x} = \frac{D_2}{h}(l_2 - x) + \frac{D_1}{h}l_1$	normal forces in spar caps
$\frac{dM_x}{dx} = -D_x$	bending moment on spar
$\tau_x = \frac{q_x}{t_x} = \frac{D_x}{ht_x}$	shear stress in webs
$\sigma_x = \pm\frac{N_x}{A_x} = \pm\frac{M_x}{hA_x}$	normal stress in spar caps
Fatigue (Miner rule)	
$D_i = \frac{n_1}{N_i}$ Failure at $\sum D_i = 0$	D_i: damage fraction, ith load cycle n_i: number of load cycles N_i: fatigue life (from S-N curve)