Chapter 1

Planetary entry environment

1.1 Planetary shapes

Even though the shape of, for instance, the Earth is determined by many factors, an approximation, which is adequate for the purpose of re-entry, is to consider the shape of the regular bodies as an ellipsoid of revolution. The parameter defining this ellipsoid is called the ellipticity e and is defined as

$$e = \frac{R_e - R_p}{R_e} \tag{1.1}$$

For the Earth, R_e is about 21 km smaller than R_p.

We can use the ellipticity to derive an expression for the radius at an arbitrary point along the surface, Rs. Starting with the general expression of an ellipse and using polar coordinates, the following general expression can be derived:

$$R_s = R_e[1 - \frac{e}{2}(1 - cos2\delta^*) + \frac{5}{16}e^2(1 - cos4\delta^*) - ...] \tag{1.2}$$

Here, δ^* is the geographic latitude which is for the ellipsoid not the same as the geocentric latitude δ (see Figure 1.1). However, since the difference is very small it is justified to approximate δ^* with δ. In addition, as R_e is not known with indefinite accuracy, for first-order analyses we may approximate Equation 1.2 by

$$R_s = R_e(1 - esin^2\delta) \tag{1.3}$$

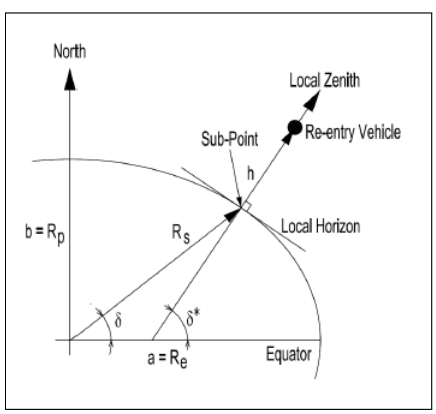

Figure 1.1: The re-entry vehicle above the reference ellipsoid; δ is the geocentric latitude for a spherical central body, $\delta*$ the geographic latitude for the ellipsoid.

For the rotation of the central body, we assume that it is rotating with a constant angular velocity ω_{cb} directed along the Z-axis of the rotating planetocentric frame, so

$$\omega_{cb} = [0, 0, \omega_{cb}]^T \tag{1.4}$$

In some situations it is possible to simplify the shape of the central body. The first simplification is to consider the central body as a (non-)rotating spheroid. A second simplification, commonly used in aircraft studies, but also a justified simplification when studying the landing of the Space Shuttle, is to assume a flat-Earth approximation.

surface models (to be added)

irregular bodies (to be added)

1.2 Gravity field

Two point masses, M and m, separated by a vector distance R, attract each other with a force given by Newton's law of gravitation as

$$F = \frac{GMm}{R^2}\hat{r} \quad = \quad \frac{\mu}{R^2}\hat{r} \tag{1.5}$$

where the hat $'$ indicates the normalised vector. In the above equation, G is the universal gravity constant ($G = 6.668 * 10^-11 m^3/kgs^2$). Let M be the mass of the central body, e.g., the Earth, then it is convenient to define the geocentric gravitational constant μ as GM. ($\mu_{earth} = 3.9860047 * 10^{14} m^3/s^2$)

Then the gravitational force may be rewritten as the gradient of the scalar U:

$$\overline{F}_G = -m\nabla U \quad with\ \ U = -\frac{\mu}{R} \tag{1.6}$$

Add spherical harmonics

1.3 Atmospheres

The equation of state is used in many derivations.

$$p = \rho RT = \rho\frac{R_A}{M}T \tag{1.7}$$

For the Earths atmosphere the following conditions hold:

$$M = 28.96 kg/kmol \qquad R = 287 J/kgK \tag{1.8}$$

The hydrostatic equation relates the pressure to the density. This equation is used to derive the exponential atmosphere.

$$dp = -\rho g dh \tag{1.9}$$

1.3.1 Exponential model

The exponential atmosphere is based on the hydrostatic equation and the gravity model of the earth. The gravitational acceleration can be modelled as a function of altitude resulting in the following derivation. The hydrostatic equation can be integrated without knowledge of ρ and g as a function of h by introducing the geopotential altitude. The geopotential altitude z, is related to the geometric altitude h by the relation:

$$g_0 dz = g dh \rightarrow z = \frac{h}{R_e + h} \Rightarrow \frac{z}{R_e} \approx \frac{h}{R_e}\left(1 - \frac{h}{R_e}\right) \tag{1.10}$$

Where the following equation is used, which is derived from Newton's law of gravitation.

$$g = \frac{g_0}{1 + \frac{h^2}{R_e^2}} \tag{1.11}$$

The exponential atmosphere is derived by integrating the hydrostatic equation, rewritten in the following form:

$$\frac{dp}{p} = -\frac{g}{RT}dh = -\frac{g_0}{RT}dz \tag{1.12}$$

Assuming $g = g_0$ yields the Exponential Atmosphere model:

$$\frac{\rho}{\rho_0} = e^{-\beta h} \tag{1.13}$$

with the scale height H defined as:

$$H = \frac{1}{\beta} = \frac{RT}{g_0} \tag{1.14}$$

The scale height for the earth is obtained using an (average?) constant temperature and approximated by:

$$H \approx 7200m \qquad with\ \ T = 246K = const. \tag{1.15}$$

1.3.2 The United States Standard Atmosphere 1962

atmosphere layers, Temperature distribution -¿ atmosphere

Chapter 2

Fundamentals of motion

Chapter 3

Aerothermodynamics

3.1 Stagnation Heatflux

The stagnation heat flux can be determined by Chapman's equation:

$$q_c = c_1 \frac{\rho^{1-n} V^m}{R_N^n} \tag{3.1}$$

Where R_N is the nose radius of the entry vehicle. The parameter depends on the type of flow, with $n = 0.5$ for laminar boundary's and $n = 0.2$ for turbulent. The value m depends on the atmosphere, with $m = 3$ for Earth.

3.2 Heatflux at an arbitrary point

The heat flux for an arbitrary point on the wall of the vehicle can be determined by the following equation:

$$\frac{dQ}{dt} = \frac{C_F}{4} \rho V^3 S_W \tag{3.2}$$

with S_W being the washed surface area and C_F a friction coefficient averaged over the surface.

Chapter 4

Planar motion

4.1 Equations of motion

The equations of motion for an entry vehicle over a spherical non-rotating Earth can be derived using the following figure:

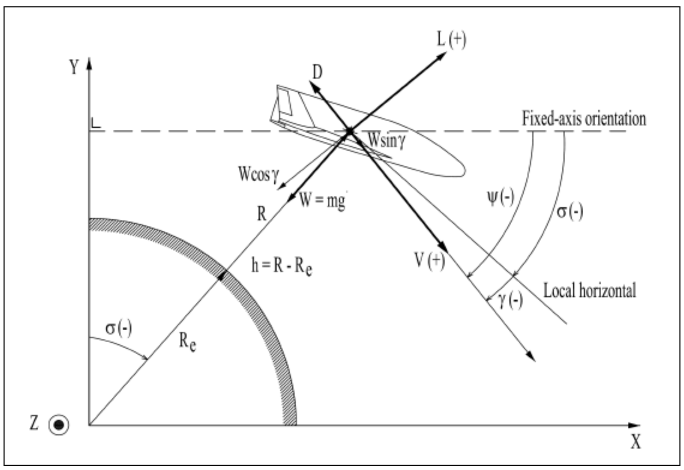

Figure 4.1: Two-dimensional planar motion

THe following relationsships hold:

$$R = R_e + h \tag{4.1}$$

$$-\Psi = -\sigma - \gamma \tag{4.2}$$

Using these relationships the following equations of motion can be derived.

$$m\frac{dV}{dt} = -D - mg\sin\gamma \tag{4.3}$$

$$mV\frac{d\gamma}{dt} = L - mg\cos\gamma\left(1 - \frac{V^2}{V_c^2}\right) \tag{4.4}$$

$$\frac{dR}{dt} = \frac{dh}{dt} = V\sin\gamma \tag{4.5}$$

Including the rotation of the Earth in the equations of motion induces an extra coriolis acceleration ($2\omega_{cb}V$) and a centripetal acceleration ($\omega_{cb}^2 R$). Where, the coriolis acceleration has a significant contribution.

For an entry velocity $V_E \approx 7450\ m/s$ and an entry height of $h_E = 120km$, the coriolis acceleration is equal to $2\omega_{cb}V \approx 1,08m/s$ and the centripetal acceleration is equal to $\omega_{cb}^2 R \approx 0,03m/s$.

4.2 Ballistic entry

A ballistic entry is by definition an entry without lift.

$$L = 0 \qquad \rightarrow \qquad mV\frac{d\gamma}{dt} = -mg\cos\gamma\left(1 - \frac{V^2}{V_c^2}\right) \tag{4.6}$$

Because of this constraint, it can be derived that the trajectory is a straight line.

$$tan(\gamma)\frac{d\gamma}{dh} = -\frac{g}{V^2} \Rightarrow \frac{d\gamma}{dh} \approx 0 \tag{4.7}$$

Because of the high velocities, the drag will be much higher than the weight. Therefore, in some cases the weight term can be neglected. This assumption is not made to determine the maximum deceleration. Using this approximation results in:

$$m\frac{dV}{dt} = -D \tag{4.8}$$

The ballistic parameter is introduced to compare different entry vehicles.

$$K = \frac{mg}{C_D S} \tag{4.9}$$

The **altitude-velocity** relation is derived by rewriting the term dV/dt and substituting the kinematic equation and the hydrostatic equation. Integration of the result and substituting the exponential atmosphere results in:

$$\frac{dV}{V} = \frac{1}{2}\frac{dp}{K\sin\gamma} \qquad \rightarrow \qquad \frac{V}{V_E} = \exp\left(\frac{1g\rho_0 e^{-\beta h}}{2K\beta\sin(\gamma_E)}\right) \tag{4.10}$$

Rewriting the velocity profile for the density ρ gives:

$$\rho = \frac{2K\beta\sin\gamma_E}{g}\ln\frac{V}{V_E} \tag{4.11}$$

The **maximum deceleration** is derived by rewriting the altitude-velocity profile for the density ρ, expanding the drag in the deceleration equation and substituting this density in the deceleration equation. Differentiating the result gives:

$$\frac{V'}{V_E} = \frac{1}{\sqrt{e}} = 0.606 \quad ; \quad \bar{a}_{max} = -\left(\frac{dV}{dt}\right)_{max} = -\frac{\beta\sin(\gamma_E)}{2e}V_E^2 \tag{4.12}$$

The maximum deceleration is independent of the ballistic parameter K, (so both weight and drag properties), but only changes with the initial parameters V_E and γ_E

Using the velocity and the exponential atmosphere, the altitude at which the maximum deceleration occurs is found to be:

$$h' = \frac{1}{\beta}\ln\left(-\frac{\rho_0 g}{K\beta\sin\gamma_E}\right) \tag{4.13}$$

The altitude does depend on the ballistic parameter K. The initial velocity has no impact on the altitude for a_{max}.

The **maximum convective heat flux in the stagnation point** is derived by using the equation for the density and substituting this into Chapman's equation, differentiating gives the velocity. The actual maximum value below is determined using a laminar boundary layer. ($n = 0.5$)

$$\frac{V''}{V_E} = e^{-\frac{1-n}{3}} \quad ; \quad q_{c_{max}} = c_1 V_E^3\sqrt{-\frac{K\beta}{3eR_N g}\sin\gamma_E} \tag{4.14}$$

which is proportional to V_E, γ_E and K. The density, and thus the altitude, at which this occurs is given by:

$$\rho'' = -\frac{2}{3}(1-n)\frac{K\beta}{g}\sin\gamma_E \quad \rightarrow \quad \frac{\rho''}{\rho'} = \frac{e^{-h''\beta}}{e^{-h'\beta}} = \frac{2}{3}(1-n) \tag{4.15}$$

The altitude at which the maximum convective heat flux in the stagnation point occurs, in relation to the one of maximum deceleration is:

$$h'' - h' = -\frac{\ln\left(\frac{2}{3}\right)(1-n)}{\beta} \tag{4.16}$$

This indicates that the altitude for $q_{c_{max}}$ is larger than the altitude for a_{max}. The altitude difference for a laminar boundary layer is larger than the one for a turbulent boundary layer.

The **maximum heat flux for an arbitrary point** can be determined by substituting the density relation it can be shown that:

$$\frac{dQ}{dt} \propto \ln\frac{V}{V_E}\left(\frac{V}{V_E}\right)^3 \quad \rightarrow \quad \frac{V'''}{V_E} = e^{-\frac{1}{3}} = 0.72 \tag{4.17}$$

Substituting this in the density relation, the altitude for the maximum heat flux and the altitude difference between the one at which the maximum convective heat flux in an arbitrary point on the vehicle occurs, and the one for maximum decelleration is:

$$\rho''' = -\frac{2}{3}\frac{K\beta}{g}\sin\gamma_E \quad \rightarrow \quad \frac{\rho'''}{\rho'} = \frac{2}{3} \quad \rightarrow \quad h''' - h' = -\frac{\ln\left(\frac{2}{3}\right)}{\beta} \tag{4.18}$$

Comparing numerical values shows that this altitude is in between the one for maximum flux in the stagnation point and the altitude for maximum deceleration.

4.3 Gliding entry

A gliding entry can be preferred over a ballistic entry, because the velocities are much lower. This results in lower heat loads, lower deceleration loads, which is more preffered for human spaceflight. A gliding entry is characterized by slow changes of the flight path angle and a small flight path angle.

$$\frac{d\gamma}{dt} \approx 0; \quad \gamma \approx 0; \quad \rightarrow \cos\gamma \approx 1; \;\; \sin\gamma \approx 0 \tag{4.19}$$

These assumptions lead to the following equations of motion:

$$m\frac{dV}{dt} = -D - mg\sin(\gamma) \tag{4.20}$$

With the equilibrium glide conditions:

$$L = mg\left(1 - \frac{V^2}{V_c^2}\right) \tag{4.21}$$

From 4.21 and the definition of the lift, the **velocity profile** can be derived, starting with:

$$\rho = 2\frac{W/S}{C_L}\left(\frac{1}{V^2} - \frac{1}{V_{c,0}^2}\right) \tag{4.22}$$

In this equation the assumption is made that the altitude is much smaller than the radius of the earth $h << R_e$ resulting in $V_c = V_{c,0}$. Using the exponential atmosphere model the **velocity profile** can be derived:

$$\frac{V^2}{V_{c,0}^2} = \frac{\frac{W/S}{C_L}}{\frac{1}{2}\rho_0 e^{-\beta h}V_{c,0}^2 + \frac{W/S}{C_L}} \tag{4.23}$$

The kinematic equation is used to derive the **equilibrium flight-path angle ,** $\overline{\gamma}$. Using the exponential atmosphere, the following equation can be derived:

$$\sin\gamma = -\frac{1}{\beta\rho}\frac{d\rho}{ds} = -\frac{1}{\beta\rho}\frac{d\rho}{dV}\frac{dV}{ds} \quad with \quad dh = -\frac{d\rho}{\beta\rho} \tag{4.24}$$

Using equation 4.22 the derivative $\frac{d\rho}{dV}$ can be found.

$$\frac{d\rho}{dV} = -4\frac{W/S}{C_L}\frac{1}{V^3} \tag{4.25}$$

Realizing γ is small so the weight term $-mg\sin(\gamma)$ can be neglected, yields the following expression which is derived from the equation of motion:

$$\frac{dV}{ds} = -\frac{\frac{D}{L}g\left(1-\frac{V^2}{V_c^2}\right)}{V} \tag{4.26}$$

Using these equations and assuming $V_c \approx \sqrt{R_e g}$ the final result is

$$\overline{\gamma} \approx \sin\overline{\gamma} = -\frac{1}{\beta R_e}\frac{2}{L/D}\frac{V_c^2}{V^2} \tag{4.27}$$

The **Flight Range** can be derived by rewriting equation 4.20 and substituting the equilibrium glide condition with the assumption that the weight term $mg\sin\gamma$ can be neglected, resulting in:

$$V dV = -\frac{D}{L}g\left(1-\frac{V^2}{V_c^2}\right)ds \tag{4.28}$$

Substituting $x = \frac{V^2}{V_c^2}$ and integrating the equation results in the following equation for the flight range, which can be simplified by assuming $V_c^2 \approx gR_e$ and $V_F << V_E$.

$$R_f = \frac{1}{2}\frac{L}{D}\frac{V_c^2}{g}\ln\left(\frac{1-\frac{V_F^2}{V_c^2}}{1-\frac{V_E^2}{V_c^2}}\right) \rightarrow \frac{R_f}{R_e} = -\frac{1}{2}\frac{L}{D}\frac{V_c^2}{g}\ln\left(1-\frac{V_E^2}{V_c^2}\right) \tag{4.29}$$

The **Flight time** is derived in approximately the same way as the flight range. Starting with the combination of the equations of motion and neglecting the weight term ,$\gamma \approx 0$.

$$\frac{dV}{dt} = -\frac{D}{L}g\frac{L}{mg} = -\frac{D}{L}g\left(1-\frac{V^2}{V_c^2}\right) \tag{4.30}$$

And integrating the following equation with the use of the substitution $x = \frac{V}{V_c}$.

$$dt = -\frac{\frac{L}{D}}{g\left(1-\frac{V^2}{V_c^2}\right)}dV \tag{4.31}$$

Yields the following expression for the flight time. This equation can be simplified by assuming that $V_F << V_c$ yielding $\frac{V_F}{V_c} \approx 0$.

$$t_{flight} = -\frac{1}{2}\frac{V_c}{g}\frac{L}{D}\ln\left(\frac{1+V_F/V_c}{1-V_F/V_c}\cdot\frac{1-V_E/V_c}{1+V_E/V_c}\right) \approx \frac{1}{2}\frac{V_c}{g}\frac{L}{D}\ln\left(\frac{1+V_E/V_c}{1-V_E/V_c}\right) \tag{4.32}$$

The **Maximum deceleration** is obtained by substituting the equilibrium glide conditions and the equilibrium glide angle in the equations of motion and differentiating this equation with respect to the velocity, yielding:

$$\frac{\overline{a}}{g} = \frac{D}{L}\left(1-x^2-\frac{2}{\beta R_e}\frac{1}{x^2}\right) \rightarrow \frac{d\overline{a}/g}{x} = \frac{D}{L}x\left(-2+\frac{4}{\beta R_e}\frac{1}{x^4}\right) = 0 \quad with \;\; x = \frac{V}{V_c} \;\; and \quad \overline{a} = -a \tag{4.33}$$

The exponential atmosphere is then used to determine the density and altitude at the point of maximum deceleration. This occurs at (on Earth):

$$\frac{V'}{V_c} = \left(\frac{2}{\beta R_e}\right)^{1/4} = 0.218 \quad and \quad \rho' = 0.639\frac{W/S}{C_L}10^{-6} \tag{4.34}$$

An error of about 10% on the maximum deceleration is introduced when the weight term is neglected in the equation of motion.

The **Maximum stagnation heat flux** is obtained by differentiating Chapman's equation. Substitution of the exponential atmosphere into Chapman's equation leads to:

$$q_c = c_1 \left(\frac{2W}{V_c^2 S C_L}\right)^{1-n} \frac{1}{R_N^n} V_c^m \left(\frac{V_c^2}{V^2} - 1\right)^{1-n} \left(\frac{V}{V_c}\right)^m \tag{4.35}$$

Rewriting the equation gives:

$$q_c = c_2 \left[\frac{1}{x^2} - 1\right]^{1-n} x^m \quad with \quad x = \frac{V}{V_c} \tag{4.36}$$

Differentiating the following equation gives the velocity where the maximum stagnation heat occurs.

$$\frac{dq_c}{dx} = 0 \rightarrow \frac{V''}{V_c} = \sqrt{\frac{m - 2(1-n)}{m}} \tag{4.37}$$

The **maximum heat flux** can be derived by substituting the density relation, yielding:

$$\frac{V'''}{V_c} = \frac{1}{\sqrt{3}} = 0.577 \tag{4.38}$$

The altitude difference with respect to the altitude for maximum deceleration will be:

$$\frac{\rho'''}{\rho'} = 2 \qquad \rightarrow \qquad h''' - h' = -\frac{\ln 2}{\beta} \tag{4.39}$$

4.3.1 Entry Corridor

The enclosure of all entry and descent trajectories that fulfill the constraints and guarantee a safe flight until landing is the so-called entry corridor, a three-dimensional spatial tube. An important feature of supercircular entries - to a lesser extent this is also true for near-circular entries - is that of the guidance accuracy acquired to survive entry. If the guidance error results in undershooting an intended trajectory too much, the vehicle will enter the atmosphere at an excessively steep ight-path angle. In that case the deceleration will be too much for either the vehicle or the ability to execute an evasive manoeuvre. If, on the other hand, the result of poor guidance is overshooting the intended trajectory, the atmospheric pull may not be strong enough to capture the vehicle, and it may disappear forever into space or enter a highly elliptic orbit prolonging dangerously the lapse time until the next return.

For initial re-entry trajectory design, usually three conditions are taken into account:

- the equilibrium-flight condition (equation 4.21). Rewrite the lift L to include the density and obtain the h-V curve.

$$\rho_{eq} = 2\frac{W/S}{C_L}\left(\frac{1}{V_{eq}^2} - \frac{1}{V_c^2}\right) \tag{4.40}$$

- the maximum allowable heat flux in the stagnation point. Rewrite the heat flux equation for the density to obtain the h-V curve.

$$\rho_{qc} = \rho_0 \left[R_N^n \frac{q_{c,max}}{c^*}\left(\frac{V_c}{V_{qc}}\right)^m\right]^{n-1} \tag{4.41}$$

- the maximum total deceleration, or g-load. Use the following equation and the definition of the lift and drag to get a C_L and C_D and rewrite for the density.

$$n_g = \frac{\sqrt{D^2 + L^2}}{mg_0} \tag{4.42}$$

$$\rho_g = 2n_{g,max}\frac{mg_0}{V_g^2 S\sqrt{C_D^2 + C_L^2}} \tag{4.43}$$

The density can be related with the height by the following equation:

$$h_1 = -H_s ln(\frac{\rho_1}{\rho_0}) \tag{4.44}$$

The equilibrium-glide condition - derived from $\frac{d\gamma}{dt}$ - should be seen as the theoretical ceiling under which no skipping flight will occur.

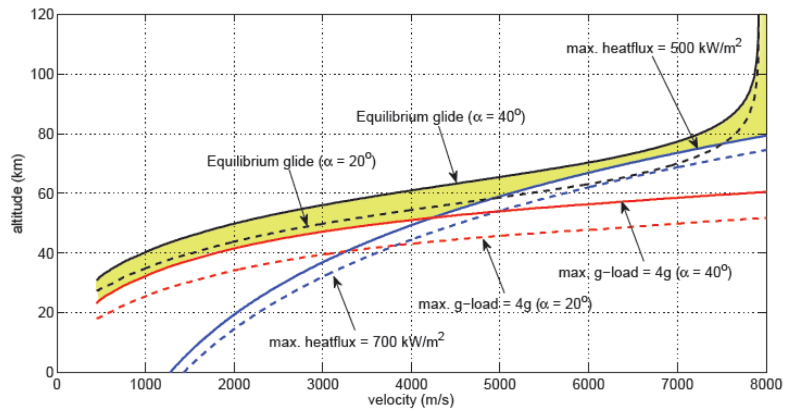

Figure 4.2: Entry corridor Horus

4.3.2 Footprint

A mapping of the maximum achievable longitudinal and lateral range boundaries is referred to as a footprint, which represents the region where recovery can be effected, or when the landing strip should be.

The footprint is obtained by systematically varying the bank angle, but keeping it constant for a single trajectory for a specified angle of attack profile.

An analytical approximation is derived by assuming a flight along the equator $\delta = 0°$. Figure 4.3 displays the range geometry for an arbitrary trajectory. Banking the vehicle with an angle σ, it starts moving away from the initial flight path, indicated by the angle Ψ. The following equation of motion can be derived from the figure:

$$mV\frac{d\psi}{dt} = Lsin\sigma \quad ; \quad mV^2\frac{d\psi}{ds} = Lsin\sigma \tag{4.45}$$

Reformulating the original equations of motion with the assumption that the the flight path angle γ is small, results in:

$$mV\frac{dV}{ds} \approx -D \tag{4.46}$$

$$mV^2\frac{d\gamma}{ds} \approx L\cos\sigma - mg\left(1 - \frac{V^2}{V_c^2}\right) \tag{4.47}$$

$$mV\frac{d\psi}{dt} = L\sin\sigma$$

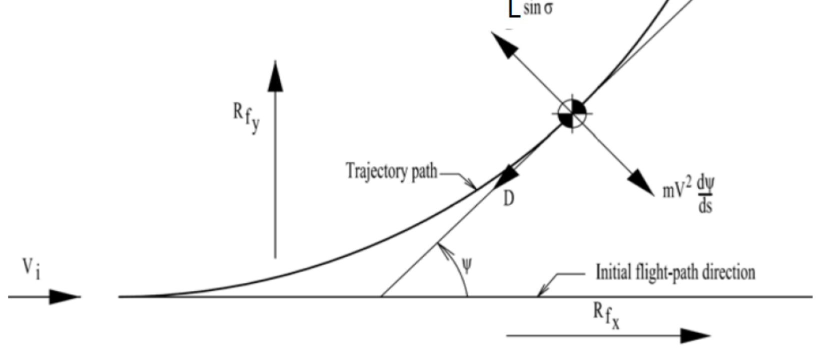

Figure 4.3: Trajectory analysis for determination of the footprint

Dividing the equations results in:

$$d\Psi = -\frac{L}{D}\sin\sigma\frac{dV}{V} \tag{4.48}$$

Integration of the equation gives the equation below, with Ψ_i and V_i being the heading and velocity at the beginning of the lateral manoeuvre. And introducing the parameter $x = \frac{V}{V_c}$, gives:

$$\Psi - \Psi_i = \frac{L}{D}\sin\sigma\ln\frac{x_i}{x} \tag{4.49}$$

For simplicity, Ψ_i can be taken as zero, as initial condition for $x = x_i$. The rewrittend equilibrium glide condition can be used to derive:

$$L\cos\sigma = mg\left(1 - \frac{V^2}{V_c^2}\right) = -mV\frac{L}{D}\frac{dV}{ds}\cos\sigma \quad \rightarrow \quad \frac{L}{D}\cos\sigma\frac{V}{\frac{V^2}{V_c^2} - 1}dV = gds \tag{4.50}$$

This equation can be rewritten with the substitution of x, as:

$$\frac{L}{D}\cos\sigma\frac{x}{x^2-1}dx = \frac{g}{V_c^2}ds = \frac{ds}{R} \approx \frac{ds}{R_e} \tag{4.51}$$

The incremental longitudinal and lateral displacements are:

$$dR_{f_x} = ds\cos\Psi \quad ; \quad dR_{f_y} = ds\sin\Psi \tag{4.52}$$

Combining the equations results in the following equation for the lateral displacement:

$$\frac{dR_{f_y}}{R_e} = \frac{L}{D}\cos\sigma\frac{x}{x^2-1}dx\sin\left(\frac{L}{D}\sin\sigma\ln\frac{x_i}{x}\right) \tag{4.53}$$

Assuming that the lateral deflection angle Ψ is small and the bank manoeuvre is initiated at the circular velocity ($x_i = 1$), the equation simplifies to:

$$\frac{dR_{f_y}}{R_e} = \left(\frac{L}{D}\right)^2\cos\sigma\sin\sigma\frac{x\ln x}{1-x^2}dx \tag{4.54}$$

Integrating with the limits $x_i = 1$ and $x = 0$, results in the final equation which is given below. The maximum lateral range is obtained with a bank angle of $\sigma = 45°$.

$$\frac{R_{f_y}}{R_e} = \frac{\pi^2}{24}\left(\frac{L}{D}\right)^2\cos\sigma\sin\sigma \qquad with \qquad \int_1^0\frac{x\ln x}{1-x^2}dx = \frac{\pi^2}{24} \tag{4.55}$$

4.4 Skipping Entry

Another option of deceleration in the atmosphere, applicable for entry velocities both greater and smaller than the circular velocity, is the skip trajectory. The vehicle dives into the atmosphere and is deflected back out of the atmosphere with a lower velocity. Outside the atmosphere, the vehicle describes a ballistic trajectory until it re-enters the atmosphere and so forth. A simple approximation of the equations of motion for a skipping entry are derived using the assumptions that the lift and drag forces are much larger than the weight. In this case, with $D >> W$ and $L >> W$, it follows that:

$$m\frac{dV}{dt} = -D \tag{4.56}$$

$$mV\frac{d\gamma}{dt} = L \tag{4.57}$$

Deviding the above equations directly yields:

$$\frac{1}{V}\frac{dV}{d\gamma} = -\frac{D}{L} \tag{4.58}$$

Integrating the above equation yields:

$$\frac{V}{V_E} = e^{-\frac{\gamma-\gamma_E}{L/D}} \tag{4.59}$$

Using the hydrostatic equation and the kinematic equation, the following result is obtained:

$$V\frac{d\gamma}{dp}\frac{dp}{dh}\frac{dh}{dt} = \frac{L}{m} \quad \rightarrow \quad cos(\gamma) - cos(\gamma_E) = \frac{p - p_E}{2\frac{W/S}{C_L}} \tag{4.60}$$

Using the exponential atmosphere, the fact that the entry pressure $p_E = 0$ and density are zero $\rho_E = 0$ and the fact that the flight path angle γ_p is zero at the lowest point p, results in:

$$\rho_p = \frac{4\beta(W/S)}{gC_L}sin^2\frac{\gamma_E}{2} \tag{4.61}$$

$$h_p = -\frac{1}{\beta}ln\left(\frac{4\beta(W/S)}{g\rho_0 C_L}sin^2\frac{\gamma_E}{2}\right) \tag{4.62}$$

$$\frac{V_p}{V_E} = e^{\frac{\gamma_E}{(L/D)}} \tag{4.63}$$

Using equation 4.60 , it can be deduced that the final climb angle is equal in magnitude to the entry angle $\gamma_F = -\gamma_E$. The final velocity at atmosphere exit is then:

$$\frac{V_F}{V_E} = e^{\frac{2\gamma_E}{(L/D)}} \tag{4.64}$$

A ballistic phase will follow resulting in a next skip trajectory. The entry angle will then be final angle of the previous skip. The entry velocity will then be the final velocity of the previous skip. Therefore the following equation will hold for n number of skip trajectories.

$$\frac{(V_F)_n}{(V_E)_1} = e^{n\frac{2\gamma_E}{(L/D)}} \tag{4.65}$$

In general the maximum thermal/mechanical loads occur in the following order:

$$q_{c,lam} \quad \rightarrow \quad q_{c,tur} \quad \rightarrow \quad dQ/dt \quad \rightarrow \quad a_{max} \tag{4.66}$$

Chapter 5

Planetary Entry and Descent

5.1 Parachutes

The need for parachutes stems basically from one or two requirements: To limit the impact speed to a range that is acceptable to the landing system (mechanical load constraint). To control the descent duration to maximize the scientific return in a given altitude range (communication constraint). Another requirement is to perform spin control, which can be a scientific constraint.

- Parachute

  A folding umbrella-shaped device of light fabric used especially for making a safe descent from an airplane.

- Pilot Parachute

  Small parachute which is attached to a deployment bag or the vent of a larger parachute, used to provide the force required to deploy this larger parachute.

- Drogue Parachute

  A parachute attached to the payload and used to provide stabilization or initial deceleration or both. Usually implies a larger parachute will be deployed later in the event sequence. Frequently used as the pilot parachute for the main parachute.

5.1.1 Key elements

The key elements of parachute systems are given in the figure below.

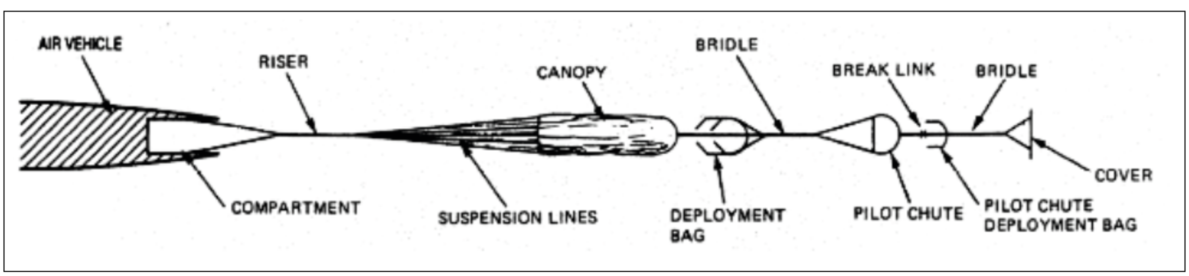

Figure 5.1: The key elements of a parachute system.

- Deployment Bag

  A textile container for a parachute from which the parachute deploys

- Riser

  A line connecting a parachute to its payload. May utilize a single or multi-point attachment scheme

- Bridle

  A means of providing a multi-point connection to a deployment bag or vehicle from a parachute or riser

- Canopy / Main chute

  The major drag producing element of the parachute

5.1.2 Operations

The parachute can be deployed using several deployment devices. A deployment device could be for example: A Drogue Gun, a Tractor Rocket or a Mortar.

After the deployment, the inflation process of the parachute will start. The maximum structural loads will occur almost always during this phase. To accurately predict the performance, the inflation time has te be known accurately.

Opening loads can be controlled by temporarily restricting canopy at the skirt - this is known as reefing. The Reefing line(s) are threaded through rings at the parachute skirt. The reefing line(s) length controll the degree of reefing and thus the drag area. By applying reefing, the descent will be more complex and possible failure modes are added. This need to be considered in the design. At the end of the reefing phase, a reefing cutter will release the parachut from its reefed state.

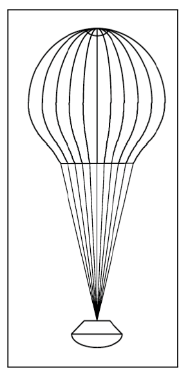

Figure 5.2: The key elements of a parachute system.

5.1.3 Terminal descent

The parachute and the entry vehicle will induce a drag force D_p and D_{EV} during the descent.

$$m\frac{dV}{dt} = -(D_p + D_{EV}) + mg \tag{5.1}$$

To determine the impact velocity it is assumed that the drag force is in equilibrium with the gravitational force.

$$\frac{dV}{dt} = 0 \tag{5.2}$$

Neglecting the drag of the entry vehicle compared to the parachut results in the following equation.

$$V_f^2 = \frac{2W}{\rho_0 S C_{D,para}} \tag{5.3}$$

5.2 Propulsive decelerators

Rockets

Chapter 6

Guidance, Navigation and Control

The Guidance, Navigation and Control system steers the vehicle such that the requirements are met with given constraints. The task of the guidance system is to generate steering commands, e.g., a commanded attitude or thrust level, taking a reference state, trajectory constraints and/or a final state into account. These data have to be provided by the navigation system, using sensor information and predefined theoretical models. The control system has to guarantee that the steering commands are carried out

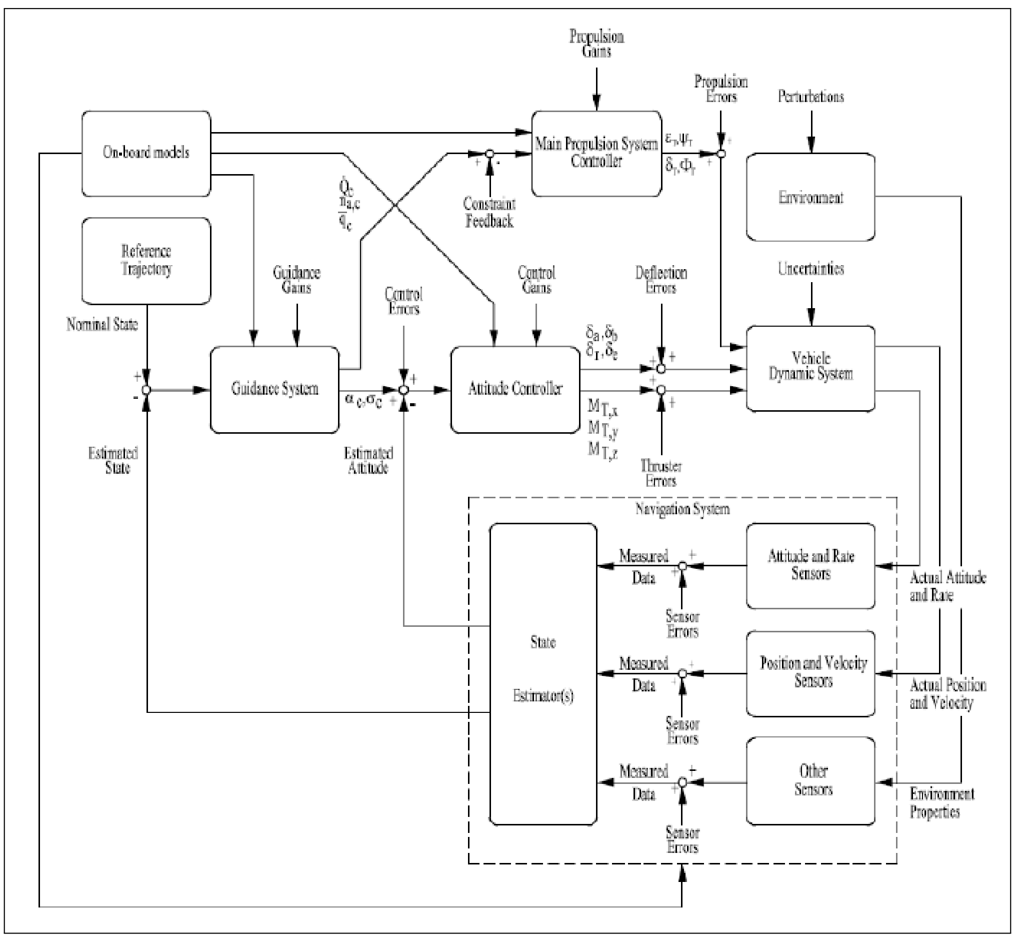

Figure 6.1: Schematic overview of a typical GNC system.

6.1 Control

6.1.1 PID Controller

A commonly used controller is the PID controller. This controller gives a control signal $u(t)$ which is dependent on the error, the derivative of the error and the integrated sum of the error. The control law is given in the following equation.

$$u(t) = K_p e(t) + K_I \int_0^t e(t) + K_d \frac{de(t)}{dt} \tag{6.1}$$

6.1.2 Optimal control

To be added

Chapter 7

Advanced descent and landing systems

Landing ellipse
The landing region will be described by a landing ellipse, which is determined such that with a probability of 3σ (or 99.7%) the vehicle will end up inside it. Why an ellipse and not a circle? The movement of the vehicle can be described down-range and cross-range. Down-range is in the direction of flight; cross-range is perpendicular to the direction of flight. As velocities are substantially larger down-range, small errors will have larger impacts. Therefore, the landing region is wider in down-range direction.

Several requirements can be set for the landing ellipse. One requirement is the maximum elevation of the landing site, because current decelerators can only decelerate the entry vehicle enough to land below a certain elevation level. Another constraint is that the landing site has to be inherently safe.

7.1 Safe landing sites

If a region is inherently safe, this describes regions, which are free of all possible surface features or conditions that could pose a hazard to the lander during descent and touch-down. Possible threats can be:

Slope The slope between two points describes how steep the terrain is in between these two points. The slope is usually not constant over an entire landing region but can vary. Usually, the slope is computed by fitting a mean-plane trough multiple neighbouring points, as this will exclude smaller outliers, such as rocks, from the slope evaluation.

Roughness Roughness describes the features that deviate from the mean plane. Simply speaking, roughness is created by small features lying on the surface, such as rocks. Roughness is equivalent to rocks and boulders lying on the surface. How much roughness a lander can handle, depends on its design. The ground clearance, a measure that indicates how much space there is in between the vehicles ̈bellyänd the ground, usually determines the maximum allowable size for roughness features.

Craters Craters do not necessarily pose a hazard. Whether or not this is the case, strongly depends on the mission itself, but also the age and size of the crater. On one hand, for a rover mission it might be very unfavourable to land inside a (smaller) crater, as the rover would be confined to the inside of the rover. On the other hand, younger craters might be very interesting geologically and could be a very interesting landing site.

Illumination As for craters, the hazardousness of certain illumination conditions depends on the mission itself. If a mission will receive its entire energy from solar power, it is of utmost importance that the landing region is illuminated. The same goes for landing systems that rely on visual imagery for certain descent systems, such as terrain-relative navigation, which we will discuss later. Last but not least, it is also important to consider that public outreach is important. If the landing site is in total darkness, no descent images could be broadcasted.

Landing in unsafe regions might even be very desirable, as crater rims, boulder fields and young craters are very interesting science locations. Therefore, there is a need to find systems that are capable of decreasing the risk of a landing failure due to surface hazards.

7.2 Advanced landing systems

Even tough many of the past missions to other Solar System bodies were successful, advancing current systems is still very important to push the current boundaries. In the previous section it was established that to increase the number of possible landing sites, and by that to increase the possible scientific return, two main points should be addressed:

1. The need to develop systems that can further decrease the landing ellipse size.
2. The need to develop a system that can decrease the risk posed by undetected surface hazards.

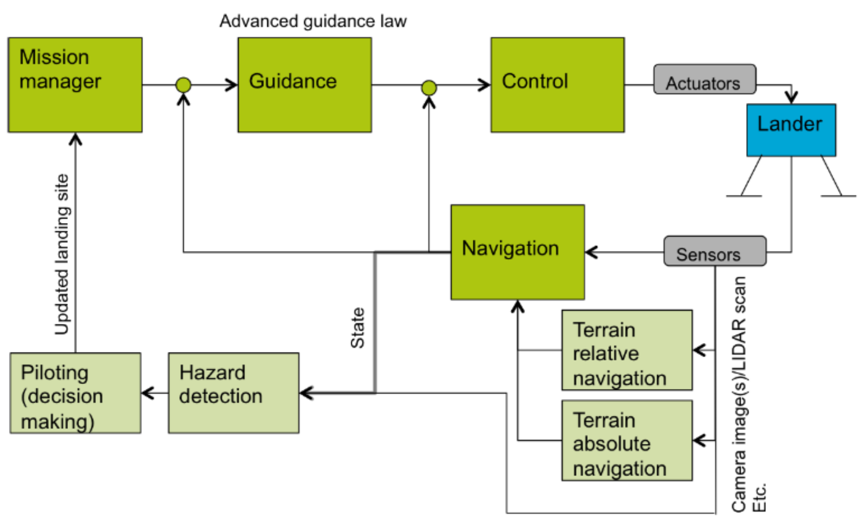

Figure 7.1: Guidance, Navigation and control system for Advanced landing systems

7.3 E-guidance

To perform precision landings and hazard avoidance, the guidance law has to be capable to land at a specified landing location independent of the initial conditions. One of the guidance laws that can achieve this is E-guidance, sometimes also called Apollo-guidance or quadratic guidance.

E-guidance is an explicit guidance method that solves the two-point boundary-value problem.It is a very simple method that can compute the trajectory in-flight.

The basic principle of E-guidance is to constrain the translational equations of motion to be a linear polynomial with time t. As such, the acceleration a of the vehicle within an arbitrary gravity fueld is defined as:

$$a(t) = a_c + g = c_1 + c_2 t_{go} \qquad t_{go} = t - t_0 \tag{7.1}$$

where a_c is the commanded acceleration and g the local gravitational vector. The coeficients c_1 and c_2 and the time-to-go t_{go} are the quantities that are to be determined such that the acceleration profile will steer the vehicle from the initial to the final conditions.

$$V - V_0 = \int_{t_0}^{t} a(\tau)d\tau = \int_{t_0}^{t} c_1 + c_2(\tau - t_0)d\tau = c_1 t_{go} + \frac{1}{2}c_2 t_{go}^2 \tag{7.2}$$

$$r - r_0 - V_0 t_{go} = \int_{t_0}^{t} (V - V_0)(\tau)d\tau = \int_{t_0}^{t} c_1(\tau - t_0) + c_2(\tau - t_0)^2 = \frac{1}{2}c_1 t_{go}^2 + \frac{1}{6}c_2 t_{go}^3 \tag{7.3}$$

Rewriting these equations in a matrix equation results in:

$$\begin{pmatrix} V - V_0 \\ r - r_0 \end{pmatrix} = \begin{bmatrix} t_{go} & \frac{1}{2}t_{go}^2 \\ \frac{1}{2}t_{go}^2 & \frac{1}{3}t_{go}^3 \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \tag{7.4}$$

Chapter 8

Equations to know by heart

Equation of state:

$$p = \rho RT \quad ; \quad \frac{d\rho}{dh} = -\beta\rho_0 e^{-\beta h} = -\rho\beta \tag{8.1}$$

Hydrostatic equation:

$$dp = -g\rho dh \tag{8.2}$$

Exponential Atmosphere:

$$\rho = \rho_0 e^{-\beta h} \tag{8.3}$$

Scale height:

$$H_s = \frac{1}{\beta} = \frac{RT}{g_0} \tag{8.4}$$

Equations of motion:

$$m\frac{dV}{dt} = -D - mg\sin\gamma \tag{8.5}$$

$$mV\frac{d\gamma}{dt} = L - mg\cos\gamma\left(1 - \frac{V^2}{V_c^2}\right) \quad ; \quad V_c = gR \ , \ V_{c,0} = g_0 R_e \tag{8.6}$$

$$\frac{dR}{dt} = \frac{dh}{dt} = V\sin\gamma \tag{8.7}$$

Ballistic parameter:

$$K = \frac{mg}{C_D S} \tag{8.8}$$