How to handle 3D problems

3D problems are rather tricky. You can't draw them well on a 2D sheet, which makes it a bit annoying. 3D problems can be very fun though. You just have to be able to imagine the problem in a good 3-dimensional way, however that is a quality some people have and others do not. But training can help.

1 Dissect forces (and of course naming them)

The first thing you often should do is name the forces and dissect them along the axes. But this is slightly more difficult than in 2D problems. There are 3 axes now, so how do you do this?

The easiest way is to make use of relations in sides of triangles. A very useful formula I often use is:

$$\frac{F_{r;x}}{l_{r;x}} = \frac{F_{r;y}}{l_{r;y}} = \frac{F_{r;z}}{l_{r;z}} = \frac{F_r}{l_r} \tag{1}$$

Looks great, but what are the meanings of those variables? Suppose there is a two-force member r with length l_r, and a force F_r in it. To know $F_{r;x}$ (the component of F_r pointing along the x-axis), you have to know $lr; x$, the length which the two-force member spans in the direction of the x-axis.

A small example might help. Suppose there is a rope (a two-force member) under a tension of $F_r = 100N$, and it goes from point (0,0,0) to point (2,3,6), thus it spans $2m$ in x-direction, $3m$ in y-direction and $6m$ in z-direction. So the total length of the rope is $l_r = \sqrt{2^2 + 3^2 + 6^2} = 7m$. Now applying the previous formula results in:

$$\frac{100}{7} = \frac{F_{r;x}}{2} = \frac{F_{r;y}}{3} = \frac{F_{r;z}}{4} \tag{2}$$

It is now quite easy to find the component of the two-force member in the direction of all axes. So dissecting forces along axes isn't so difficult really, not even in 3D or 4D, but 4D is something we won't be talking about. And when you've dissected forces, you can simply add them up in the way you're used to in 2D problems, as long as you've got the signs right.

2 But what about torques?

Now the next question arises, how do you use torques? That is quite tricky, until you get the hang of it. In 2D you rotate about a point. As you know a point is 1-dimensional. When we go to 3 dimension, we rotate about a 2-dimensional point, which is in fact a line. So we rotate about a line.

Suppose we have a line which we'll be rotating about. How do we do that? We first have to know which forces even cause a torque. In 2D there is a rule that says that a force of which the line of action goes through the point of rotation has an arm of 0. In 3D that's exactly the same, except for the fact that now the line of action of a force has to go through the line of rotation, for the force to have an arm of 0. There is just one "exception" to this rule (even though mathematicians don't find this an exception). A line which is parallel to the line of rotation has no arm too. (Mathematicians often say that two parallel lines cross each other in exactly one point infinitely far away. Don't ask them whether that point is to the left or to the right, because they don't know.) Anyway, forces with a line of action parallel to the line of rotation have no arm and thus cause no torque.

To calculate the torque caused by a force, you have to find the magnitude of the force and the arm of it. First make sure all the forces are dissected along axes perpendicular to the line of rotation (in most cases it's handy to choose a line of rotation parallel to 1 of the existing axes). The arm of the force is the minimal distance between the line of action of the force, and the line of rotation. This might sound a bit confusing, but if it does, just draw it on a paper and it'll make sense.

So, calculating torques isn't so difficult either. If you do calculate torques, do make sure that you choose a clever line of rotation. This is incredibly important, and might safe you a lot of work, so it might be well worth it to stare 5 minutes at a drawing looking for the right line of rotation, instead of immediately losing yourself in calculations leading to nothing.

3 Equilibrium equations in 3D

As you know (or should know), there are 3 equilibrium equations in 2D-situations. You that $F_x = 0$, $F_y = 0$ and $T = 0$. In 3D, there are more equilibrium equations. There are 6, to be exact. You can guess the first three: $F_x = 0$, $F_y = 0$ and $F_z = 0$.

Torques are something different though. Instead of rotating about a point, you rotate about a line. In 2D-situations, any point is just a point, but in 3D situations, lines have a direction (points don't). So instead of having 1 torque equation, you got 3: $T_x = 0$, $T_y = 0$ and $T_z = 0$. With M_x, we mean the torques about the x-axis or a line parallel to the x-axis. It is of course also possible to take the torque about a line not parallel to any of the axes. That will work out eventually (as long as you don't take torques about two lines that are parallel, since that will give the same equations), but it will make most statics problems rather complicated, so it is not advised.