Statics definitions

1 Definition list

- **Two-force member** - An object (like a rope, or a bar) on which two forces are acting. Both forces have the same magnitude, but opposite directions. These forces seem to cancel eachother, but they work on different positions, and therefore they can often not be ignored. When a two-force member is present, you often ought to determine whether tension or compression is present. Tension has a positive sign, compression negative. So when making calculations, always assume tension.
- **Zero-force member** - An object is a zero-force member, if there is no force acting on it. If, for any hinge in the object, there is only one two-force member which has a component along a certain axis through that point, then that member must be a zero-force member.
- **Truss structure** - A framework composed of members, joined at their ends to form a rigid structure. Even though in real life, the members of the truss structure might be connected by any connection imaginable, you always ought to assume the members are connected by pins/hinges. This means that the connection can not apply a moment on the truss members connected to it, which implies that the forces acting on the truss members are always in the exact direction of the truss members.
- **Rigid truss** - A truss which can not change shape (unless materials in it change shape, but it's assumed that that doesn't occur).
- **Flexible / Non-rigid truss** - The opposite of a rigid truss. A flexible truss can change shape when small forces are acting on the right positions.
- **Frame** - A frame is a structure where at least one of its individual members is a multi-force member (neither a two-force member or a zero-force member).
- **Kinematical determinancy** - An object is kinematically determinate if all free movements are prohibited (it can not move). This is in a 2D situations the case when the line of action of the forces caused by the solid connections are not parallel, nor cross eachother in the same point. Of course, in 2D, there must be at least 3 connections. In 3D there must be at least 6 connections for kinematical determinancy. When an object is not kinematically determinant, it's called kinematically indeterminant.
- **Statical determinancy** - If an object is kinematically determinant, we can check whether it's also statically determinant. An object is statically determinant when it is kinematically determinant, and all the connections of the object are needed to maintain that kinematic determinancy. Otherwise the object is statically indeterminant. So suppose r is the amount of reaction forces that the connections on the object cause. If the object is kinematically determinant, then, in a 2D situation, 3 forces are needed for kinematical determinancy. (In a 3D situation 6 forces are needed, in a 4D situation 10 forces, and in dD, $d * (d + 1)/2$ forces are needed.)
- **Degree of statical** indeterminancy - When an object is statically indeterminant, the degree of the statical indeterminancy is the amount of forces caused by connections that can be removed, until the object becomes statically determinant. So when there are r forces in a 2D situation, then the degree is $r - 3$.

2 Formulas about determinancy

Suppose m is the number of members in a truss structure, j is the number of joints/hinges (including constrained hinges), and r is the number of forces caused by constraints (note that in a 2-dimensional situation a fixed hinge has 2 support forces (forces in horizontal and vertical direction) and a clamped bar has 3 support forces (also a support moment)). The following equation can be derived:

$$m \geq 2j - 3 \tag{1}$$

If this equation is true, the truss is a rigid truss. If it is false (and thus $m < 2 * j - 3$), the truss is a flexible truss.

Let's define n as:

$$n = r + m - 2j \tag{2}$$

Now look at n:

- $n < 0 \Rightarrow$ The structure is kinematically indeterminate (non-rigid truss).
- $n = 0 \Rightarrow$ The structure is kinematically determinate and statically determinate (rigid truss).
- $n > 0 \Rightarrow$ The structure is kinematically determinate, but statically indeterminate (rigid truss).

In statics, you almost always deal with situations where $n = 0$. It can also be shown that $n = e - u$, where e is the number of equilibrium equations (for 2D situations this is 3), and u is the number of unknowns in all equations.