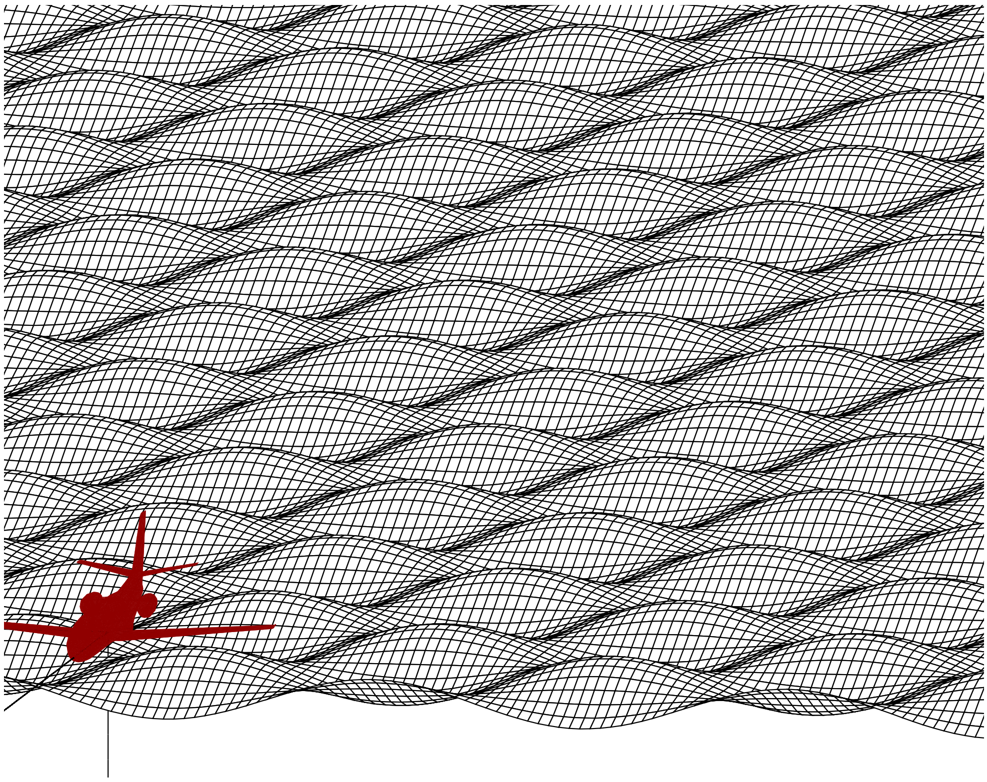


AIRCRAFT RESPONSES
TO
ATMOSPHERIC TURBULENCE

January 26th, 1999

Examination for the lectures LR4-4

This examination contains **9** problems.

1. Given the autocovariance function,

$$C_{\bar{x}\bar{x}}(\tau) = \frac{1}{2}\, cos(2\pi\tau)$$

of stochastic variable $\bar{x}$. Calculate the autospectrum $S_{\bar{x}\bar{x}}(\omega)$.

NOTE

Assume that,

$$cos(2\pi\tau) = \frac{e^{-j2\pi\tau} + e^{j2\pi\tau}}{2}$$

and,

$$\int_{-\infty}^{+\infty} e^{-j\omega\tau}\, d\tau = \delta(\omega)$$

2. Proof that,

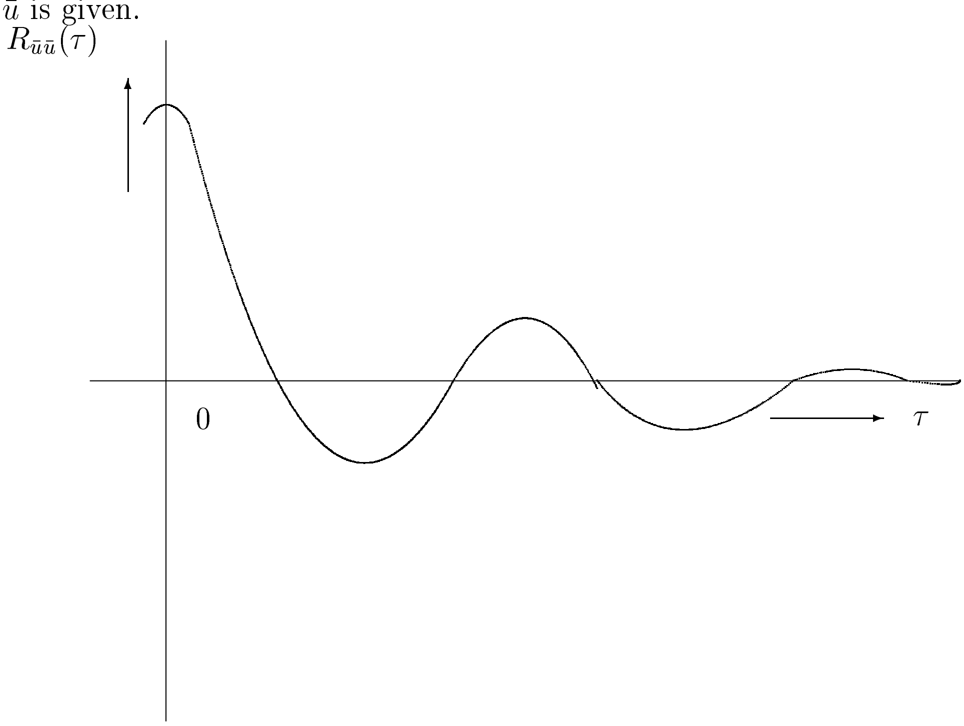
   (a) the variance of the "stochastic" variable $\bar{y} = c$ equals $\sigma_{\bar{y}}^2 = 0$

   (b) $\mu_{\bar{x}}^2 = E\{\bar{x}^2\} - \sigma_{\bar{x}}^2$

   (c) if $\bar{y} = b\,\bar{x} + c$ then $\sigma_{\bar{y}}^2 = b^2\sigma_{\bar{x}}^2$

   In the above mentioned questions b and c are constants.

3. In the following figure the product function $R_{\bar{u}\bar{u}}(\tau)$ of the stationairy stochastic process $\bar{u}$ is given.



What can be said about the properties of the stochastic variable $\bar{u}$?

(a) It is white noise.

(b) It is noise with a small bandwidth.

(c) It is white noise plus a sinus.

(d) It is a sinus

Explain your answer.

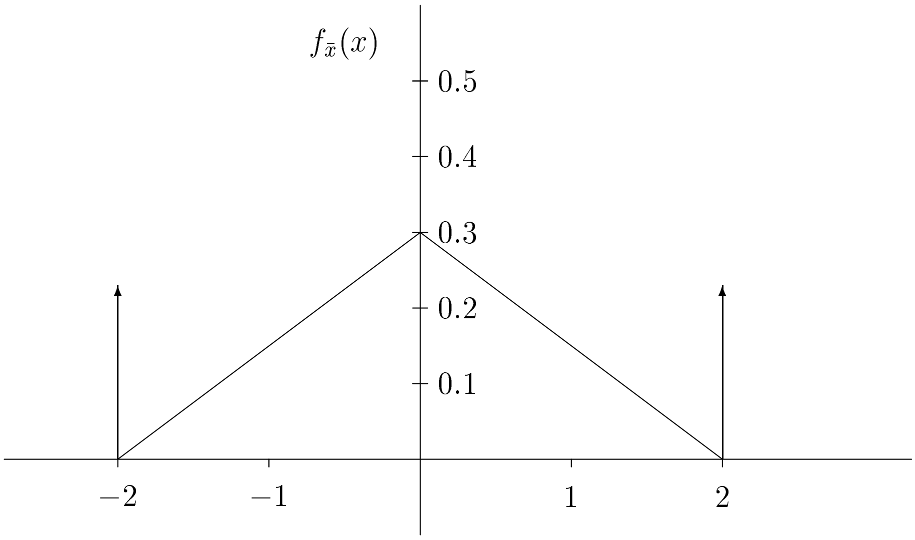
4. Given the probability density function of the stochastic variable $\bar{x}$ with parameter λ ($\lambda > 0$),

$$\begin{cases} f_{\bar{x}}(x) = \lambda\, e^{-\lambda\, x} & (x > 0) \\ f_{\bar{x}}(x) = 0 & (x < 0) \end{cases}$$

Calculate the probability distribution function $F_{\bar{x}}(x)$, and proof that the mean value $\mu_{\bar{x}}$ and the variance $\sigma^2_{\bar{x}}$ are equal to,

$$\mu_{\bar{x}} = \frac{1}{\lambda} \qquad \sigma^2_{\bar{x}} = \frac{1}{\lambda^2}$$

5. The random variable $\bar{x}$ has a probability density function $f_{\bar{x}}(x)$ as depicted in the following figure.



What is the probability of $P(x \geq -1)$?

(a) 0.125
(b) 0.275
(c) 0.725
(d) 0.750
(e) 0.875
(f) Not enough data available

<u>**Explain your answer.**</u>

6. Proof that the Fourier transform of the signal $x(t - t_0)$ equals,

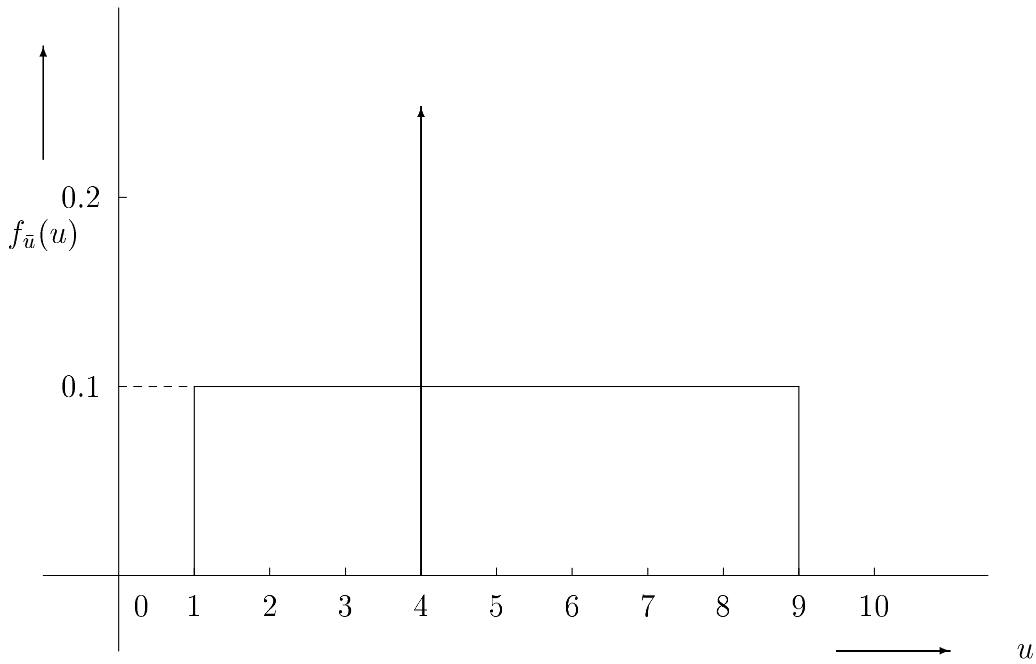
$$F\{x(t - t_0)\} = \{e^{-j\omega t_0}\} X(\omega)$$

7. Which of the following statements are true?

   (a) $R_{\bar{x}\bar{y}}(\tau) = R_{\bar{y}\bar{x}}(\tau)$
   (b) $C_{\bar{x}\bar{y}}(\tau) = C_{\bar{y}\bar{x}}(\tau)$
   (c) $K_{\bar{x}\bar{x}}(\tau) = K_{\bar{x}\bar{x}}(-\tau)$
   (d) $K_{\bar{x}\bar{x}}(0) = 1$
   (e) $S_{\bar{x}\bar{y}}(\omega) = S_{\bar{y}\bar{x}}(\omega)$
   (f) $S_{\bar{x}\bar{x}}(\omega) = S_{\bar{x}\bar{x}}(-\omega)$

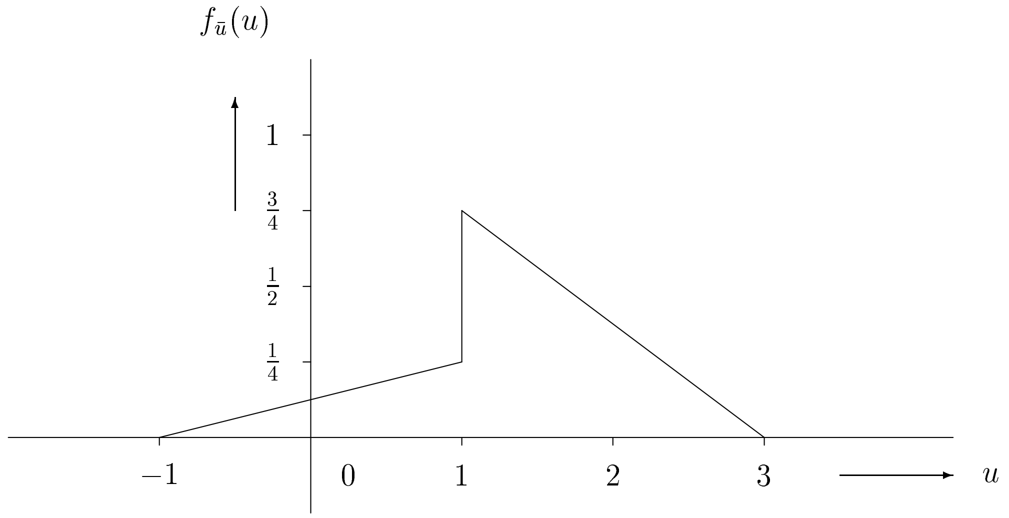
   Explain your answer.

8. (a) Given the probability density function $f_{\bar{u}}(u)$ of the stochastic process $\bar{u}$ as depicted in the following picture.



   Calculate the probability $P(u = 4)$.

(b) Given the probability density function $f_{\bar{u}}(u)$ of the stochastic process $\bar{u}$ as depicted in the following picture.



Calculate the probability $P(u = 1)$.

Explain your answer.

9. Proof that the periodogram $I_{\bar{y}\bar{y}}[k]$ of the signal $y[n] = a\ x[n] + b$ equals,

$$I_{\bar{y}\bar{y}}[k] = a^2 I_{\bar{x}\bar{x}}[k] + (2a\ Re\{X[k]\} + b)\, b\ \delta[k]$$

with,

$$I_{\bar{x}\bar{x}}[k] = X^*[k] \cdot X[k]$$

and $Re\{X[k]\}$ the real part of the Fourier transform of $x[n]$.

NOTE

The discrete Finite Fourier Transform (FFT) of a constant b equals,

$$FFT\{b\} = \left(\frac{1}{N} \sum_{n=0}^{N-1} b \cdot e^{-j \frac{2\pi k}{N} n} \right) = b \cdot \delta[k]$$

with $\delta[k]$ the Kronecker delta function. Use the result $FFT\{b\} = b \cdot \delta[k]$ in your proof. Remember that $\delta[k]$ equals zero for $k = 0$, and $\delta[k]$ equals one for $k = 1$.