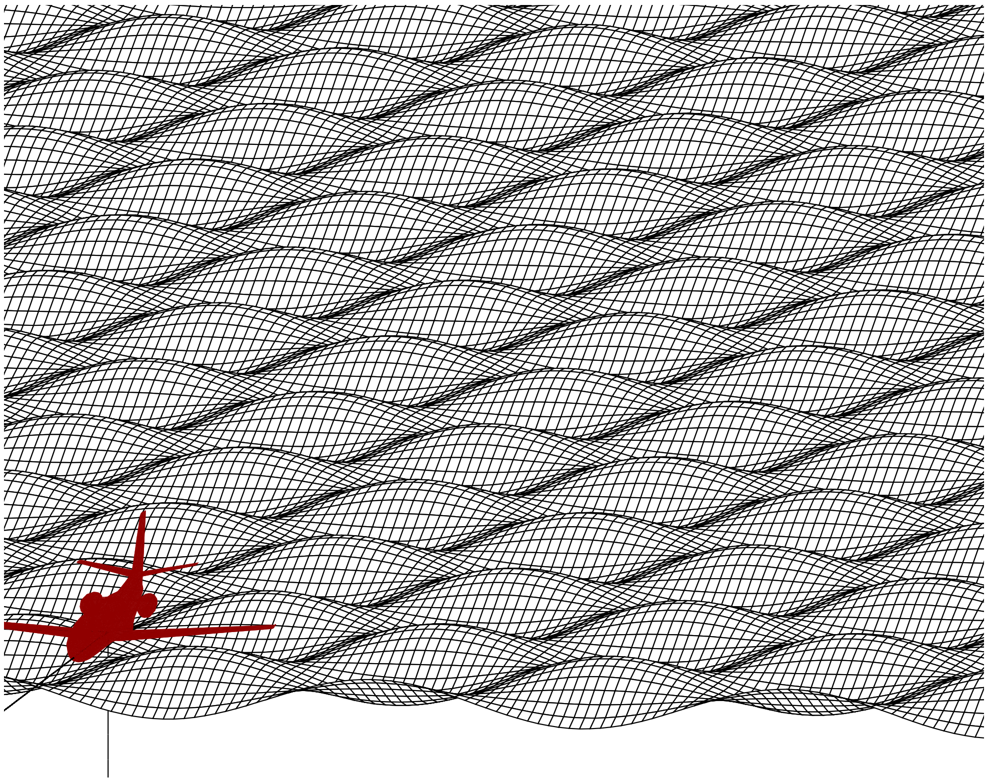

AIRCRAFT RESPONSES TO ATMOSPHERIC TURBULENCE

March 26th, 2001

Examination for the lectures AE4-304

This examination contains **9** problems.

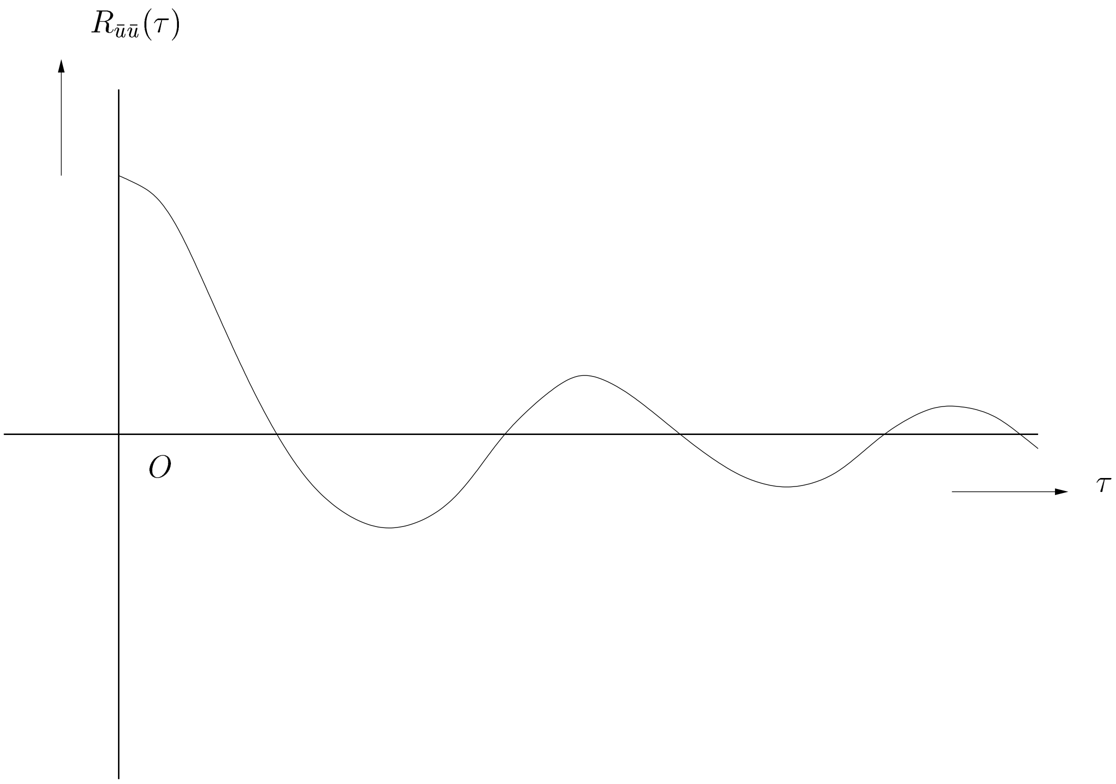

Figure 1: Product function $R_{\bar{u}\bar{u}}(\tau)$

1. In figure 1 the product function $R_{\bar{u}\bar{u}}(\tau)$ of the stationary stochastic process $\bar{u}$ is given. What can be said about the properties of the stochastic variable $\bar{u}$?

   (a) It is white noise.

   (b) It is noise with a small bandwidth.

   (c) It is white noise plus a sinus.

   (d) It is a sinus

   Explain your answer.

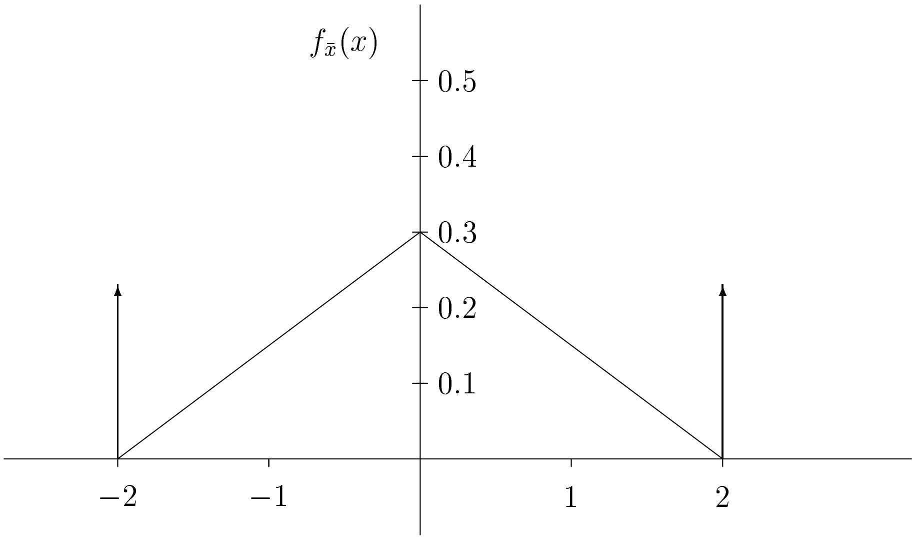

Figure 2: Probability density function $f_{\bar{x}}(x)$

2. The random variable $\bar{x}$ has a probability density function $f_{\bar{x}}(x)$ as depicted in figure 2.

   What is the probability of $P(x \geq -1)$?

   (a) 0.125
   (b) 0.275
   (c) 0.725
   (d) 0.750
   (e) 0.875
   (f) Not enough data available

   Explain your answer.

3. Proof that the Fourier transform of the signal $\dot{y}(t)$ $(= \frac{dy(t)}{dt})$ equals,

$$F\{\dot{y}(t)\} = j\omega\, Y(\omega)$$

   with $Y(\omega)$ the Fourier transform of $y(t)$.

4. Proof that the Fourier transform of the product of two functions,

$$F\{x(t)y(t)\} = \frac{1}{2\pi} X(\omega) \star Y(\omega)$$

Note: the symbol "$\star$" represents the convolution operator.

5. Proof that the periodogram $I_{\bar{y}\bar{y}}[k]$ of the signal $y[n] = a\, x[n] + b$ equals,

$$I_{\bar{y}\bar{y}}[k] = a^2 I_{\bar{x}\bar{x}}[k] + (2a\ Re\{X[k]\} + b \cdot N)\, b\, \delta[k]$$

with,

$$I_{\bar{x}\bar{x}}[k] = X^*[k] \cdot X[k]/N$$

and $Re\{X[k]\}$ the real part of the Fourier transform of $x[n]$.

Note: the Discrete Fourier Transform (FFT) of a constant b equals,

$$FFT\{b\} = \left(\sum_{n=0}^{N-1} b \cdot e^{-j\frac{2\pi k}{N} n} \right) = b \cdot N \cdot \delta[k]$$

with $\delta[k]$ the Kronecker delta function. Use the result $FFT\{b\} = b \cdot N \cdot \delta[k]$ in your proof. Remember that $\delta[k]$ equals 0 for $k \neq 0$ and $\delta[k]$ equals 1 for $k = 0$.

6. Make a qualitative sketch for the auto power spectral density functions of the following signals,

   (a) $y_1(t) = sin(\omega_0\ t)$

   (b) $y_2(t) = cos(\omega_1\ t) + 1$

   (c) $y_3(t) = sin(\omega_1\ t) + 1$

   Explain your answer.

7. Assume the stochastic process $\bar{x}$ which is defined as the wave-height in the North-Sea at a certain position. At a certain instant in time, t_1, this stochastic process has a certain probability density function $f_{\bar{x}}(x; t_1)$, with t_1 the time during the day when hardly any wind is present. At time instant t_2, representing a time in a period with strong winds, the probability density function is written as $f_{\bar{x}}(x; t_2)$.

   Make a qualitative sketch of the probability density functions $f_{\bar{x}}(x; t_1)$ and $f_{\bar{x}}(x; t_2)$.

   Note: assume that $\int_{-\infty}^{+\infty} f_{\bar{x}}(x; t)dx = 1\ \forall\ t$.

   Explain your answer.

8. Given the system in figure 3 of which the frequency response functions $H_1(\omega)$ and $H_2(\omega)$ are known. The input $U(\omega)$ and the noise on the output $N(\omega)$ are stochastic and their power spectral density functions are also known (both $U(\omega)$ and $N(\omega)$ are white noise).

   (a) Calculate the power spectral density function of the output: $S_{yy}(\omega)$.

   (b) Calculate the power spectral density function of the output for the case the input $U(\omega)$ and the noise $N(\omega)$ do not resemble white noise ánd may even be correlated.

9. The probability distribution function $F_{\bar{x}}(x)$ of a uniformly distributed stochastic variable $\bar{x}$ is written as (with $b > a$, see also figure 4),

$$
\begin{cases}
F_{\bar{x}}(x) = 0 & \text{for x} \leq \text{a,} \\
\qquad\quad = \frac{x-a}{b-a} & \text{for a} < \text{x} \leq \text{b,} \\
\qquad\quad = 1 & \text{for x} > \text{b}
\end{cases}
$$

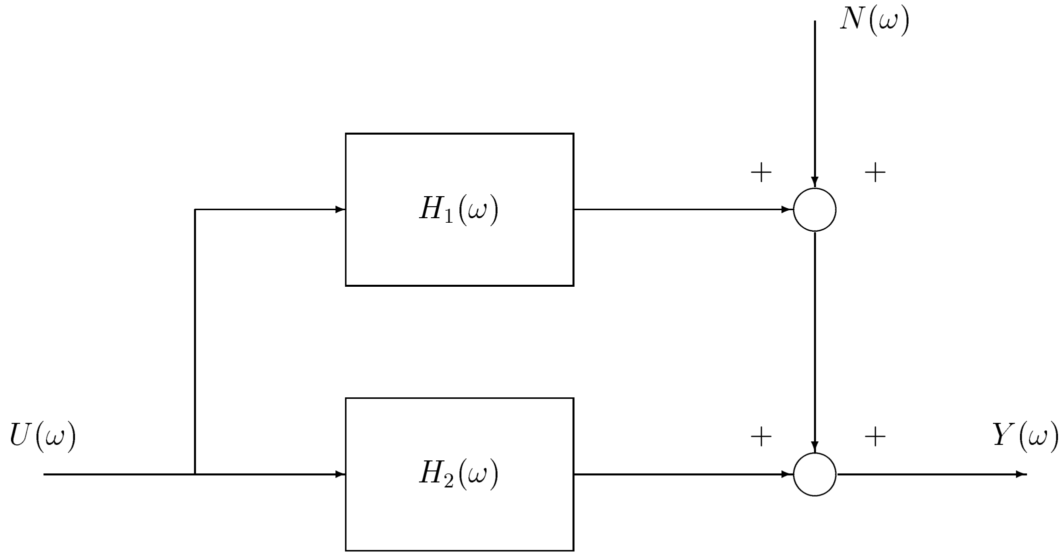

Figure 3: System description

(**Question 9 continued**)
Calculate the probability density function $f_{\bar{x}}(x)$ and proof that the stochastic variable's mean value and variance are respectively,

$$\mu_{\bar{x}} = \frac{a+b}{2} \quad \text{and} \quad \sigma_{\bar{x}}^2 = \frac{1}{12}\,(b-a)^2\ .$$

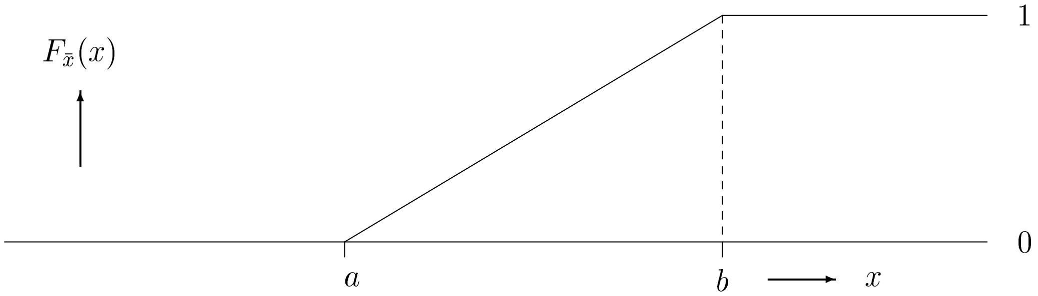

Figure 4: The probability distribution function $F_{\bar{x}}(x)$ for a uniformly distributed stochastic variable $\bar{x}$