FORMULAE AE4-304 (v2.0)
STOCHASTIC
AEROSPACE SYSTEMS

Delft University of Technology
Faculty of Aerospace Engineering
Control and Simulation Division

- The Fourier series

$$\tilde{\bar{x}}(t) = \sum_{k=0}^{N-1} \left[a_k \ \cos(k\omega_0 t) + b_k \ \sin(k\omega_0 t)\right]$$

with:

$$\begin{aligned}
\omega_0 &= \frac{2\pi}{T} \\
a_0 &= \frac{1}{T} \int_{t_0}^{t_0+T} \bar{x}(t)\ dt \\
a_k &= \frac{2}{T} \int_{t_0}^{t_0+T} \bar{x}(t)\ \cos(k\omega_0 t) dt \\
b_k &= \frac{2}{T} \int_{t_0}^{t_0+T} \bar{x}(t)\ \sin(k\omega_0 t) dt
\end{aligned}$$

- Complex form of the Fourier series

$$\begin{aligned}
\tilde{\bar{x}}(t) &= \sum_{k=-N+1}^{N-1} c_k \ e^{jk\omega_0 t} \\
c_k &= \frac{1}{T} \int_{t_0}^{t_0+T} \bar{x}(t)\ e^{-jk\omega_0 t}\ dt
\end{aligned}$$

- <u>The Fourier transform and its inverse</u>

$$\bar{X}(\omega) = \mathcal{F}\{\bar{x}(t)\} = \int\limits_{-\infty}^{+\infty} \bar{x}(t)\, e^{-j\omega t}\, dt$$

$$\bar{x}(t) = \mathcal{F}^{-1}\{\bar{X}(\omega)\} = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty} \bar{X}(\omega)\, e^{j\omega t}\, d\omega$$

- <u>The cross Power Spectral Density function in continuous time</u>

$$S_{\bar{x}\bar{y}}(\omega) = \mathcal{F}\{C_{\bar{x}\bar{y}}(\tau)\} = \int\limits_{-\infty}^{+\infty} C_{\bar{x}\bar{y}}(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$C_{\bar{x}\bar{y}}(\tau) = \mathcal{F}^{-1}\{S_{\bar{x}\bar{y}}(\omega)\} = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty} S_{\bar{x}\bar{y}}(\omega)\, e^{j\omega\tau}\, d\omega$$

- <u>The Discrete Fourier transform (DFT)</u>

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi kn}{N}}$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{+j\frac{2\pi kn}{N}}$$

- <u>The estimator for the discrete-time PSD: periodogram</u>

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi kn}{N}}$$

$$Y[k] = \sum_{n=0}^{N-1} y[n]\, e^{-j\frac{2\pi kn}{N}}$$

$$I_{xy}[k] = \frac{1}{N} X^*[k]\, Y[k]$$