# FORMULAE AE4304 (v5.0, Nov. 2016)
# STOCHASTIC AEROSPACE SYSTEMS

## TU Delft, Faculty of Aerospace Engineering

- The Trigonometric Fourier series

$$\tilde{\bar{x}}(t) = \sum_{k=0}^{N-1} \left[a_k \ \cos(k\omega_0 t) + b_k \ \sin(k\omega_0 t)\right]$$

where ($T$ is the period, and $\omega_0$ the fundamental frequency):

$$\begin{aligned}
\omega_0 &= \frac{2\pi}{T} \\
a_0 &= \frac{1}{T}\int_{t_0}^{t_0+T} \bar{x}(t)\,\mathrm{d}t \\
a_k &= \frac{2}{T}\int_{t_0}^{t_0+T} \bar{x}(t)\ \cos(k\omega_0 t)\mathrm{d}t \\
b_k &= \frac{2}{T}\int_{t_0}^{t_0+T} \bar{x}(t)\ \sin(k\omega_0 t)\mathrm{d}t
\end{aligned}$$

- Complex form of the Fourier series

$$\begin{aligned}
\tilde{\bar{x}}(t) &= \sum_{k=-N+1}^{N-1} c_k\, e^{jk\omega_0 t} \\
c_k &= \frac{1}{T}\int_{t_0}^{t_0+T} \bar{x}(t)\, e^{-jk\omega_0 t}\,\mathrm{d}t
\end{aligned}$$

- The Fourier transform and its inverse

$$\begin{aligned}
\bar{X}(\omega) &= \mathcal{F}\{\bar{x}(t)\} = \int_{-\infty}^{+\infty} \bar{x}(t)\, e^{-j\omega t}\,\mathrm{d}t \\
\bar{x}(t) &= \mathcal{F}^{-1}\{\bar{X}(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \bar{X}(\omega)\, e^{j\omega t}\,\mathrm{d}\omega
\end{aligned}$$

- The cross Power Spectral Density function in continuous time

Assume $\bar{x}(t)$ and $\bar{y}(t)$ to be zero mean:

$$S_{\bar{x}\bar{y}}(\omega) = \mathcal{F}\{C_{\bar{x}\bar{y}}(\tau)\} = \int_{-\infty}^{+\infty} C_{\bar{x}\bar{y}}(\tau)\, e^{-j\omega\tau}\, \mathrm{d}\tau$$

$$C_{\bar{x}\bar{y}}(\tau) = \mathcal{F}^{-1}\{S_{\bar{x}\bar{y}}(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{\bar{x}\bar{y}}(\omega)\, e^{j\omega\tau}\, \mathrm{d}\omega$$

- The Discrete Fourier transform (DFT) and its inverse

$$\bar{X}[k] = \mathrm{DFT}\{\bar{x}[n]\} = \sum_{n=0}^{N-1} \bar{x}[n]\, e^{-jk\frac{2\pi}{N}n}$$

$$\bar{x}[n] = \mathrm{iDFT}\{\bar{X}[k]\} = \frac{1}{N}\sum_{k=0}^{N-1} \bar{X}[k]\, e^{+jk\frac{2\pi}{N}n}$$

- The estimator for the discrete-time PSD: periodogram

When $\bar{X}[k] = \mathrm{DFT}\{\bar{x}[n]\}$ and $\bar{Y}[k] = \mathrm{DFT}\{\bar{y}[n]\}$:

$$\bar{I}_{\bar{x}\bar{y}}[k] = \frac{1}{N}\bar{X}^*[k]\, \bar{Y}[k]$$

- Some trigonometric relations

$$\begin{aligned}
\sin(u \pm v) &= \sin(u)\cos(v) \pm \cos(u)\sin(v) \\
\cos(u \pm v) &= \cos(u)\cos(v) \mp \sin(u)\sin(v) \\
\cos^2(u) &= \frac{1}{2}(1 + \cos(2u)) \\
\sin^2(u) &= \frac{1}{2}(1 - \cos(2u))
\end{aligned}$$