Atmospheric Flight Dynamics
Example Exam 1 – Problems

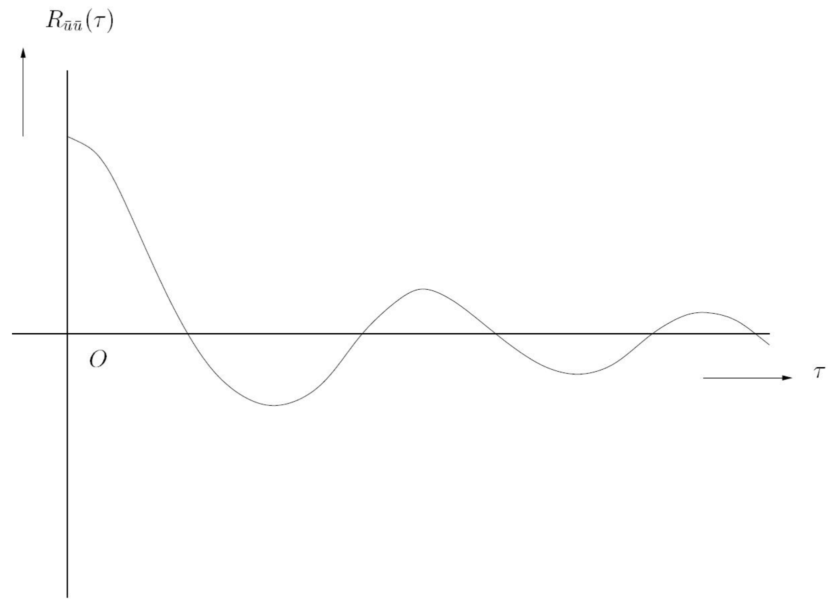

Figure 1: Product function $R_{\bar{u}\bar{u}}(\tau)$

1 Question

In figure 1 the product function $R_{\bar{u}\bar{u}}(\tau)$ of the stationary stochastic process $\bar{u}$ is given. What can be said about the properties of the stochastic variable $\bar{u}$?

(a) It is white noise.

(b) It is noise with a small bandwidth.

(c) It is white noise plus a sinus.

(d) It is a sinus.

2 Question

The random variable $\bar{x}$ has a probability density function $f_{\bar{x}}(x)$ as depicted in figure 2.
What is the probability of $P(\bar{x} \geq -1)$?

(a) 0.125

(b) 0.275

(c) 0.725

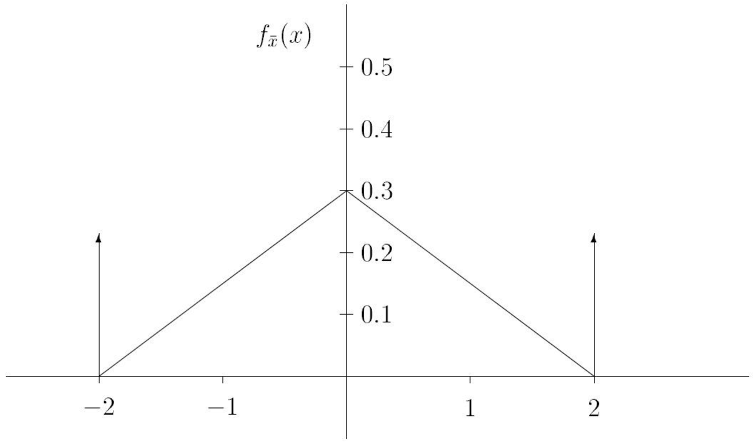

Figure 2: Probability density function $f_{\bar{x}}(x)$

(d) 0.750

(e) 0.875

(f) Not enough data available

3 Question

Proof that the Fourier transform of the signal $y(t)$ $\left(= \frac{dy(t)}{dt}\right)$ equals,

$$\mathcal{F}\left\{\dot{y}(t)\right\} = j\omega\, Y(\omega) \tag{3.1}$$

with $Y(\omega)$ the Fourier transform of $y(t)$.

4 Question

Proof that the Fourier transform of the product of two functions,

$$\mathcal{F}\left\{x(t)y(t)\right\} = \frac{1}{2\pi} X(\omega) * Y(\omega) \tag{4.1}$$

Note: the symbol "$*$" represents the convolution operator.

5 Question

Proof that the periodogram $I_{\bar{y}\bar{y}}[k]$ of the signal $y[n] = ax[n] + b$ equals,

$$I_{\bar{y}\bar{y}}[k] = a^2 I_{\bar{x}\bar{x}}[k] + (2a\mathrm{Re}\left\{X[k]\right\} + bN)b\delta[k] \tag{5.1}$$

with,

$$I_{\bar{x}\bar{x}}[k] = X^*[k]X[k]/N \tag{5.2}$$

and $\mathrm{Re}\left\{X[k]\right\}$ the real part of the Fourier transform of $x[n]$.

Note: the Discrete Fourier Transform (FFT) of a constant b equals,

$$FFT\{b\} = \left(\sum_{n=0}^{N-1} be^{-j\frac{2\pi k}{N}n}\right) = bN\delta[k] \tag{5.3}$$

with $\delta[k]$ the Kronecker delta function. Use the result $FFT\{b\} = bN\delta[k]$ in your proof. Remember that $\delta[k]$ equals 0 for $k \neq 0$ and $\delta[k]$ equals 1 for $k = 0$.

6 Question

Make a qualitative sketch for the auto power spectral density functions of the following signals,

(a) $y_1(t) = \sin(\omega_0 t)$

(b) $y_2(t) = \cos(\omega_1 t) + 1$

(c) $y_3(t) = \sin(\omega_1 t) + 1$

7 Question

Assume the stochastic process $\bar{x}$ which is defined as the wave-height in the North-Sea at a certain position. At a certain instant in time t_1, this stochastic process has a certain probability density function $f_{\bar{x}}(x; t_1)$, with t_1 the time during the day when hardly any wind is present. At time instant t_2, representing a time in a period with strong winds, the probability density function is written as $f_{\bar{x}}(x; t_2)$.

Make a qualitative sketch of the probability density functions $f_{\bar{x}}(x; t_1)$ and $f_{\bar{x}}(x; t_2)$.

Note: assume that $\int_{-\infty}^{+\infty} f_{\bar{x}}(x; t)\, dx = 1 \forall t$.

8 Question

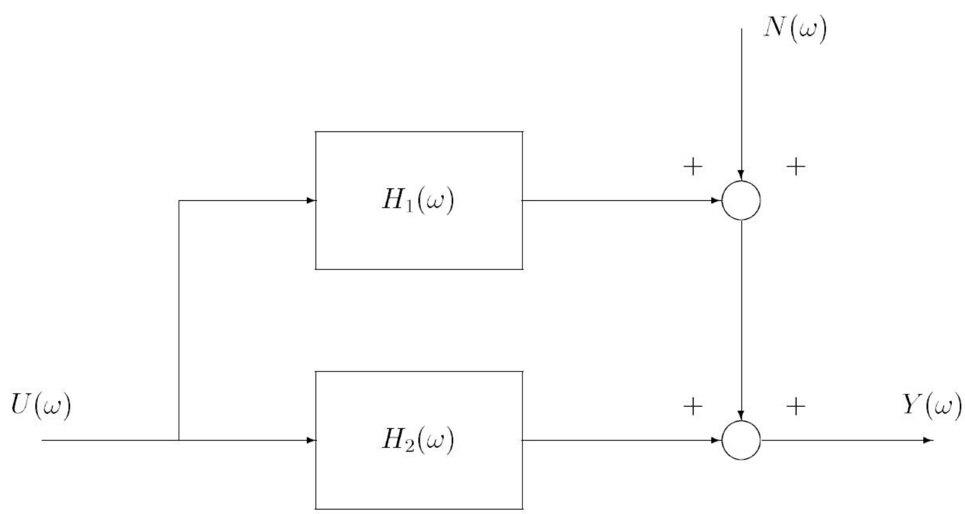

Figure 3: System description

Given the system in figure 3 of which the frequency response functions $H_1(\omega)$ and $H_2(\omega)$ are known. The input $U(\omega)$ and the noise on the output $N(\omega)$ are stochastic and their power spectral density functions are also known (both $U(\omega)$ and $N(\omega)$ are white noise).

(a) Calculate the power spectral density function of the output $S_{yy}(\omega)$.

(b) Calculate the power spectral density function of the output for the case the input $U(\omega)$ and the noise $N(\omega)$ do not resemble white noise ánd may even be correlated.

9 Question

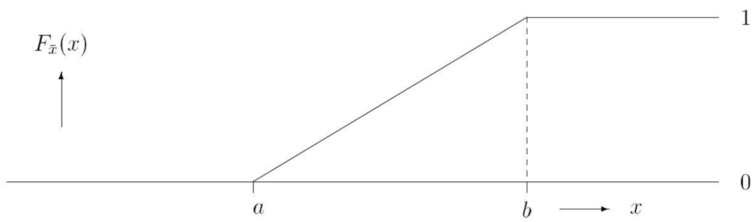

Figure 4: The probability distribution function $F_{\bar{x}}(x)$ for a uniformly distributed stochastic variable $\bar{x}$

The probability distribution function $F_{\bar{x}}(x)$ of a uniformly distributed stochastic variable $\bar{x}$ is written as (with $b > a$, see also figure 4),

$$F_{\bar{x}}(x) = \begin{cases} 0 & \text{for } x \leq a, \\ \frac{x-a}{b-a} & \text{for } a < x \leq b, \\ 1 & \text{for } x > b \end{cases} \tag{9.1}$$

Calculate the probability density function $f_{\bar{x}}(x)$ and proof that the stochastic variable's mean value and variance are respectively,

$$\mu_x = \frac{a+b}{2} \quad \text{and} \quad \sigma_x^2 \frac{1}{12}(b-a)^2. \tag{9.2}$$