

Delft University of Technology DEPARTMENT OF AEROSPACE ENGINEERING	
Course: Physics I (AE1-104)	Course year: 1
Date: 17-01-2011	Time: 14:00-17:00
Student name and initials (capital letters):.....	
Student number: .....	
You have attended the lectures given in (cross as appropriate) <input type="checkbox"/> Dutch <input type="checkbox"/> English	
<p><u>Instructions</u></p> <p>Write your answers using the blank space on the page where the problem is given.</p> <p>Or use the back of the page, if necessary.</p> <p>Deliver <b>ONLY</b> the present booklet at the end of the exam. (do not include scrap paper or any other sheet)</p>	

**Problem 1: (10 points)**

True or false exercise (mark the appropriate box after the statement)

Statement	True	False
1 – A Carnot heat engine is an isolated system		X
2 – When compressing a gas adiabatically its energy increases	X	
3 – A modern diesel engine can be more efficient than a Carnot heat engine		X
4 – No device can steadily produce work out of a single temperature reservoir	X	
5 – Adiabatic compression of a gas will increase its entropy		X
6 – An object that absorbs no energy from the incident radiation is termed a <i>blackbody</i>		X
7 – Heating superheated vapor will increase its quality		X
8 – Air at standard conditions can be considered a <i>pure substance</i>	X	
9 – Convective heat transfer in a boiling fluid is more efficient than in a gas flow	X	
10 – Enthalpy and internal energy of an ideal gas have the same units	X	
11 – When you mix together hot and cold water entropy is produced	X	
12 – The kinetic energy is the most important term in the analysis of gas turbines		X

## Problem 2 (15 points)

Electric power is to be generated by installing a hydraulic turbine-generator at a site 70 m below the free surface of a large water reservoir that can supply water at a rate of 1500 kg/s steadily. If the mechanical power output of the turbine is 800 kW and the electric power generation is 750 kW, determine the turbine efficiency and the combined turbine-generator efficiency of this plant. Neglect losses in the pipes.

**Solution (2-71)** A hydraulic turbine-generator is generating electricity from the water of a large reservoir. The combined turbine-generator efficiency and the turbine efficiency are to be determined.

**Assumptions 1** The elevation of the reservoir remains constant. **2** The mechanical energy of water at the turbine exit is negligible.

**Analysis** We take the free surface of the reservoir to be point 1 and the turbine exit to be point 2. We also take the turbine exit as the reference level ( $z_2 = 0$ ), and thus the potential energy at points 1 and 2 are  $pe_1 = gz_1$  and  $pe_2 = 0$ . The flow energy  $P/\rho$  at both points is zero since both 1 and 2 are open to the atmosphere ( $P_1 = P_2 = P_{\text{atm}}$ ). Further, the kinetic energy at both points is zero ( $ke_1 = ke_2 = 0$ ) since the water at point 1 is essentially motionless, and the kinetic energy of water at turbine exit is assumed to be negligible. The potential energy of water at point 1 is

$$pe_1 = gz_1 = (9.81 \text{ m/s}^2)(70 \text{ m}) \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 0.687 \text{ kJ/kg}$$

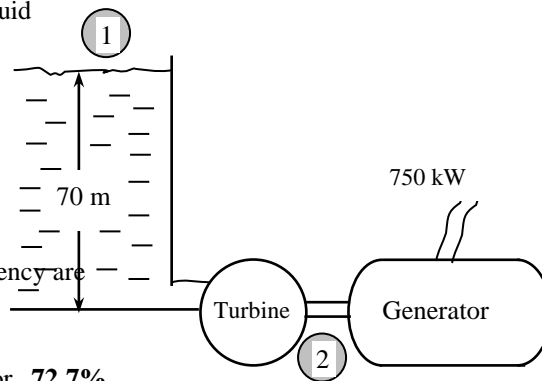
Then the rate at which the mechanical energy of the fluid is supplied to the turbine become

$$\begin{aligned} |\Delta \dot{E}_{\text{mech,fluid}}| &= \dot{m}(e_{\text{mech,in}} - e_{\text{mech,out}}) = \dot{m}(pe_1 - 0) \\ &= \dot{m}pe_1 \\ &= (1500 \text{ kg/s})(0.687 \text{ kJ/kg}) \\ &= 1031 \text{ kW} \end{aligned}$$

The combined turbine-generator and the turbine efficiency are determined from their definitions,

$$\eta_{\text{turbine-gen}} = \frac{\dot{W}_{\text{elect,out}}}{|\Delta \dot{E}_{\text{mech,fluid}}|} = \frac{750 \text{ kW}}{1031 \text{ kW}} = 0.727 \text{ or } 72.7\%$$

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft,out}}}{|\Delta \dot{E}_{\text{mech,fluid}}|} = \frac{800 \text{ kW}}{1031 \text{ kW}} = 0.776 \text{ or } 77.6\%$$



Therefore, the reservoir supplies 1031 kW of mechanical energy to the turbine, which converts 800 kW of it to shaft work that drives the generator, which generates 750 kW of electric power.

**Discussion** This problem can also be solved by taking point 1 to be at the turbine inlet, and using flow energy instead of potential energy. It would give the same result since the flow energy at the turbine inlet is equal to the potential energy at the free surface of the reservoir.

### Problem 3 (15 points)

A 1-m<sup>3</sup> tank containing air at 25°C and 500 kPa is connected through a valve to another tank containing 5 kg of air at 35°C and 200 kPa. Now the valve is opened, and the entire system is allowed to reach thermal equilibrium with the surroundings, which are at 20°C. Determine the volume of the second tank and the final equilibrium pressure of air.

**Solution (3-80)** Two rigid tanks connected by a valve to each other contain air at specified conditions. The volume of the second tank and the final equilibrium pressure when the valve is opened are to be determined.

**Assumptions** At specified conditions, air behaves as an ideal gas.

**Properties** The gas constant of air is  $R = 0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$  (Table A-1).

**Analysis** Let's call the first and the second tanks A and B. Treating air as an ideal gas, the volume of the second tank and the mass of air in the first tank are determined to be

$$V_B = \left( \frac{m_B R T_B}{P_B} \right) = \frac{(5 \text{ kg})(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(308 \text{ K})}{200 \text{ kPa}} = 2.21 \text{ m}^3$$

$$m_A = \left( \frac{P_A V_A}{R T_A} \right) = \frac{(500 \text{ kPa})(1.0 \text{ m}^3)}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298 \text{ K})} = 5.846 \text{ kg}$$

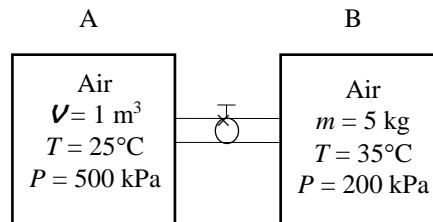
Thus,

$$V = V_A + V_B = 1.0 + 2.21 = 3.21 \text{ m}^3$$

$$m = m_A + m_B = 5.846 + 5.0 = 10.846 \text{ kg}$$

Then the final equilibrium pressure becomes

$$P_2 = \frac{m R T_2}{V} = \frac{(10.846 \text{ kg})(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293 \text{ K})}{3.21 \text{ m}^3} = 284.1 \text{ kPa}$$



#### Problem 4 (15 points)

A piston–cylinder device contains 0.15 kg of air initially at 2 MPa and 350°C. The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure, and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the net work of the cycle.

**Solution (4-25)** A piston-cylinder device contains air gas at a specified state. The air undergoes a cycle with three processes. The boundary work for each process and the net work of the cycle are to be determined.

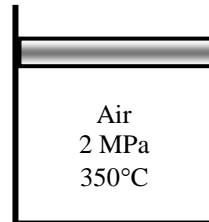
**Properties** The properties of air are  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ ,  $k = 1.4$  (Table A-2a).

**Analysis** For the isothermal expansion process:

$$v_1 = \frac{mRT}{P_1} = \frac{(0.15 \text{ kg})(0.287 \text{ kJ/kg}\cdot\text{K})(350 + 273 \text{ K})}{(2000 \text{ kPa})} = 0.01341 \text{ m}^3$$

$$v_2 = \frac{mRT}{P_2} = \frac{(0.15 \text{ kg})(0.287 \text{ kJ/kg}\cdot\text{K})(350 + 273 \text{ K})}{(500 \text{ kPa})} = 0.05364 \text{ m}^3$$

$$W_{b,1-2} = P_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = (2000 \text{ kPa})(0.01341 \text{ m}^3) \ln\left(\frac{0.05364 \text{ m}^3}{0.01341 \text{ m}^3}\right) = \mathbf{37.18 \text{ kJ}}$$



For the polytropic compression process:

$$P_2 v_2^n = P_3 v_3^n \longrightarrow (500 \text{ kPa})(0.05364 \text{ m}^3)^{1.2} = (2000 \text{ kPa})v_3^{1.2} \longrightarrow v_3 = 0.01690 \text{ m}^3$$

$$W_{b,2-3} = \frac{P_3 v_3 - P_2 v_2}{1-n} = \frac{(2000 \text{ kPa})(0.01690 \text{ m}^3) - (500 \text{ kPa})(0.05364 \text{ m}^3)}{1-1.2} = \mathbf{-34.86 \text{ kJ}}$$

For the constant pressure compression process:

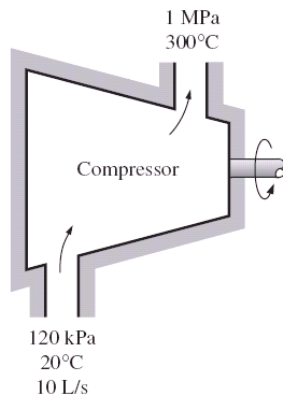
$$W_{b,3-1} = P_3 (v_1 - v_3) = (2000 \text{ kPa})(0.01341 - 0.01690) \text{ m}^3 = \mathbf{-6.97 \text{ kJ}}$$

The net work for the cycle is the sum of the works for each process

$$W_{\text{net}} = W_{b,1-2} + W_{b,2-3} + W_{b,3-1} = 37.18 + (-34.86) + (-6.97) = \mathbf{-4.65 \text{ kJ}}$$

### Problem 5 (15 points)

An adiabatic air compressor compresses 10 L/s of air at 120 kPa and 20°C to 1000 kPa and 300°C. Determine (a) the work required by the compressor, in kJ/kg, and (b) the power required to drive the air compressor, in kW.



**Solution (5-51)** Air is compressed at a rate of 10 L/s by a compressor. The work required per unit mass and the power required are to be determined.

**Assumptions 1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

**Properties** The constant pressure specific heat of air at the average temperature of  $(20+300)/2=160^\circ\text{C}=433\text{ K}$  is  $c_p = 1.018\text{ kJ/kg}\cdot\text{K}$  (Table A-2b). The gas constant of air is  $R = 0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$  (Table A-1).

**Analysis** (a) There is only one inlet and one exit, and thus  $\dot{m}_1 = \dot{m}_2 = \dot{m}$ . We take the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\dot{\Delta E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} \stackrel{\approx 0 \text{ (steady)}}{=} 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1)$$

Thus,

$$w_{\text{in}} = c_p(T_2 - T_1) = (1.018\text{ kJ/kg}\cdot\text{K})(300 - 20)\text{K} = \mathbf{285.0\text{ kJ/kg}}$$

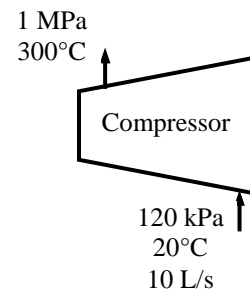
(b) The specific volume of air at the inlet and the mass flow rate are

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20 + 273\text{ K})}{120\text{ kPa}} = 0.7008\text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{0.010\text{ m}^3/\text{s}}{0.7008\text{ m}^3/\text{kg}} = 0.01427\text{ kg/s}$$

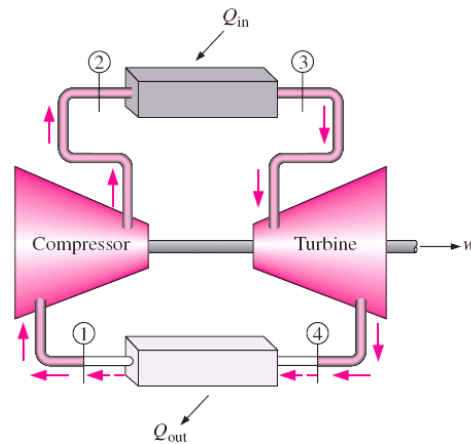
Then the power input is determined from the energy balance equation to be

$$\dot{W}_{\text{in}} = \dot{m}c_p(T_2 - T_1) = (0.01427\text{ kg/s})(1.018\text{ kJ/kg}\cdot\text{K})(300 - 20)\text{K} = \mathbf{4.068\text{ kW}}$$



### Problem 6 (15 points)

A simple ideal Brayton cycle operates with air with minimum and maximum temperatures of 27°C and 727°C. It is designed so that the maximum cycle pressure is 2000 kPa and the minimum cycle pressure is 100 kPa. Determine the net work produced per unit mass of air each time this cycle is executed and the cycle's thermal efficiency. Use constant specific heats at room temperature.



**9-94** A simple ideal Brayton cycle with air as the working fluid operates between the specified temperature and pressure limits. The net work and the thermal efficiency are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 The air-standard assumptions are applicable. 3 Kinetic and potential energy changes are negligible. 4 Air is an ideal gas with constant specific heats.

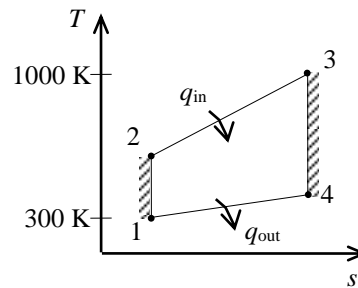
**Properties** The properties of air at room temperature are  $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$  and  $k = 1.4$  (Table A-2a).

**Analysis** Using the isentropic relations for an ideal gas,

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{(k-1)/k} = (300 \text{ K}) \left( \frac{2000 \text{ kPa}}{100 \text{ kPa}} \right)^{0.4/1.4} = 706.1 \text{ K}$$

Similarly,

$$T_4 = T_3 \left( \frac{P_4}{P_3} \right)^{(k-1)/k} = (1000 \text{ K}) \left( \frac{100 \text{ kPa}}{2000 \text{ kPa}} \right)^{0.4/1.4} = 424.9 \text{ K}$$



Applying the first law to the constant-pressure heat addition process 2-3 produces

$$q_{\text{in}} = h_3 - h_2 = c_p (T_3 - T_2) = (1.005 \text{ kJ/kg}\cdot\text{K})(1000 - 706.1) \text{ K} = 295.4 \text{ kJ/kg}$$

Similarly,

$$q_{\text{out}} = h_4 - h_1 = c_p (T_4 - T_1) = (1.005 \text{ kJ/kg}\cdot\text{K})(424.9 - 300) \text{ K} = 125.5 \text{ kJ/kg}$$

The net work production is then

$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 295.4 - 125.5 = \mathbf{169.9 \text{ kJ/kg}}$$

and the thermal efficiency of this cycle is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{169.9 \text{ kJ/kg}}{295.4 \text{ kJ/kg}} = \mathbf{0.575}$$

#### APPENDIX

Universal gas constant:  $R = 8.314 \text{ kJ/kmol}\cdot\text{K}$ ,

Gas constant for air:  $R_{\text{air}} = 287 \text{ kJ/kg}\cdot\text{K}$

Constant pressure specific heat for air:  $c_p = 1008 \text{ kJ/kg}\cdot\text{K}$

Ratio of specific heats for air:  $k = 1.4$

