

AE1104 Physics 1

List of equations

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Chapter 1

Introduction and basic concepts

Newton's second law

$$F = M \cdot a \quad (N)$$

Weight

$$W = m \cdot g \quad (N)$$

$$1J = 1N \cdot m$$

Density

$$\rho = \frac{m}{V} \quad \left(\frac{kg}{m^3} \right)$$

Specific volume

$$v = \frac{V}{m} = \frac{1}{\rho}$$

Specific weight

$$\gamma_s = \rho \cdot g \quad \left(\frac{N}{m^3} \right)$$

Kelvin to Celcius

$$T(K) = T(^{\circ}C) + 273.15 \rightarrow \Delta T(K) = \Delta T(^{\circ}C)$$

Rankine to Fahrenheit

$$T(R) = T(^{\circ}F) + 459.67 \rightarrow \Delta T(R) = \Delta T(^{\circ}F)$$

$$(R) = 1.8T(K)$$

$$T(^{\circ}F) = 1.8T(^{\circ}C) + 32$$

$$1 Pa = 1 \frac{N}{m^2}$$

$$1 bar = 10^5 Pa = 0.1 MPa = 100 kPa$$

Absolute, gage and vacuum pressure

$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$

The pressure at depth h from the free surface is

$$P = P_{atm} + \rho gh \quad \text{or} \quad P_{gage} = \rho gh$$

Relation for the variation of pressure with elevation

$$\frac{dP}{dz} = -\rho g$$
$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g dz$$

The atmospheric pressure is measured by a barometer and is given by

$$P_{atm} = \rho gh$$

Chapter 2

Energy, Energy Transfer and General Energy Analysis

The total energy of a system on a unit mass basis

$$e = \frac{E}{m} \quad \left(\frac{kJ}{kg} \right)$$

Kinetic Energy

$$KE = m \frac{V^2}{2} \quad (kJ)$$

Kinetic Energy on a unit mass basis

$$ke = \frac{V^2}{2} \quad \left(\frac{kJ}{kg} \right)$$

Potential Energy

$$PE = mgz \quad (kJ)$$

Potential Energy on a unit mass basis

$$pe = gz \quad \left(\frac{kJ}{kg} \right)$$

Total Energy of a system

$$E = U + KE + PE = U + m \frac{V^2}{2} + mgz$$

Total Energy of a system on a unit mass basis

$$e = u + ke + pe = u + \frac{V^2}{2} + gz$$

Mass flow rate

$$\dot{m} = \rho \dot{V} = \rho A_c V_{avg} \quad \left(\frac{kg}{s} \right)$$

Energy flow rate

$$\dot{E} = \dot{m}e \quad \left(\frac{kJ}{s} \quad \text{or} \quad kW \right)$$

Mechanical Energy of a flowing fluid on a unit mass basis

$$e_{mech} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$

Mechanical Energy of a flowing fluid expressed in rate form

$$\dot{E}_{mech} = \dot{m}e_{mech} = \dot{m} \left(\frac{P}{\rho} + \frac{V^2}{2} + gz \right)$$

Mechanical Energy change of a fluid during incompressible flow

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \quad \left(\frac{kJ}{kg} \right)$$

And

$$\Delta \dot{E}_{mech} = \dot{m}e_{mech} = \dot{m} \left(\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right) \quad (kW)$$

Heat transfer per unit mass of a system

$$q = \frac{Q}{m} \quad \left(\frac{kJ}{kg} \right)$$

Amount of heat transfer during a process

$$Q = \int_{t_1}^{t_2} \dot{Q} dt \quad (kJ)$$

When Q remains constant

$$Q = \dot{Q} \Delta t \quad (kJ)$$

Work done per unit mass of a system

$$w = \frac{W}{m} \quad \left(\frac{kJ}{kg} \right)$$

The total volume change during a process between states 1 and 2

$$\int_1^2 dV = V_2 - V_1 = \Delta V$$

The total work done during process 1-2

$$\int_1^2 \delta W = W_{12} \quad (\text{Not } \Delta W)$$

Electrical work (where N is the amount of coulombs and V is a potential difference)

$$W_e = VN$$

Electrical work expressed in rate form (Electrical Power)

$$\dot{W}_e = \mathbf{V}I \quad (W)$$

Electrical work done during time interval Δt

$$W_e = \int_1^2 \mathbf{V}I dt \quad (kJ)$$

Work done by a constant force

$$W = Fs \quad (kJ)$$

Work done by a not constant force

$$W = \int_1^2 F ds \quad (kJ)$$

Torque

$$T = Fr \rightarrow F = \frac{T}{r}$$

This force acts through a distance s , which is related to the radius r by

$$s = (2\pi r)n$$

Shaft Work

$$W_{sh} = Fs = \left(\frac{T}{r}\right)(2\pi rn) = 2\pi nT \quad (kJ)$$

Power transmitted through the shaft

$$\dot{W}_{sh} = 2\pi nT \quad (kW)$$

Spring Work

$$\delta W_{spring} = F dx$$

Total spring work

$$W_{spring} = \frac{1}{2}k(x_2^2 - x_1^2) \quad (kJ)$$

Work associated with the expansion or contraction of a solid bar

$$W_{elastic} = \int_1^2 F dx = \int_1^2 \sigma_n A dx \quad (kJ)$$

Work associated with the stretching of a film (also called surface tension work)

$$W_{surface} = \int_1^2 \sigma_s A dx \quad (kJ)$$

Energy balance

$$E_{in} - E_{out} = \Delta E_{system}$$

The change in the total energy of a system during a process (in the absence of electric, magnetic and surface tension effects)

$$\Delta E = \Delta U + \Delta KE + \Delta PE$$

Where

$$\Delta U = m(u_2 - u_1)$$

$$\Delta KE = \frac{1}{2}m(V_2^2 - V_1^2)$$

$$\Delta PE = mg(z_2 - z_1)$$

Energy balance more explicitly

$$E_{in} - E_{out} = (Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass,in} - E_{mass,out}) = \Delta E_{system}$$

Energy balance for any system undergoing any kind of process

$$E_{in} - E_{out} = \Delta E_{system} \quad (kJ)$$

Or in the rate form

$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{system}}{dt} \quad (kW)$$

For constant rates, the total quantities during a time interval Δt are related to the quantities per unit time as

$$Q = \dot{Q}\Delta t \quad (kJ)$$

$$W = \dot{W}\Delta t \quad (kJ)$$

$$\Delta E = \frac{dE}{dt}\Delta t \quad (kJ)$$

Energy balance on a unit mass basis

$$e_{in} - e_{out} = \Delta e_{system} \quad \left(\frac{kJ}{kg}\right)$$

Energy balance in differential form

$$\delta E_{in} - \delta E_{out} = dE_{system} \quad \text{or} \quad \delta e_{in} - \delta e_{out} = de_{system}$$

The Energy balance for a cycle

$$W_{net,out} = Q_{net,in} \quad \text{or} \quad \dot{W}_{net,out} = \dot{Q}_{net,in}$$

Performance or efficiency

$$Performance = \frac{Desired\ output}{Required\ input}$$

Combustion efficiency

$$\eta_{combustion} = \frac{Q}{HV} = \frac{\text{Amount of heat released during combustion}}{\text{Heating value of the fuel burned}}$$

Mechanical efficiency

$$\eta_{mech} = \frac{E_{mech,out}}{E_{mech,in}} = 1 - \frac{E_{mech,loss}}{E_{mech,in}}$$

Pump efficiency

$$\eta_{pump} = \frac{\Delta \dot{E}_{mech,fluid}}{\dot{W}_{shaft,in}} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump}}$$

Turbine efficiency

$$\eta_{turbine} = \frac{\dot{W}_{shaft,out}}{|\Delta \dot{E}_{mech,fluid}|} = \frac{\dot{W}_{turbine}}{\dot{W}_{turbine,e}}$$

Motor efficiency

$$\eta_{motor} = \frac{\dot{W}_{shaft,out}}{\dot{W}_{elect,in}}$$

Generator efficiency

$$\eta_{generator} = \frac{\dot{W}_{elect,out}}{\dot{W}_{shaft,in}}$$

Combined efficiency of a pump-motor combination

$$\eta_{pump-motor} = \eta_{pump} \eta_{motor} = \frac{\dot{W}_{pump,u}}{\dot{W}_{elect,in}} = \frac{\Delta \dot{E}_{mech,fluid}}{\dot{W}_{elect,in}}$$

Combined efficiency of a turbine-generator combination

$$\eta_{turbine-generator} = \eta_{turbine} \eta_{generator} = \frac{\dot{W}_{elect,out}}{\dot{W}_{turb,in}} = \frac{\dot{W}_{elect,out}}{|\Delta \dot{E}_{mech,fluid}|}$$

Rate of heat conduction

$$\dot{Q}_{cond} = k_t A \frac{\Delta T}{\Delta x}$$

In the limiting case of $\Delta x \rightarrow 0$ (Fouriers law)

$$\dot{Q}_{cond} = -k_t A \frac{dT}{dx} \quad (W)$$

Rate of heat transfer by convection

$$\dot{Q}_{conv} = hA(T_s - T_f) \quad (W)$$

Maximum rate of radiation

$$\dot{Q}_{emit,max} = \sigma A T_s^4$$

Radiation emitted by a real surface

$$\dot{Q}_{emit} = \varepsilon \sigma A T_s^4 \quad (W)$$

Rate at which a surface absorbs radiation

$$\dot{Q}_{abs} = \alpha \dot{Q}_{incident} \quad (W)$$

Net rate of radiation heat transfer

$$\dot{Q}_{rad} = \epsilon \sigma A (T_s^4 - T_s^4) \quad (W)$$

Chapter 3

Properties of Pure Substances

The quality x as the ratio of the mass of vapor to the total mass of the mixture (for saturated mixtures only)

$$x = \frac{m_{\text{vapor}}}{m_{\text{total}}}$$

Where

$$m_{\text{total}} = m_{\text{liquid}} + m_{\text{vapor}} = m_f + m_g$$

The total volume in a tank containing a saturated liquid-vapor mixture is

$$V = V_f + V_g$$

$$V = mv \rightarrow m_t v_{\text{avg}} = m_f v_f + m_g v_g$$

$$m_f = m_t - m_g \rightarrow m_t v_{\text{avg}} = (m_t - m_g) v_f + m_g v_g$$

Dividing by m_t yields

$$v_{\text{avg}} = (1 - x)v_f + xv_g$$

Since $x = m_g/m_t$. This relation can also be expressed as

$$v_{\text{avg}} = v_f + xv_{fg} \quad \left(\frac{m^3}{kg} \right)$$

Where $v_{fg} = v_g - v_f$. Solving for quality we obtain

$$x = \frac{v_{\text{avg}} - v_f}{v_{fg}}$$

The analysis given above can be repeated for internal energy and enthalpy with the following results

$$u_{\text{avg}} = u_f + xu_{fg} \quad \left(\frac{kJ}{kg} \right)$$

$$h_{\text{avg}} = h_f + xh_{fg} \quad \left(\frac{kJ}{kg} \right)$$

All the results are of the same format, and they can be summarized in a single equation as

$$y_{\text{avg}} = y_f + xy_{fg}$$

Where y is v , u or h .

Ideal-gas Equation of State

$$P = R \left(\frac{T}{v} \right) \rightarrow Pv = RT$$

The gas constant R is determined from

$$R = \frac{R_u}{M} \quad \left(\frac{kJ}{kg \cdot K} \text{ or } \frac{kPa \cdot m^3}{kg \cdot K} \right)$$

Where R_u is the universal gas constant The mass of a system

$$m = MN \quad (kg)$$

The ideal-gas Equation of State can be written in several different forms

$$\begin{aligned} V = mv &\rightarrow PV = mRT \\ mR = (MN)R = NR_u &\rightarrow PV = NR_u T \\ V = N\bar{v} &\rightarrow P\bar{v} = R_u T \end{aligned}$$

The properties of an ideal gas at two different states are related to each other by

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Compressibility factor

$$Z = \frac{Pv}{RT}$$

or

$$Pv = ZRT$$

Gases behave differently at a given temperature and pressure, but they behave very much the same at temperatures and pressure normalized with respect to their critical temperatures and pressures. The normalization is done as

$$P_R = \frac{P}{P_{cr}} \quad \text{and} \quad T_R = \frac{T}{T_{cr}}$$

Pseudo-reduced specific volume

$$v_R = \frac{v_{actual}}{RT_{cr}/P_{cr}}$$

Van der Waals Equation of State

$$\left(P + \frac{a}{v^2} \right) (v - b) = RT$$

The determination of the two constants appearing in this equation is based on the observation that the critical isotherm on a $P - v$ diagram has a horizontal inflection point of the critical point. Thus, the first and the second derivatives of P with respect to v at the critical point must be zero. That is

$$\left(\frac{\partial P}{\partial v} \right)_{T=T_{cr}=const} = 0 \quad \text{and} \quad \left(\frac{\partial^2 P}{\partial v^2} \right)_{T=T_{cr}=const} = 0$$

By performing the differentiations and eliminating v_{cr} , the constants a and b are determined to be

$$a = \frac{27R^2T_{cr}^2}{64P_{cr}} \quad \text{and} \quad b = \frac{RT_{cr}}{8P_{cr}}$$

Beattie-Bridgeman Equation of State

$$P = \frac{R_u T}{\bar{v}^2} \left(1 - \frac{c}{\bar{v} T^3}\right) (\bar{v} + B) - \frac{A}{\bar{v}^2}$$

where

$$A = A_0 \left(1 - \frac{a}{\bar{v}}\right) \quad \text{and} \quad B = B_0 \left(1 - \frac{b}{\bar{v}}\right)$$

Benedict-Webb-Rubin Equation of State

$$P = \frac{R_u T}{\bar{v}} + \left(B_0 R_u T - A_0 - \frac{C_0}{T^2}\right) \frac{1}{\bar{v}^2} + \frac{b R_u T - a}{\bar{v}^3} + \frac{a \alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3 T^2} \left(1 + \frac{\gamma}{\bar{v}^2}\right) e^{-\frac{\gamma}{\bar{v}^2}}$$

Virial Equation of State

$$P = \frac{RT}{v} + \frac{a(T)}{v^2} + \frac{b(T)}{v^3} + \frac{c(T)}{v^4} + \frac{d(T)}{v^5} + \dots$$

Vapor Pressure

$$P_{atm} = P_a + P_v$$

Chapter 4

Energy Analysis of Closed Systems

Boundary work

$$\delta W_b = F ds = P Ad s = P dv$$

The total boundary work

$$W_b = \int_1^2 P dV \quad (kJ)$$

The total area under the process curve 1-2

$$Area = A = \int_1^2 dA = \int_1^2 p dV$$

Pressure for a polytropic process

$$P = CV^{-n}$$

Work done during a polytropic process

$$W_b = \int_1^2 p dV = \int_1^2 CV^{-n} dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

Since $C = P_1 V_1^n = P_2 V_2^n$. For an ideal gas ($PV = mRT$), this equation can also be written as

$$W_b = \frac{mR(T_2 - T_1)}{1-n} \quad n \neq 1 \quad (kJ)$$

For the special case of $n = 1$ the boundary work becomes

$$W_b = \int_1^2 p dV = \int_1^2 CV^{-1} dV = PV \ln \left(\frac{V_2}{V_1} \right)$$

Energy balance for a closed system undergoing a cycle

$$W_{net,out} = Q_{net,in} \quad \text{or} \quad \dot{W}_{net,out} = \dot{Q}_{net,in} \quad (\text{for a cycle})$$

Various forms of the first-law relation for closed systems

$$\begin{array}{ll} \text{General} & Q - W = \Delta E \\ \text{Stationary systems} & Q - W = \Delta U \\ \text{Per unit mass} & q - w = \Delta e \\ \text{Differential form} & \delta q - \delta w = de \end{array}$$

Specific heat at constant volume (= the change in internal energy with temperature at constant volume)

$$c_v = \left(\frac{\delta u}{\delta T} \right)_v \quad \left(\frac{kJ}{kg \cdot ^\circ C} \text{ or } \frac{kJ}{kg \cdot ^\circ K} \right)$$

Specific heat at constant pressure (= the change in enthalpy with temperature at constant pressure)

$$c_p = \left(\frac{\delta h}{\delta T} \right)_p \quad \left(\frac{kJ}{kg \cdot ^\circ C} \text{ or } \frac{kJ}{kg \cdot ^\circ K} \right)$$

Using the definition of enthalpy and the equation of state of an ideal gas, we have

$$\left. \begin{array}{l} h = u + PV \\ Pv = RT \end{array} \right\} h = u + RT$$

The differential changes in the internal energy and enthalpy of an ideal gas

$$du = c_v(T)dT$$

and

$$dh = c_p(T)dT$$

the change in internal energy or enthalpy for an ideal gas during a process from state 1 to state 2 is determined by integrating these equations

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T)dT \quad \left(\frac{kJ}{kg} \right)$$

and

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)dT \quad \left(\frac{kJ}{kg} \right)$$

A special relationship between C_p and C_v for ideal gasses can be obtained by differentiating the relation $h = u + RT$, which yields

$$dh = du + RdT$$

Replacing dh by $c_p dT$ and du by $c_v dT$ and dividing the resulting expression by dT , we obtain

$$c_p = c_v + R \quad \left(\frac{kJ}{kg \cdot K} \right)$$

Specific heat ratio

$$k = \frac{c_p}{c_v}$$

For incompressible substances (solids and liquids)

$$c_p = C_v = c$$

The change in internal energy of incompressible substances between states 1 and states 2 is obtained by

$$\Delta u = u_2 - u_1 = \int_1^2 c(T) dT \cong c_{avg}(T_2 - T_1) \quad \left(\frac{kJ}{kg} \right)$$

The change in enthalpy of incompressible substances between states 1 and states 2 is obtained by

$$\Delta h = \Delta u + v\Delta P \quad \left(\frac{kJ}{kg} \right)$$

Chapter 5

Mass and Energy Analysis of Control Volumes

Conservation of mass principle

$$m_{in} - m_{out} = \Delta m_{system} \quad \text{and} \quad \dot{m}_{in} - \dot{m}_{out} = \frac{dm_{system}}{dt}$$

Mass flow rate

$$\dot{m} = \rho VA$$

Volume flow rate

$$\dot{V} = VA = \frac{\dot{m}}{\rho}$$

The total energy of a flowing fluid

$$\theta = h + ke + pe = h + \frac{V^2}{2} + gz$$

The general mass and energy balances for any system undergoing any process can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Changes in internal, kinetic, potential, etc., energies}}$$

It can also be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{\Delta E_{system}}{dt}}_{\text{Rate of changes in internal, kinetic, potential, etc., energies}}$$

Conservation of mass and energy equations for steady-flow processes

$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$
$$\dot{Q} - \dot{W} = \underbrace{\sum_{out} \dot{m} \left(h + \frac{V^2}{2} + gz \right)}_{\text{for each exit}} - \underbrace{\sum_{in} \dot{m} \left(h + \frac{V^2}{2} + gz \right)}_{\text{for each inlet}}$$

For single-stream (one-inlet-one-exit) systems they simplify to

$$\dot{m}_1 = \dot{m}_2 \rightarrow \frac{1}{v_1} V_1 A_1 = \frac{1}{v_2} V_2 A_2$$
$$\dot{Q} - \dot{W} = \dot{m} \left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$

When kinetic and potential energy changes associated with the control volume and the fluid streams are negligible, the mass and energy balance relations for a uniform-flow system are expressed as

$$m_{in} - m_{out} = \Delta m_{system}$$
$$Q - W = \sum_{out} mh - \sum_{in} mh + (m_2 u_2 - m_1 u_1)_{system}$$

Chapter 6

The Second Law of Thermodynamics

The thermal efficiency of a heat engine

$$\eta_{th} = \frac{W_{net,out}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Coefficient of performance

$$COP_R = \frac{Q_L}{W_{net,in}} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$
$$COP_{HP} = \frac{Q_H}{W_{net,in}} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

Thermodynamic temperature scale related to the heat transfers between a reversible device and the high- and low-temperature reservoirs

$$\left(\frac{Q_H}{Q_L} \right)_{rev} = \frac{T_H}{T_L}$$

Thermal efficiency of a Carnot heat engine, as well as all other reversible heat engines

$$\eta_{th,rev} = 1 - \frac{T_L}{T_H}$$

Coefficient of performance of reversible refrigerators and heat pumps

$$COP_{R,rev} = \frac{1}{\frac{T_H}{T_L} - 1}$$

and

$$COP_{HP,rev} = \frac{1}{1 - \frac{T_L}{T_H}}$$

Chapter 7

Entropy

Definition of entropy

$$dS = \left(\frac{dQ}{T} \right)_{int,rev}$$

For the special case of an internally reversible, isothermal process, this gives

$$\Delta S = \frac{Q}{T_0}$$

Increase of entropy principle

$$S_{gen} \geq 0$$

The entropy change and isentropic relations for a process with pure substances

$$\begin{aligned} \text{Any process:} \quad \Delta s &= s_2 - s_1 \\ \text{Isentropic process:} \quad s_2 &= s_1 \end{aligned}$$

The entropy change and isentropic relations for a process with incompressible substances

$$\begin{aligned} \text{Any process:} \quad s_2 - s_1 &= c_{avg} \ln \frac{T_2}{T_1} \\ \text{Isentropic process:} \quad T_2 &= T_1 \end{aligned}$$

The entropy change and isentropic relations for a process with ideal gases with constant specific heats (approximate treatment)

$$\begin{aligned} \text{Any process:} \quad s_2 - s_1 &= c_{v,avg} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \\ s_2 - s_1 &= c_{p,avg} \ln \frac{T_2}{T_1} + R \ln \frac{P_2}{P_1} \\ \text{Isentropic process:} \quad \left(\frac{T_2}{T_1} \right)_{s=constant} &= \left(\frac{v_1}{v_2} \right)^{k-1} \\ \left(\frac{T_2}{T_1} \right)_{s=constant} &= \left(\frac{P_1}{P_2} \right)^{\frac{k-1}{k}} \\ \left(\frac{P_2}{P_1} \right)_{s=constant} &= \left(\frac{v_1}{v_2} \right)^k \end{aligned}$$

The entropy change and isentropic relations for a process with ideal gases with variable specific heats (exact treatment)

$$\text{Any process: } s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}$$

$$\begin{aligned} \text{Isentropic process: } s_2^\circ &= s_1^\circ + R \ln \frac{P_2}{P_1} \\ \left(\frac{P_2}{P_1} \right)_{s=\text{constant}} &= \frac{P_{r2}}{P_{r1}} \\ \left(\frac{v_2}{v_1} \right)_{s=\text{constant}} &= \frac{v_{r2}}{v_{r1}} \end{aligned}$$

Steady flow work for a reversible process

$$w_{rev} = - \int_1^2 v dP - \Delta ke - \Delta pe$$

For incompressible substances it simplifies to

$$w_{rev} = -v(P_2 - P_1) - \Delta ke - \Delta pe$$

The reversible work inputs to a compressor compressing an ideal gas from T_1, P_1 to P_2 in an isentropic, polytropic or isothermal manner

$$\begin{aligned} \text{Isentropic: } w_{comp,in} &= \frac{kR(T_2 - T_1)}{k-1} = \frac{kRT_1}{k-1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right] \\ \text{Polytropic: } w_{comp,in} &= \frac{nR(T_2 - T_1)}{n-1} = \frac{nRT_1}{n-1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \\ \text{Isothermal: } w_{comp,in} &= RT \ln \frac{P_2}{P_1} \end{aligned}$$

Isentropic or adiabatic efficiency for turbines, compressors and nozzles

$$\begin{aligned} \eta_T &= \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} \\ \eta_C &= \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \\ \eta_N &= \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2} \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} \end{aligned}$$

The entropy balance for any system undergoing any process can be expressed in the general form as

$$\underbrace{S_{in} - S_{out}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{gen}}_{\text{Entropy generation}} = \underbrace{\Delta S_{system}}_{\text{Change in entropy}}$$

or, in rate form as

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{gen}}_{\text{Rate of entropy generation}} = \underbrace{\frac{dS_{system}}{dt}}_{\text{Rate of change in entropy}}$$

For a general steady-flow process it simplifies to

$$\dot{S}_{gen} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

Chapter 8

Exergy: A Measure of Work Potential

Not mandatory for the exam

Chapter 9

Gas Power Cycles

Thermal efficiency of the Carnot cycle

$$\eta_{th,Carnot} = 1 - \frac{T_L}{T_H}$$

In reciprocating engines, the compression ratio r and the mean effective pressure MEP are defined as

$$r = \frac{V_{max}}{V_{min}} = \frac{V_{BDC}}{V_{TDC}}$$
$$MEP = \frac{w_{net}}{v_{max} - v_{min}}$$

The thermal efficiency of the ideal Otto cycle (spark-ignition reciprocating engines) under cold-air-standard assumptions is

$$\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}}$$

The thermal efficiency of the ideal Diesel cycle (compression-ignition reciprocating engines) under cold-air-standard assumptions is

$$\eta_{th,Diesel} = 1 - \frac{1}{r^{k-1}} \left[\frac{r_c^k - 1}{k(r_c - 1)} \right]$$

The thermal efficiency of the ideal Brayton cycle (modern gas-turbine engines) under cold-air-standard assumptions is

$$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

The deviation of the actual compressor and the turbine from the idealized isentropic ones can be accurately accounted for by utilizing their isentropic efficiencies, defined as

$$\eta_C = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

and

$$\eta_T = \frac{w_a}{w_s} \cong \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

Where states 1 and 3 are the inlet states, 2a and 4a are the actual exit states, and 2s and 4s are the isentropic exit states.

Effectiveness (the extent to which a regenerator approaches an ideal regenerator)

$$\varepsilon = \frac{q_{regen,act}}{q_{regen,max}}$$

Under cold-air-standard assumptions, the thermal efficiency of an ideal Brayton cycle with regeneration becomes

$$\eta_{th,regen} = 1 - \left(\frac{T_1}{T_3}\right) (r_P)^{\frac{k-1}{k}}$$

The net thrust developed by the ideal jet-propulsion cycle

$$F = \dot{m}(V_{exit} - V_{inlet})$$

Propulsive power

$$\dot{W}_P = \dot{m}(V_{exit} - V_{inlet})V_{aircraft}$$

Propulsive efficiency

$$\eta_P = \frac{\text{Propulsive power}}{\text{Energy input rate}} = \frac{\dot{W}_P}{\dot{Q}_{in}}$$

For an ideal cycle that involves heat transfer only with a source at T_H and a sink at T_L , the exergy destruction is

$$x_{dest} = T_0 \left(\frac{q_{out}}{T_L} - \frac{q_{in}}{T_H} \right)$$