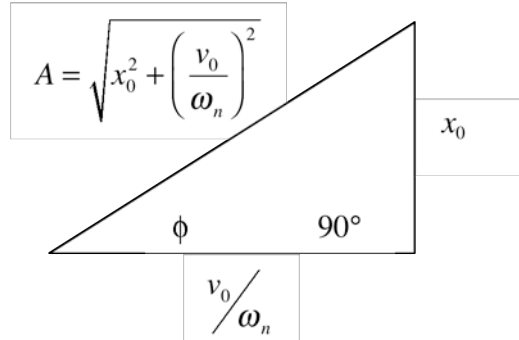


**1.10** Using Figure 1.6, verify that equation (1.10) satisfies the initial velocity condition.

**Solution:** Following the lead given in Example 1.1.2, write down the general expression of the velocity by differentiating equation (1.10):

$$x(t) = A \sin(\omega_n t + \phi) \Rightarrow \dot{x}(t) = A \omega_n \cos(\omega_n t + \phi) \\ \Rightarrow v(0) = A \omega_n \cos(\omega_n 0 + \phi) = A \omega_n \cos(\phi)$$

From the figure:



**Figure 1.6**

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}, \quad \cos \phi = \frac{\frac{v_0}{\omega_n}}{\sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}}$$

Substitution of these values into the expression for  $v(0)$  yields

$$v(0) = A \omega_n \cos \phi = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2} (\omega_n) \frac{\frac{v_0}{\omega_n}}{\sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}} = v_0$$

verifying the agreement between the figure and the initial velocity condition.

**1.11** (a) A 0.5 kg mass is attached to a linear spring of stiffness 0.1 N/m. Determine the natural frequency of the system in hertz. b) Repeat this calculation for a mass of 50 kg and a stiffness of 10 N/m. Compare your result to that of part a.

**Solution:** From the definition of frequency and equation (1.12)

$$\begin{aligned} (a) \quad \omega_n &= \sqrt{\frac{k}{m}} = \sqrt{\frac{.5}{.1}} = 0.447 \text{ rad/s} \\ f_n &= \frac{\omega_n}{2\pi} = \frac{2.236}{2\pi} = \underline{0.071 \text{ Hz}} \\ (b) \quad \omega_n &= \sqrt{\frac{50}{10}} = 0.447 \text{ rad/s}, f_n = \frac{\omega_n}{2\pi} = \underline{0.071 \text{ Hz}} \end{aligned}$$

Part (b) is the same as part (a) thus very different systems can have same natural frequencies.

- 1.12** Derive the solution of the single degree of freedom system of Figure 1.4 by writing Newton's law,  $ma = -kx$ , in differential form using  $adx = vdv$  and integrating twice.

**Solution:** Substitute  $a = vdv/dx$  into the equation of motion  $ma = -kx$ , to get  $mv dv = -kx dx$ . Integrating yields:

$$\frac{v^2}{2} = -\omega_n^2 \frac{x^2}{2} + c^2, \text{ where } c \text{ is a constant}$$

$$\text{or } v^2 = -\omega_n^2 x^2 + c^2 \Rightarrow$$

$$v = \frac{dx}{dt} = \sqrt{-\omega_n^2 x^2 + c^2} \Rightarrow$$

$$dt = \frac{dx}{\sqrt{-\omega_n^2 x^2 + c^2}}, \text{ write } u = \omega_n x \text{ to get:}$$

$$t - 0 = \frac{1}{\omega_n} \int \frac{du}{\sqrt{c^2 - u^2}} = \frac{1}{\omega_n} \sin^{-1} \left( \frac{u}{c} \right) + c_2$$

Here  $c_2$  is a second constant of integration that is convenient to write as  $c_2 = -\phi/\omega_n$ . Rearranging yields

$$\omega_n t + \phi = \sin^{-1} \left( \frac{\omega_n x}{c} \right) \Rightarrow$$

$$\frac{\omega_n x}{c} = \sin(\omega_n t + \phi) \Rightarrow$$

$$\underline{x(t) = A \sin(\omega_n t + \phi)}, \quad A = \frac{c}{\omega_n}$$

in agreement with equation (1.19).

**1.13** Determine the natural frequency of the two systems illustrated.

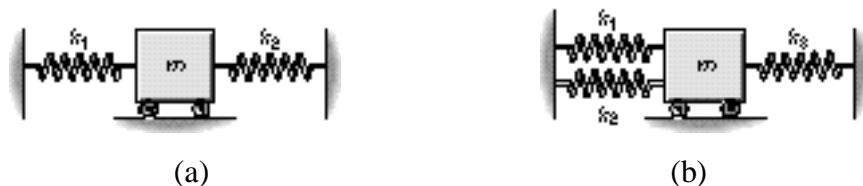


Figure P1.13

**Solution:**

(a) Summing forces from the free-body diagram in the  $x$  direction yields:

$$m\ddot{x} = -k_1x - k_2x \Rightarrow$$

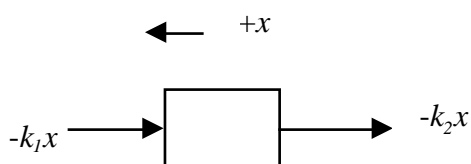
$$m\ddot{x} + k_1x + k_2x = 0$$

$$m\ddot{x} + x(k_1 + k_2) = 0, \text{ dividing by } m \text{ yields:}$$

$$\ddot{x} + \left( \frac{k_1 + k_2}{m} \right) x = 0$$

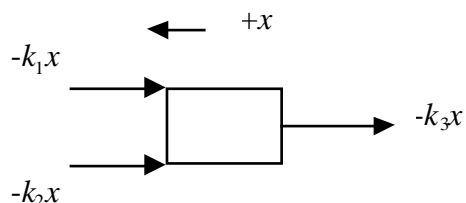
Examining the coefficient of  $x$  yields:

$$\omega_n = \sqrt{\frac{k_1 + k_2}{m}}$$



Free-body diagram for part a

(b) Summing forces from the free-body diagram in the  $x$  direction yields:



Free-body diagram for part b

$$m\ddot{x} = -k_1x - k_2x - k_3x, \Rightarrow$$

$$m\ddot{x} + k_1x + k_2x + k_3x = 0 \Rightarrow$$

$$m\ddot{x} + (k_1 + k_2 + k_3)x = 0 \Rightarrow \ddot{x} + \frac{(k_1 + k_2 + k_3)}{m}x = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{k_1 + k_2 + k_3}{m}}$$

**1.14\*** Plot the solution given by equation (1.10) for the case  $k = 1000$  N/m and  $m = 10$  kg for two complete periods for each of the following sets of initial conditions: a)  $x_0 = 0$  m,  $v_0 = 1$  m/s, b)  $x_0 = 0.01$  m,  $v_0 = 0$  m/s, and c)  $x_0 = 0.01$  m,  $v_0 = 1$  m/s.

**Solution:** Here we use Mathcad:

a) all units in m, kg, s

$$m := 10 \quad k := 1000$$

$$x_0 := 0.0$$

$$v_0 := 1$$

$$\omega_n := \sqrt{\frac{k}{m}}$$

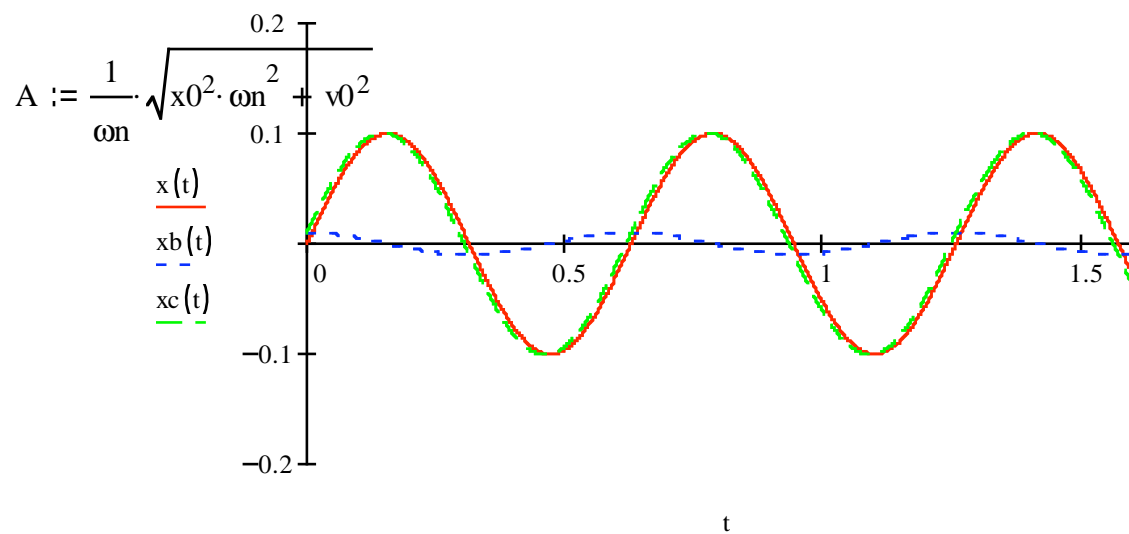
$$f_n := \frac{\omega_n}{2 \cdot \pi}$$

$$T := \frac{2 \cdot \pi}{\omega_n}$$

$$\phi := \operatorname{atan}\left(\frac{\omega_n \cdot x_0}{v_0}\right)$$

$$x(t) := A \cdot \sin(\omega_n \cdot t + \phi)$$

parts b and c are plotted in the above by simply changing the initial conditions as appropriate



**1.15\*** Make a three dimensional surface plot of the amplitude  $A$  of an undamped oscillator given by equation (1.9) versus  $x_0$  and  $v_0$  for the range of initial conditions given by  $-0.1 \leq x_0 \leq 0.1$  m and  $-1 \leq v_0 \leq 1$  m/s, for a system with natural frequency of 10 rad/s.

**Solution:** Working in Mathcad the solution is generated as follows:

$$\omega_n := 10$$

$$N := 25$$

$$i := 0..N$$

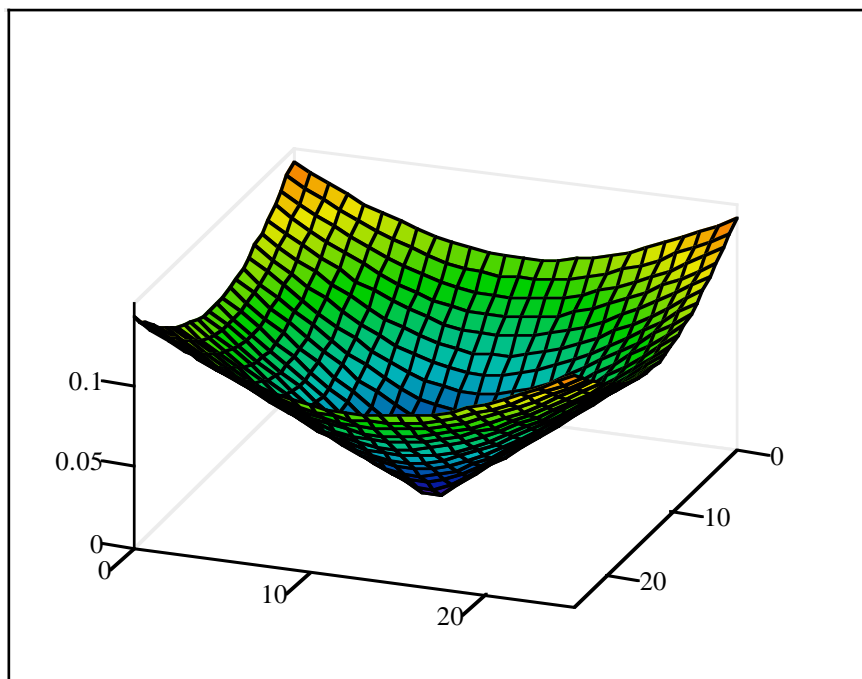
$$j := 0..N$$

$$v_{0j} := -1 + \left(\frac{2}{N} \cdot j\right)$$

$$x_{0i} := -0.1 + \left(\frac{0.2}{N}\right) \cdot i$$

$$A(x_0, v_0) := \frac{1}{\omega_n} \cdot \sqrt{\omega_n^2 \cdot (x_0)^2 + (v_0)^2}$$

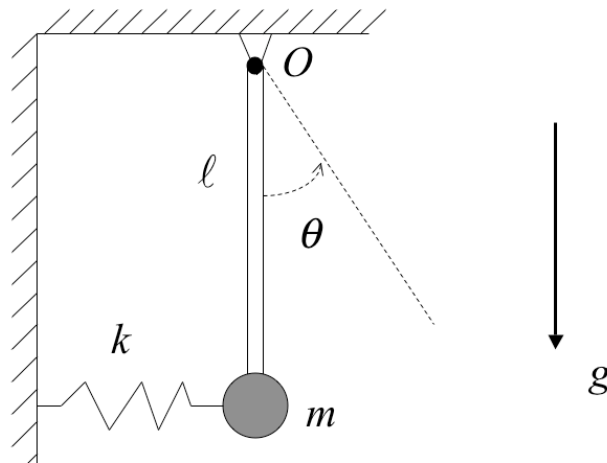
$$M_{i,j} := A(x_{0i}, v_{0j})$$



M

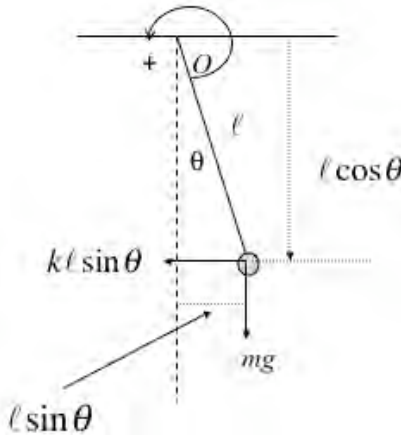
Amplitude vs initial conditions

- 1.16** A machine part is modeled as a pendulum connected to a spring as illustrated in Figure P1.16. Ignore the mass of pendulum's rod and derive the equation of motion. Then following the procedure used in Example 1.1.1, linearize the equation of motion and compute the formula for the natural frequency. Assume that the rotation is small enough so that the spring only deflects horizontally.



**Figure P1.16**

**Solution:** Consider the free body diagram of the mass displaced from equilibrium:



There are two forces acting on the system to consider, if we take moments about point  $O$  (then we can ignore any forces at  $O$ ). This yields

$$\begin{aligned}\sum M_O &= J_O \alpha \Rightarrow m\ell^2 \ddot{\theta} = -mg\ell \sin\theta - k\ell \sin\theta \cdot \ell \cos\theta \\ &\Rightarrow \underline{m\ell^2 \ddot{\theta} + mg\ell \sin\theta + k\ell^2 \sin\theta \cos\theta = 0}\end{aligned}$$

Next consider the small  $\theta$  approximations to that  $\sin\theta \sim \theta$  and  $\cos\theta=1$ . Then the linearized equation of motion becomes:

$$\ddot{\theta}(t) + \left( \frac{mg + k\ell}{m\ell} \right) \theta(t) = 0$$

Thus the natural frequency is

$$\omega_n = \sqrt{\frac{mg + k\ell}{m\ell}} \text{ rad/s}$$

**1.17** A pendulum has length of 250 mm. What is the system's natural frequency in Hertz?

**Solution:**

Given:  $l=250 \text{ mm}$

Assumptions: small angle approximation of sin

From Window 1.1, the equation of motion for the pendulum is as follows:

$$I_o \ddot{\theta} + mg\theta = 0, \text{ where } I_o = ml^2 \Rightarrow \ddot{\theta} + \frac{g}{l} \theta = 0$$

The coefficient of  $\theta$  yields the natural frequency as:

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8 \text{ m/s}^2}{0.25 \text{ m}}} = 6.26 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = 0.996 \text{ Hz}$$

**1.18** The pendulum in Example 1.1.1 is required to oscillate once every second. What length should it be?

**Solution:**

Given:  $f = 1 \text{ Hz}$  (one cycle per second)

$$\omega_n = 2\pi f = \sqrt{\frac{g}{l}}$$



$$\therefore l = \frac{g}{(2\pi f)^2} = \frac{9.81}{4\pi^2} = 0.248 \text{ m}$$

- 1.19** The approximation of  $\sin \theta = \theta$ , is reasonable for  $\theta$  less than  $10^\circ$ . If a pendulum of length 0.5 m, has an initial position of  $\theta(0) = 0$ , what is the maximum value of the initial angular velocity that can be given to the pendulum without violating this small angle approximation? (be sure to work in radians)

**Solution:** From Window 1.1, the linear equation of the pendulum is

$$\ddot{\theta}(t) + \frac{g}{\ell} \theta(t) = 0$$

For zero initial position, the solution is given in equation (1.10) by

$$\theta(t) = \frac{v_0 \sqrt{\ell}}{\sqrt{g}} \sin\left(\sqrt{\frac{g}{\ell}} t\right) \Rightarrow |\theta| \leq \frac{v_0 \sqrt{\ell}}{\sqrt{g}}$$

since  $\sin$  is always less than one. Thus if we need  $\theta < 10^\circ = 0.175$  rad, then we need to solve:

$$\frac{v_0 \sqrt{0.5}}{\sqrt{9.81}} = 0.175$$

for  $v_0$  which yields:

$$v_0 \leq \mathbf{0.773 \text{ rad/s.}}$$

## Problems and Solutions for Section 1.2 and Section 1.3 (1.20 to 1.51)

### Problems and Solutions Section 1.2 (Numbers 1.20 through 1.30)

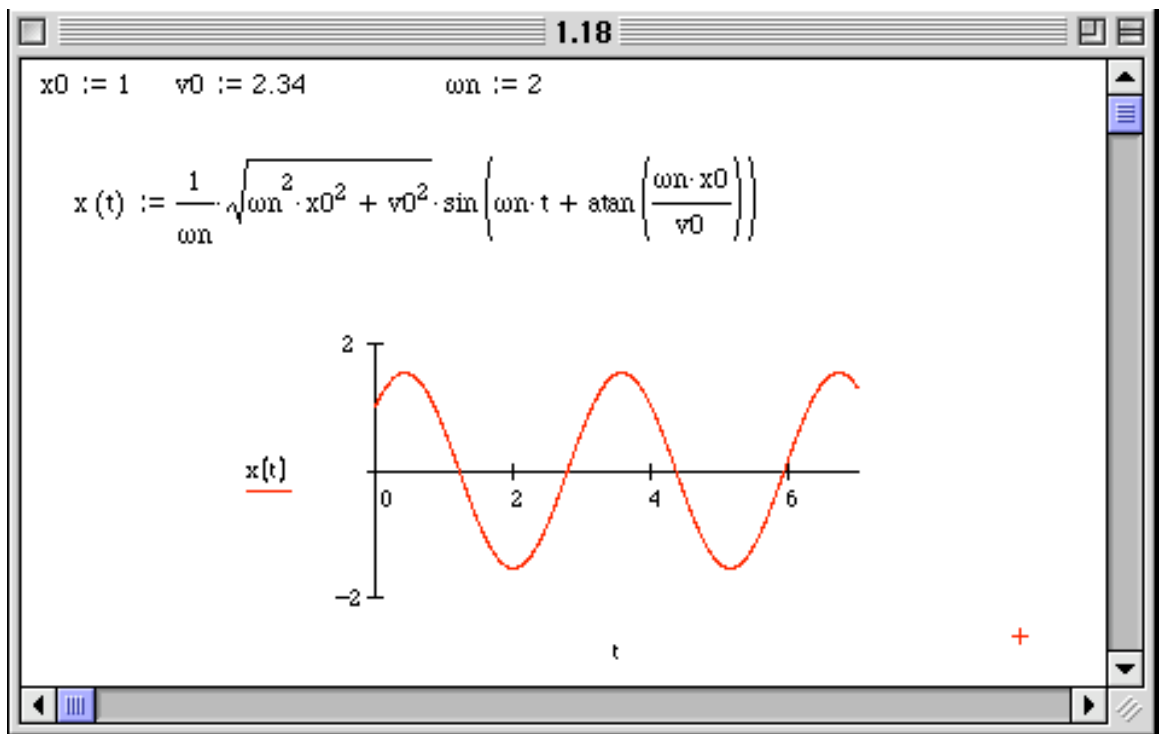
**1.20\*** Plot the solution of a linear, spring and mass system with frequency  $\omega_n = 2$  rad/s,  $x_0 = 1$  mm and  $v_0 = 2.34$  mm/s, for at least two periods.

**Solution:** From Window 1.18, the plot can be formed by computing:

$$A = \frac{1}{\omega_n} \sqrt{\omega_n^2 x_0^2 + v_0^2} = 1.54 \text{ mm}, \quad \phi = \tan^{-1}\left(\frac{\omega_n x_0}{v_0}\right) = 40.52^\circ$$

$$x(t) = A \sin(\omega_n t + \phi)$$

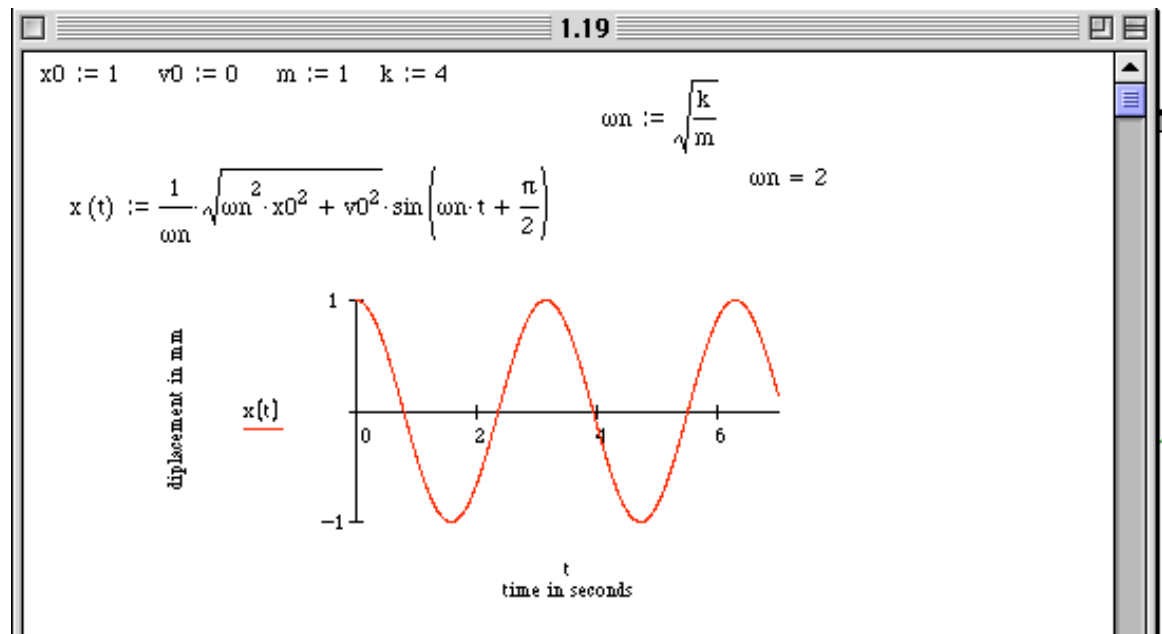
This can be plotted in any of the codes mentioned in the text. In Mathcad the program looks like.



In this plot the units are in mm rather than meters.

**1.21\*** Compute the natural frequency and plot the solution of a spring-mass system with mass of 1 kg and stiffness of 4 N/m, and initial conditions of  $x_0 = 1$  mm and  $v_0 = 0$  mm/s, for at least two periods.

**Solution:** Working entirely in Mathcad, and using the units of mm yields:



Any of the other codes can be used as well.

- 1.22** To design a linear, spring-mass system it is often a matter of choosing a spring constant such that the resulting natural frequency has a specified value. Suppose that the mass of a system is 4 kg and the stiffness is 100 N/m. How much must the spring stiffness be changed in order to increase the natural frequency by 10%?

**Solution:** Given  $m = 4$  kg and  $k = 100$  N/m the natural frequency is

$$\omega_n = \sqrt{\frac{100}{4}} = 5 \text{ rad/s}$$

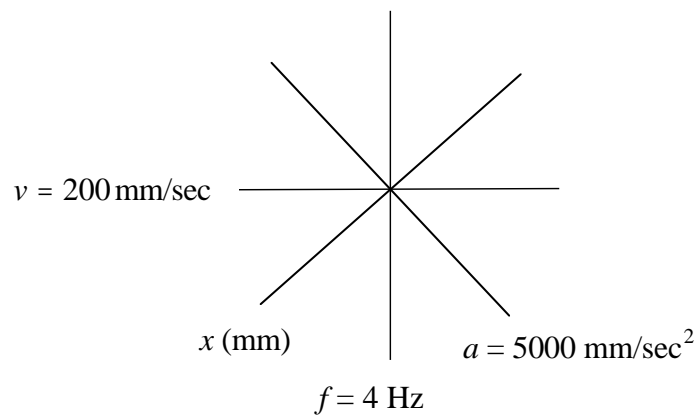
Increasing this value by 10% requires the new frequency to be  $5 \times 1.1 = 5.5$  rad/s.

Solving for  $k$  given  $m$  and  $\omega_n$  yields:

$$5.5 = \sqrt{\frac{k}{4}} \Rightarrow k = (5.5)^2(4) = 121 \text{ N/m}$$

Thus the stiffness  $k$  must be increased by about 20%.

- 1.23** Referring to Figure 1.8, if the maximum peak velocity of a vibrating system is 200 mm/s at 4 Hz and the maximum allowable peak acceleration is 5000 mm/s<sup>2</sup>, what will the peak displacement be?



**Solution:**

Given:  $v_{\max} = 200 \text{ mm/s @ } 4 \text{ Hz}$

$a_{\max} = 5000 \text{ mm/s @ } 4 \text{ Hz}$

$$x_{\max} = A$$

$$v_{\max} = A\omega_n$$

$$a_{\max} = A\omega_n^2$$

$$\therefore x_{\max} = \frac{v_{\max}}{\omega_n} = \frac{v_{\max}}{2\pi f} = \frac{200}{8\pi} = 7.95 \text{ mm}$$

At the center point, the peak displacement will be **x = 7.95 mm**

- 1.24** Show that lines of constant displacement and acceleration in Figure 1.8 have slopes of +1 and -1, respectively. If rms values instead of peak values are used, how does this affect the slope?

**Solution:** Let

$$x = x_{\max} \sin \omega_n t$$

$$\dot{x} = x_{\max} \omega_n \cos \omega_n t$$

$$\ddot{x} = -x_{\max} \omega_n^2 \sin \omega_n t$$

Peak values:

$$\dot{x}_{\max} = x_{\max} \omega_n = 2\pi f x_{\max}$$

$$\ddot{x}_{\max} = x_{\max} \omega_n^2 = (2\pi f)^2 x_{\max}$$

Location:

$$\ln \dot{x}_{\max} = \ln x_{\max} + \ln 2\pi f$$

$$\ln \ddot{x}_{\max} = \ln \dot{x}_{\max} - \ln 2\pi f$$

Since  $x_{\max}$  is constant, the plot of  $\ln \dot{x}_{\max}$  versus  $\ln 2\pi f$  is a straight line of slope +1. If  $\ln \ddot{x}_{\max}$  is constant, the plot of  $\ln \dot{x}_{\max}$  versus  $\ln 2\pi f$  is a straight line of slope -1. Calculate RMS values

Let

$$x(t) = A \sin \omega_n t$$

$$\dot{x}(t) = A \omega_n \cos \omega_n t$$

$$\ddot{x}(t) = -A \omega_n^2 \sin \omega_n t$$

Mean Square Value:  $\bar{x}^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x^2(t) dt$

$$\bar{x}^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T A^2 \sin^2 \omega_n t dt = \lim_{T \rightarrow \infty} \frac{A^2}{T} \int_0^T (1 - \cos 2\omega_n t) dt = \frac{A^2}{2}$$

$$\dot{\bar{x}}^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T A^2 \omega_n^2 \cos^2 \omega_n t dt = \lim_{T \rightarrow \infty} \frac{A^2 \omega_n^2}{T} \int_0^T \frac{1}{2} (1 + \cos 2\omega_n t) dt = \frac{A^2 \omega_n^2}{2}$$

$$\ddot{\bar{x}}^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T A^2 \omega_n^4 \sin^2 \omega_n t dt = \lim_{T \rightarrow \infty} \frac{A^2 \omega_n^4}{T} \int_0^T \frac{1}{2} (1 - \cos 2\omega_n t) dt = \frac{A^2 \omega_n^4}{2}$$

Therefore,

$$x_{rms} = \sqrt{\bar{x}^2} = \frac{\sqrt{2}}{2} A$$

$$\dot{x}_{rms} = \sqrt{\dot{\bar{x}}^2} = \frac{\sqrt{2}}{2} A \omega_n$$

$$\ddot{x}_{rms} = \sqrt{\ddot{\bar{x}}^2} = \frac{\sqrt{2}}{2} A \omega_n^2$$

The last two equations can be rewritten as:

$$\dot{x}_{rms} = x_{rms} \omega = 2\pi f x_{rms}$$

$$\ddot{x}_{rms} = x_{rms} \omega^2 = 2\pi f \dot{x}_{rms}$$

The logarithms are:

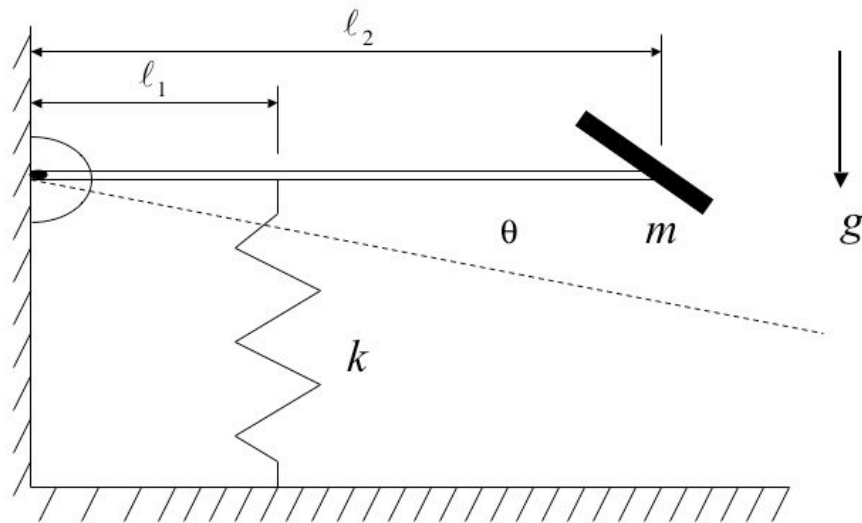
$$\ln \dot{x}_{rms} = \ln x_{rms} + \ln 2\pi f$$

$$\ln \ddot{x}_{rms} = \ln \dot{x}_{rms} + \ln 2\pi f$$

The plots of  $\ln \dot{x}_{rms}$  versus  $\ln 2\pi f$  is a straight line of slope +1 when  $x_{rms}$  is constant, and -1 when  $\ddot{x}_{rms}$  is constant. Therefore **the slopes are unchanged**.

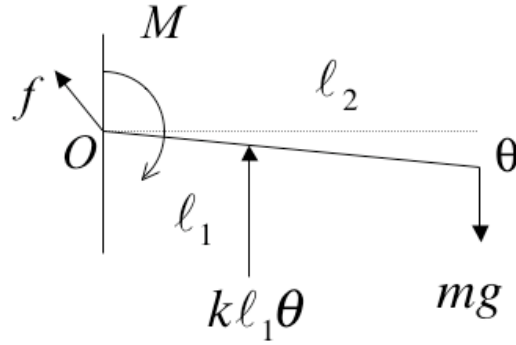


- 1.25** A foot pedal mechanism for a machine is crudely modeled as a pendulum connected to a spring as illustrated in Figure P1.25. The purpose of the spring is to keep the pedal roughly vertical. Compute the spring stiffness needed to keep the pendulum at  $1^\circ$  from the horizontal and then compute the corresponding natural frequency. Assume that the angular deflections are small, such that the spring deflection can be approximated by the arc length, that the pedal may be treated as a point mass and that pendulum rod has negligible mass. The values in the figure are  $m = 0.5$  kg,  $g = 9.8$  m/s<sup>2</sup>,  $\ell_1 = 0.2$  m and  $\ell_2 = 0.3$  m.



**Figure P1.25**

**Solution:** You may want to note to your students, that many systems with springs are often designed based on static deflections, to hold parts in specific positions as in this case, and yet allow some motion. The free-body diagram for the system is given in the figure.



For static equilibrium the sum of moments about point  $O$  yields ( $\theta_1$  is the static deflection):

$$\begin{aligned}\sum M_O &= -\ell_1 \theta_1 (\ell_1) k + mg \ell_2 = 0 \\ \Rightarrow \ell_1^2 \theta_1 k &= mg \ell_2 \\ \Rightarrow k &= \frac{mg \ell_2}{\ell_1^2 \theta_1} = \frac{0.5 \cdot 0.3}{(0.2)^2 \frac{\pi}{180}} = \underline{2106 \text{ N/m}}\end{aligned}\quad (1)$$

Again take moments about point  $O$  to get the dynamic equation of motion:

$$\sum M_O = J\ddot{\theta} = m\ell_2^2 \ddot{\theta} = -\ell_1^2 k(\theta + \theta_1) + mg \ell_2 = -\ell_1^2 k\theta + \ell_1^2 k\theta_1 - mg \ell_2 \theta$$

Next using equation (1) above for the static deflection yields:

$$\begin{aligned}m\ell_2^2 \ddot{\theta} + \ell_1^2 k\theta &= 0 \\ \Rightarrow \ddot{\theta} + \left( \frac{\ell_1^2 k}{m\ell_2^2} \right) \theta &= 0 \\ \Rightarrow \omega_n &= \frac{\ell_1}{\ell_2} \sqrt{\frac{k}{m}} = \frac{0.2}{0.3} \sqrt{\frac{2106}{0.5}} = \underline{43.27 \text{ rad/s}}\end{aligned}$$

**1.26** An automobile is modeled as a 1000-kg mass supported by a spring of stiffness  $k = 400,000 \text{ N/m}$ . When it oscillates it does so with a maximum deflection of 10 cm. When loaded with passengers, the mass increases to as much as 1300 kg. Calculate the change in frequency, velocity amplitude, and acceleration amplitude if the maximum deflection remains 10 cm.

**Solution:**

Given:  $m_1 = 1000 \text{ kg}$

$m_2 = 1300 \text{ kg}$

$k = 400,000 \text{ N/m}$

$$x_{max} = A = 10 \text{ cm}$$

$$\omega_{n1} = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{400,000}{1000}} = 20 \text{ rad / s}$$

$$\omega_{n2} = \sqrt{\frac{k}{m_2}} = \sqrt{\frac{400,000}{1300}} = 17.54 \text{ rad / s}$$

$$\Delta\omega = 17.54 - 20 = -2.46 \text{ rad / s}$$

$$\Delta f = \frac{\Delta\omega}{2\pi} = \left| \frac{-2.46}{2\pi} \right| = 0.392 \text{ Hz}$$

$$v_1 = A\omega_{n1} = 10 \text{ cm} \times 20 \text{ rad/s} = 200 \text{ cm/s}$$

$$v_2 = A\omega_{n2} = 10 \text{ cm} \times 17.54 \text{ rad/s} = 175.4 \text{ cm/s}$$

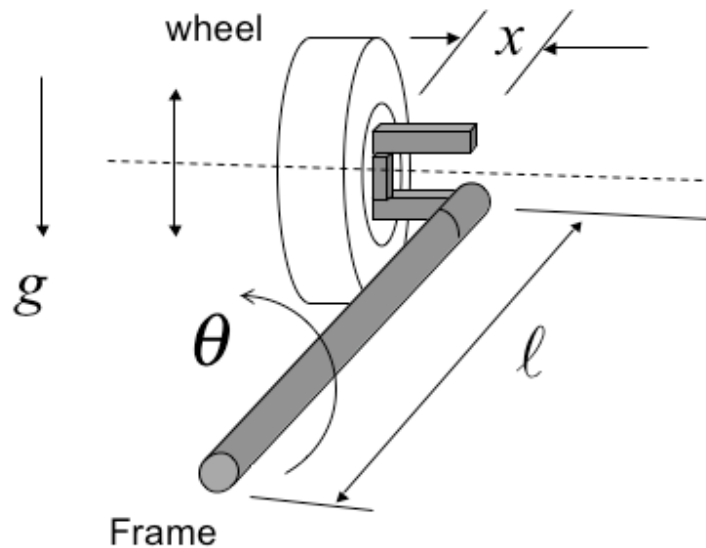
$$\Delta v = 175.4 - 200 = \textbf{-24.6 cm/s}$$

$$a_1 = A\omega_{n1}^2 = 10 \text{ cm} \times (20 \text{ rad/s})^2 = 4000 \text{ cm/s}^2$$

$$a_2 = A\omega_{n2}^2 = 10 \text{ cm} \times (17.54 \text{ rad/s})^2 = 3077 \text{ cm/s}^2$$

$$\Delta a = 3077 - 4000 = \textbf{-923 cm/s}^2$$

- 1.27** The front suspension of some cars contains a torsion rod as illustrated in Figure P1.27 to improve the car's handling. (a) Compute the frequency of vibration of the wheel assembly given that the torsional stiffness is 2000 N m/rad and the wheel assembly has a mass of 38 kg. Take the distance  $x = 0.26$  m. (b) Sometimes owners put different wheels and tires on a car to enhance the appearance or performance. Suppose a thinner tire is put on with a larger wheel raising the mass to 45 kg. What effect does this have on the frequency?



**Figure P1.27**

**Solution:** (a) Ignoring the moment of inertia of the rod, and computing the moment of inertia of the wheel as  $mx^2$ , the frequency of the shaft mass system is

$$\omega_n = \sqrt{\frac{k}{mx^2}} = \sqrt{\frac{2000 \text{ N} \cdot \text{m}}{38 \cdot \text{kg} (0.26 \text{ m})^2}} = \underline{27.9 \text{ rad/s}}$$

(b) The same calculation with 45 kg will *reduce* the frequency to

$$\omega_n = \sqrt{\frac{k}{mx^2}} = \sqrt{\frac{2000 \text{ N} \cdot \text{m}}{45 \cdot \text{kg} (0.26 \text{ m})^2}} = \underline{25.6 \text{ rad/s}}$$

This corresponds to about an 8% change in unsprung frequency and could influence wheel hop etc. You could also ask students to examine the effect of increasing  $x$ , as commonly done on some trucks to extend the wheels out for appearance sake.

- 1.28** A machine oscillates in simple harmonic motion and appears to be well modeled by an undamped single-degree-of-freedom oscillation. Its acceleration is measured to have an amplitude of 10,000 mm/s<sup>2</sup> at 8 Hz. What is the machine's maximum displacement?

**Solution:**

Given:  $a_{max} = 10,000 \text{ mm/s}^2 @ 8 \text{ Hz}$

The equations of motion for position and acceleration are:

$$x = A \sin(\omega_n t + \phi) \quad (1.3)$$

$$\ddot{x} = -A\omega_n^2 \sin(\omega_n t + \phi) \quad (1.5)$$

The amplitude of acceleration is  $A\omega_n^2 = 10,000 \text{ mm/s}^2$  and  $\omega_n = 2\pi f = 2\pi(8) = 16\pi \text{ rad/s}$ , from equation (1.12).

The machine's displacement is  $A = \frac{10,000}{\omega_n^2} = \frac{10,000}{(16\pi)^2}$

$$A = 3.96 \text{ mm}$$

- 1.29** A simple undamped spring-mass system is set into motion from rest by giving it an initial velocity of 100 mm/s. It oscillates with a maximum amplitude of 10 mm. What is its natural frequency?

**Solution:**

Given:  $x_0 = 0$ ,  $v_0 = 100 \text{ mm/s}$ ,  $A = 10 \text{ mm}$

From equation (1.9),  $A = \frac{v_0}{\omega_n}$  or  $\omega_n = \frac{v_0}{A} = \frac{100}{10}$ , so that:  $\omega_n = 10 \text{ rad/s}$

- 1.30** An automobile exhibits a vertical oscillating displacement of maximum amplitude 5 cm and a measured maximum acceleration of 2000 cm/s<sup>2</sup>. Assuming that the automobile can be modeled as a single-degree-of-freedom system in the vertical direction, calculate the natural frequency of the automobile.

**Solution:**

Given:  $A = 5$  cm. From equation (1.15)

$$|\ddot{x}| = A\omega_n^2 = 2000 \text{ cm/s}^2$$

Solving for  $\omega_n$  yields:

$$\omega_n = \sqrt{\frac{2000}{A}} = \sqrt{\frac{2000}{5}}$$
$$\omega_n = \mathbf{20 \text{ rad/s}}$$