Problems and Solutions Section 1.7 (1.82 through 1.89)

1.82 Choose a dashpot's viscous damping value such that when placed in parallel with the spring of Example 1.7.2 reduces the frequency of oscillation to 9 rad/s.

Solution:

The frequency of oscillation is $\omega_d = \omega_n\sqrt{1-\zeta^2}$

From example 1.7.2: $\omega_n = 10$ rad/s, $m = 10$ kg, and $k = 10^3$ N/m

So,
$$9 = 10\sqrt{1-\zeta^2}$$
$$\Rightarrow 0.9 = \sqrt{1-\zeta^2} \Rightarrow (0.9)^2 = 1-\zeta^2$$
$$\zeta = \sqrt{1-(0.9)^2} = 0.436$$

Then
$$c = 2m\omega_n\zeta = 2(10)(10)(0.436) = \underline{87.2 \text{ kg/s}}$$

1.83 For an underdamped system, $x_0 = 0$ and $v_0 = 10$ mm/s. Determine m, c, and k such that the amplitude is less than 1 mm.

Solution: Note there are multiple correct solutions. The expression for the amplitude is:

$$A^2 = x_0^2 + \frac{(v_o + \zeta\omega_n x_o)^2}{\omega_d^2}$$

$$\text{for } x_o = 0 \Rightarrow A = \frac{v_o}{\omega_d} < 0.001 \text{ m} \Rightarrow \omega_d > \frac{v_o}{0.001} = \frac{0.01}{0.001} = 10$$

So

$$\omega_d = \sqrt{\frac{k}{m}\left(1-\zeta^2\right)} > 10$$

$$\Rightarrow \frac{k}{m}\left(1-\zeta^2\right) > 100, \Rightarrow k = m\frac{100}{1-\zeta^2}$$

(1) Choose $\zeta = 0.01 \Rightarrow \dfrac{k}{m} > 100.01$

(2) Choose $\underline{m = 1 \text{ kg}}$ $\Rightarrow k > 100.01$

(3) Choose $k = 144$ N/m > 100.01

$$\Rightarrow \omega_n = \sqrt{144}\frac{\text{rad}}{\text{s}} = 12\frac{\text{rad}}{\text{s}}$$

$$\Rightarrow \omega_d = 11.99\frac{\text{rad}}{\text{s}}$$

$$\Rightarrow c = 2m\zeta\omega_n = \underline{\underline{0.24 \ \frac{\text{kg}}{\text{s}}}}$$

1.84 Repeat problem 1.83 if the mass is restricted to lie between 10 kg < *m* < 15 kg.

Solution: Referring to the above problem, the relationship between *m* and *k* is

$$k > 1.01\text{x}10^{-4}\, m$$

after converting to meters from mm. Choose *m* =10 kg and repeat the calculation at the end of Problem 1.82 to get ω_n (again taking ζ = 0.01). Then *k* = 1000 N/m and:

$$\Rightarrow \omega_n = \sqrt{\frac{1.0\times 10^3}{10}}\ \frac{\text{rad}}{\text{s}} = 10\ \frac{\text{rad}}{\text{s}}$$

$$\Rightarrow \omega_d = 9.998\ \frac{\text{rad}}{\text{s}}$$

$$\Rightarrow c = 2m\zeta\omega_n = \underline{\underline{2.000\ \frac{\text{kg}}{\text{s}}}}$$

1.85 Use the formula for the torsional stiffness of a shaft from Table 1.1 to design a 1-m shaft with torsional stiffness of 10^5 N·m/rad.

Solution: Referring to equation (1.64) the torsional stiffness is

$$k_t = \frac{GJ_p}{\ell}$$

Assuming a solid shaft, the value of the shaft polar moment is given by

$$J_p = \frac{\pi d^4}{32}$$

Substituting this last expression into the stiffness yields:

$$k_t = \frac{G\pi d^4}{32\ell}$$

Solving for the diameter d yields

$$d = \left(\frac{k_t(32)\ell}{G\pi}\right)^{1/4}$$

Thus we are left with the design variable of the material modulus (G). Choose steel, then solve for d. For steel $G = 8 \times 10^{10}$ N/m^2. From the last expression the numerical answer is

$$d = \left[\frac{10^5 \frac{\text{Nm}}{\text{rad}}(32)(1\text{m})}{\left(8\times 10^{10}\frac{\text{N}}{\text{m}^2}\right)(\pi)}\right]^{1/4} = 0.0597 \text{ m}$$

1.86 Repeat Example 1.7.2 using aluminum. What difference do you note?

Solution:

For aluminum $G = 25 \times 10^9$ N/m^2

From example 1.7.2, the stiffness is $k = 10^3 = \dfrac{Gd^4}{64nR^3}$ and $d = .01$ m

So, $10^3 = \dfrac{(25\times 10^9)(.01)^4}{64nR^3}$

Solving for nR^3 yields: $nR^3 = 3.906 \times 10^{-3}\text{m}^3$

Choose $R = 10$ cm $= 0.1$ m, so that

$$n = \frac{3.906 \times 10^{-3}}{(0.1)^3} = 4\,\text{turns}$$

Thus, aluminum requires 1/3 fewer turns than steel.

1.87 Try to design a bar (see Figure 1.21) that has the same stiffness as the spring of Example 1.7.2. Note that the bar must remain at least 10 times as long as it is wide in order to be modeled by the formula of Figure 1.21.

Solution:

From Figure 1.21, $k = \dfrac{EA}{l}$

For steel, $E = 210 \times 10^9\ \text{N/m}^2$

From Example 1.7.2, $k = 10^3$ N/m

So, $10^3 = \dfrac{(210 \times 10^9)A}{l}$

$$l = (2.1 \times 10^8)A$$

If $A = 0.0001\ \text{m}^2$ ($1\ \text{cm}^2$), then

$$l = (2.1 \times 10^8)(10^{-4}) = 21{,}000\ \text{m}\ \ (21\text{km or 13 miles})$$

Not very practical at all.

1.88 Repeat Problem 1.87 using plastic (E = 1.40 × 10^9 N/m^2) and rubber (E = 7 × 10^6 N/m^2). Are any of these feasible?

Solution:

From problem 1.53, $k = 10^3 \text{ N/m} = \dfrac{EA}{l}$

For plastic, $E = 1.40 \times 10^9 \text{ N/m}^2$

So, $l = 140 \text{ m}$

For rubber, $E = 7 \times 10^6 \text{ N/m}^2$

So, $l = 0.7 \text{ m}$

Rubber may be feasible, plastic would not.

1.89 Consider the diving board of Figure P1.89. For divers, a certain level of static deflection is desirable, denoted by Δ. Compute a design formula for the dimensions of the board (b, h and ℓ) in terms of the static deflection, the average diver's mass, m, and the modulus of the board.

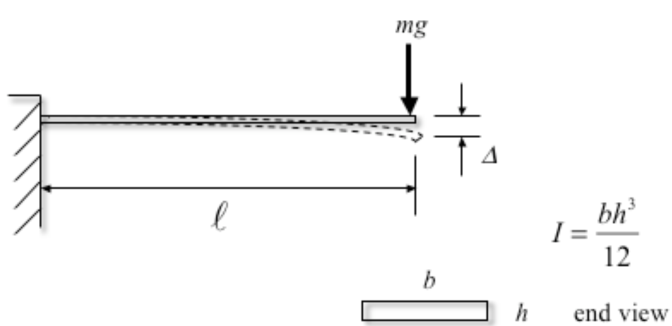

Figure P1.89

Solution: From Figure 1.15 (b), $\Delta k = mg$ holds for the static deflection. The period is:

$$T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/\Delta}} = 2\pi\sqrt{\frac{\Delta}{g}} \quad (1)$$

From Figure 1.24, we also have that

$$T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m\ell^3}{3EI}} \quad (2)$$

Equating (1) and (2) and replacing I with the value from the figure yields:

$$2\pi\sqrt{\frac{m\ell^3}{3EI}} = 2\pi\sqrt{\frac{12m\ell^3}{3Ebh^3}} = 2\pi\sqrt{\frac{\Delta}{g}} \Rightarrow \underline{\frac{\ell^3}{bh^3} = \frac{\Delta E}{4mg}}$$

Alternately just use the static deflection expression and the expression for the stiffness of the beam from Figure 1.24 to get

$$\Delta k = mg \Rightarrow \Delta\frac{3EI}{\ell^3} = mg \Rightarrow \underline{\frac{\ell^3}{bh^3} = \frac{\Delta E}{4mg}}$$