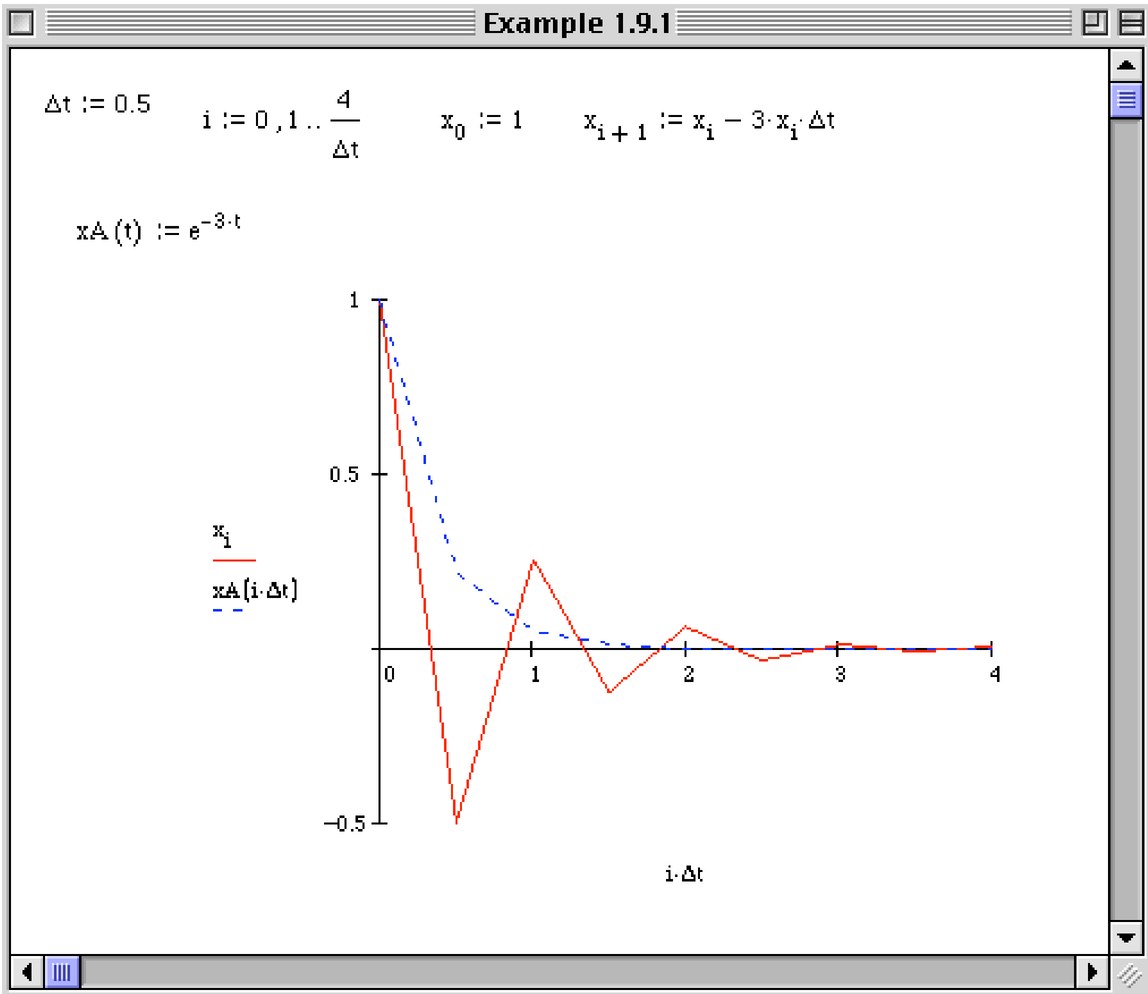
Problems and Solutions Section 1.9 (1.94 through 1.101)

1.94* Reproduce Figure 1.38 for the various time steps indicated.

Solution: The code is given here in Mathcad, which can be run repeatedly with different Δt to see the importance of step size. Matlab and Mathematica can also be used to show this.



1.95* Use numerical integration to solve the system of Example 1.7.3 with m = 1361 kg, k = 2.688 x 10^5 N/m, c = 3.81 x 10^3 kg/s subject to the initial conditions $x(0) = 0$ and $v(0)$ = 0.01 mm/s. Compare your result using numerical integration to just plotting the analytical solution (using the appropriate formula from Section 1.3) by plotting both on the same graph.

Solution: The solution is shown here in Mathcad using an Euler integration. This can also been done in the other codes or the Toolbox:

1.84

$k := 2.688\cdot 10^5 \quad m := 1361 \quad x_0 := 0.0 \quad v_0 := 0.01 \quad c := 3.81\cdot 10^3$ all units in m, kg, s

$$\omega n := \sqrt{\frac{k}{m}} \qquad fn := \frac{\omega n}{2\cdot\pi} \qquad \zeta := \frac{c}{2\cdot\sqrt{m\cdot k}} \qquad \omega d := \omega n\cdot\sqrt{1-\zeta^2}$$

$$A := \frac{1}{\omega d}\cdot\sqrt{\left(v_0+\zeta\cdot x_0\cdot\omega n\right)^2+\left(x_0\cdot\omega d\right)^2} \qquad T := \frac{2\cdot\pi}{\omega d}$$

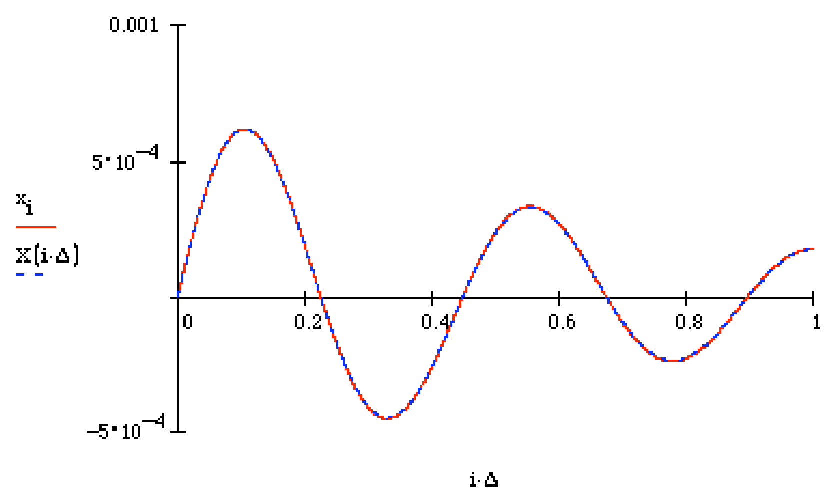
$$\phi := \operatorname{atan}\left[\frac{\omega d\cdot x_0}{v_0+\zeta\cdot\omega n\cdot x_0}\right] \qquad \zeta = 0.1$$

$$X(t) := A\cdot\sin(\omega d\cdot t+\phi)\cdot e^{-\zeta\cdot\omega n\cdot t}$$

$$N := 10000 \qquad \Delta := \frac{6\cdot\pi}{\omega n\cdot N} \qquad y := \begin{bmatrix}0\\0.01\end{bmatrix}$$

$$i := 0..N$$

$$\begin{bmatrix}x_{i+1}\\v_{i+1}\end{bmatrix} := \begin{bmatrix}v_i\cdot\Delta+x_i\\-\omega n^2\cdot(\Delta)\cdot x_i-2\cdot\zeta\cdot\omega n\cdot v_i\cdot\Delta+v_i\end{bmatrix}$$



1.96* Consider again the damped system of Problem 1.95 and design a damper such that the oscillation dies out after 2 seconds. There are at least two ways to do this. Here it is intended to solve for the response numerically, following Examples 1.9.2, 1.9.3 or 1.9.4, using different values of the damping parameter *c* until the desired response is achieved.

Solution: Working directly in Mathcad (or use one of the other codes). Changing *c* until the response dies out within about 2 sec yields *c* =6500 kg/s or ζ = 0.17.

1.85

$k := 2.688\cdot 10^5$ $\quad m := 1361$ $\quad x_0 := 0.0$ $\quad v_0 := 0.01$ $\quad c := 6.5\cdot 10^3$ $\quad$ all units in m, kg, s

$$\omega n := \sqrt{\frac{k}{m}} \qquad fn := \frac{\omega n}{2\cdot\pi} \qquad \zeta := \frac{c}{2\cdot\sqrt{m\cdot k}} \qquad \omega d := \omega n\cdot\sqrt{1-\zeta^2} \qquad T := \frac{2\cdot\pi}{\omega d}$$

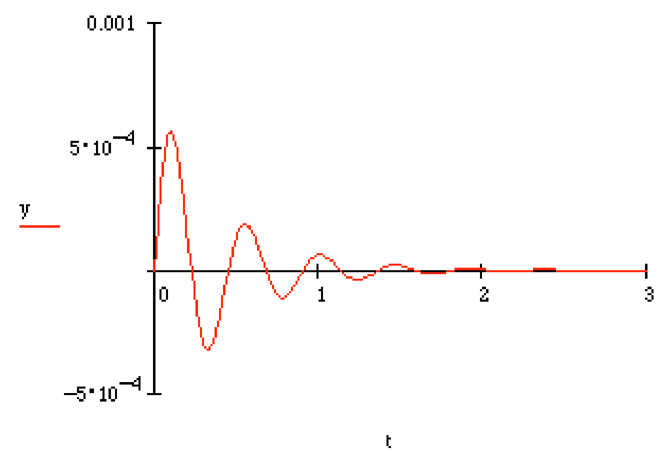
$$A := \frac{1}{\omega d}\cdot\sqrt{\left(v_0+\zeta\cdot x_0\cdot\omega n\right)^2+\left(x_0\cdot\omega d\right)^2} \qquad \phi := \operatorname{atan}\left[\frac{\omega d\cdot x_0}{v_0+\zeta\cdot\omega n\cdot x_0}\right] \qquad y := \begin{bmatrix} x_0 \\ v_0 \end{bmatrix}$$

$$D(t,y) := \begin{bmatrix} y_1 \\ -\left(2\cdot\zeta\cdot\omega n\cdot y_1\right)-\left(\omega n^2\right)\cdot y_0 \end{bmatrix} \qquad Z := \mathrm{rkfixed}(y,0,20,1000,D)$$

$A = 7.221\cdot 10^{-4}$ in meters $\qquad \dfrac{\phi}{deg} = 0$ in degrees $\qquad t := Z^{\langle 0\rangle}$

$\omega n = 14.054$ in rad/s $\qquad fn = 2.237$ in Hz $\qquad T = 0.454$ in sec $\qquad \zeta = 0.17$ $\qquad y := Z^{\langle 1\rangle}$

$\omega d = 13.849$



1.97* Consider again the damped system of Example 1.9.2 and design a damper such that the oscillation dies out after 25 seconds. There are at least two ways to do this. Here it is intended to solve for the response numerically, following Examples 1.9.2, 1.9.3 or 1.9.4, using different values of the damping parameter *c* until the desired response is achieved. Is your result overdamped, underdamped or critically damped?

Solution: The following Mathcad program is used to change *c* until the desired response results. This yields a value of c = 1.1 kg/s or $\zeta = 0.225$, an underdamped solution.

1.86_1.9.2

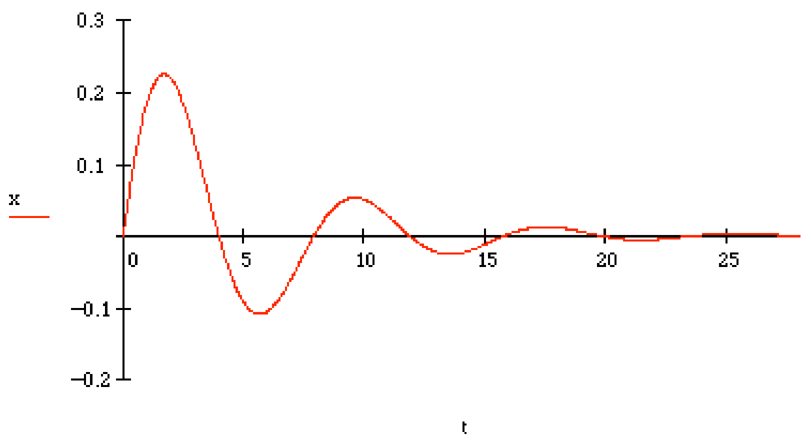
$k := 2 \qquad m := 3 \qquad x_0 := 0.0 \qquad v_0 := 0.25 \qquad c := 1.1$

all units in m, kg, s

$$\omega n := \sqrt{\frac{k}{m}} \qquad fn := \frac{\omega n}{2\cdot\pi} \qquad \zeta := \frac{c}{2\cdot\sqrt{m\cdot k}} \qquad \omega d := \omega n\cdot\sqrt{1-\zeta^2} \qquad T := \frac{2\cdot\pi}{\omega d}$$

$$x := \begin{bmatrix} x_0 \\ v_0 \end{bmatrix} \qquad D(t,x) := \begin{bmatrix} x_1 \\ -(2\cdot\zeta\cdot\omega n\cdot x_1) - (\omega n^2)\cdot x_0 \end{bmatrix} \qquad Z := \mathrm{rkfixed}(x, 0, 28, 1000, D)$$

$$t := Z^{\langle 0 \rangle} \qquad x := Z^{\langle 1 \rangle}$$



$\omega d = 0.796 \qquad \omega n = 0.816 \qquad fn = 0.13 \qquad T = 7.897 \qquad \zeta = 0.225$

in rad/s _ _ _ _ _ in Hz . . in sec .

1.98* Repeat Problem 1.96 for the initial conditions $x(0) = 0.1$ m and $v(0) = 0.01$ mm/s.

Solution: Using the code in 1.96 and changing the initial conditions does not change the settling time, which is just a function of ζ and ω_n. Hence the value of $c = 6.5\text{x}10^3$ kg/s ($\zeta = 0.17$) as determined in problem 1.96 will still reduce the response within 2 seconds.

1.99* A spring and damper are attached to a mass of 100 kg in the arrangement given in Figure 1.9. The system is given the initial conditions $x(0) = 0.1$ m and $v(0) = 1$ mm/s. Design the spring and damper (i.e. choose k and c) such that the system will come to rest in 2 s and not oscillate more than two complete cycles. Try to keep c as small as possible. Also compute ζ.

Solution: In performing this numerical search on two parameters, several underdamped solutions are possible. Students will note that increasing k will decrease ζ. But increasing k also increases the number of cycles which is limited to two. A solution with $c = 350$ kg/s and k =2000 N/m is illustrated.

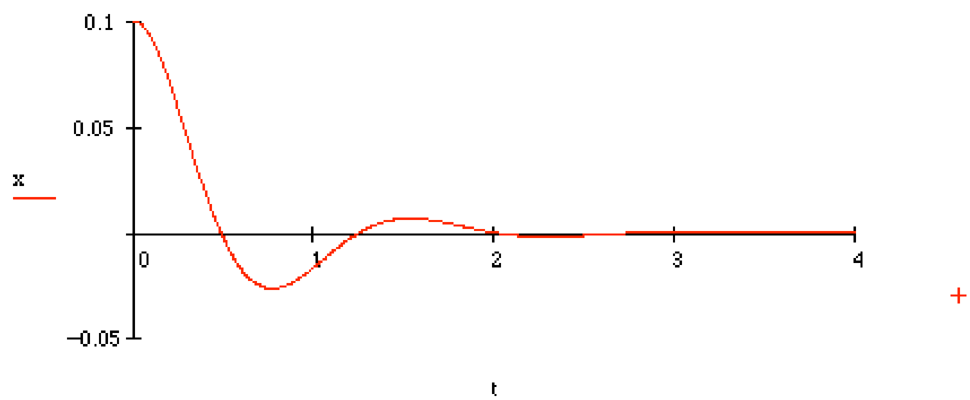
1.88

$k := 2000 \quad m := 100 \quad x_0 := 0.1 \quad v_0 := 0.01 \quad c := 350$ all units in m, kg, s

$$\omega n := \sqrt{\frac{k}{m}} \quad fn := \frac{\omega n}{2\cdot\pi} \quad \zeta := \frac{c}{2\cdot\sqrt{m\cdot k}} \quad \omega d := \omega n\cdot\sqrt{1-\zeta^2} \quad T := \frac{2\cdot\pi}{\omega d}$$

$$x := \begin{bmatrix} x_0 \\ v_0 \end{bmatrix} \quad D(t,x) := \begin{bmatrix} x_1 \\ -(2\cdot\zeta\cdot\omega n\cdot x_1) - (\omega n^2)\cdot x_0 \end{bmatrix} \quad Z := \mathrm{rkfixed}(x, 0, 4, 1000, D)$$

$$t := Z^{\langle 0 \rangle} \quad x := Z^{\langle 1 \rangle}$$

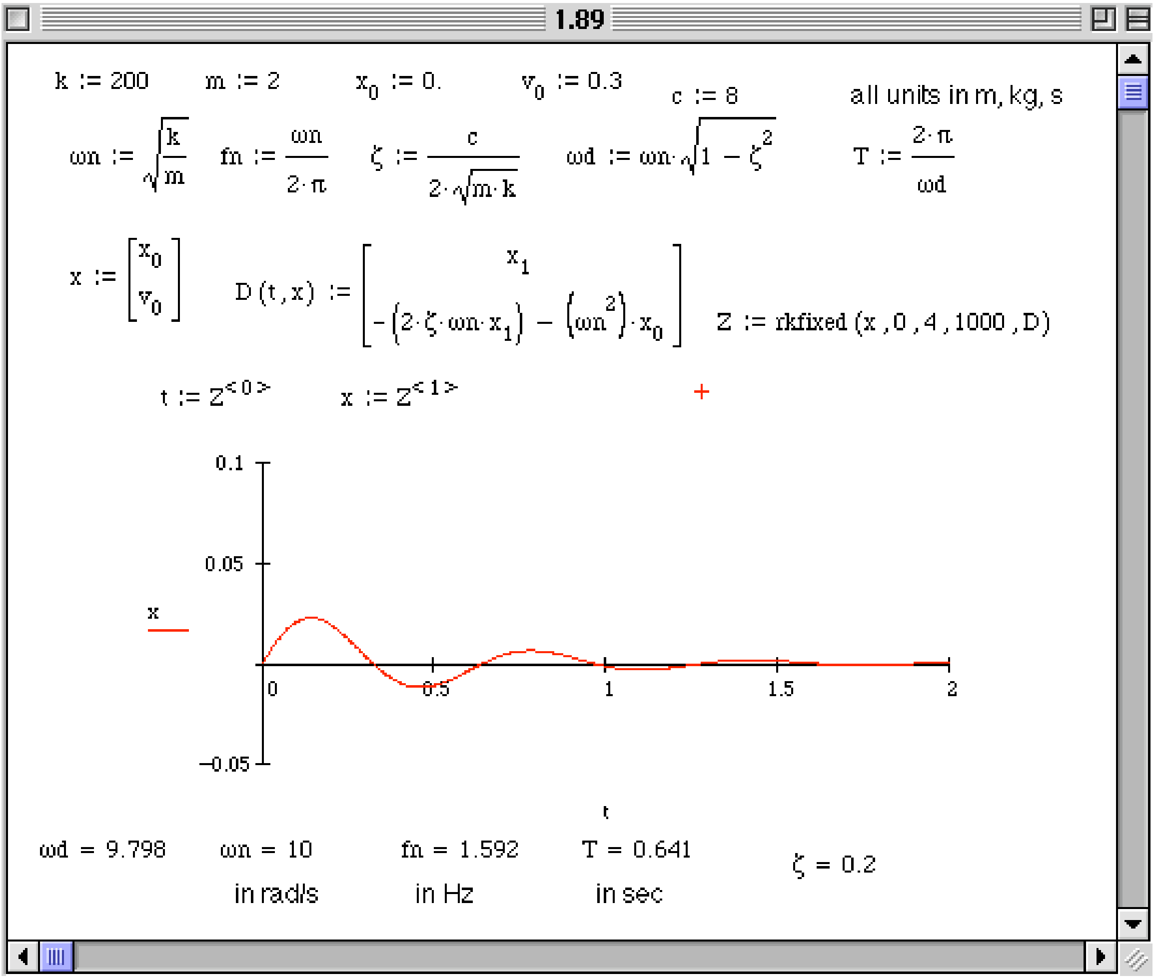


$\omega d = 4.116 \quad \omega n = 4.472 \quad fn = 0.712 \quad T = 1.527 \quad \zeta = 0.391$

in rad/s in Hz in sec

1.100* Repeat Example 1.7.1 by using the numerical approach of the previous 5 problems.

Solution: The following Mathcad session can be used to solve this problem by varying the damping for the fixed parameters given in Example 1.7.1.



The other codes or the toolbox may also be used to do this.

1.101* Repeat Example 1.7.1 for the initial conditions $x(0) = 0.01$ m and $v(0) = 1$ mm/s.

Solution: The above Mathcad session can be used to solve this problem by varying the damping for the fixed parameters given in Example 1.7.1. For the given values of initial conditions, the solution to Problem 1.100 also works in this case. Note that if $x(0)$ gets too large, this problem will not have a solution.