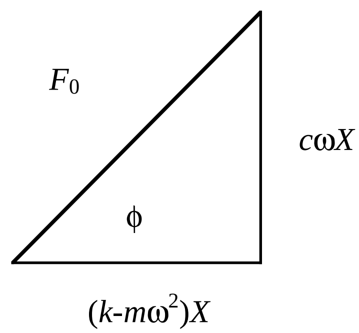
Problems and Solutions Section 2.3 (2.32 through 2.36)

2.32 Referring to Figure 2.10, draw the solution for the magnitude *X* for the case *m* = 100 kg, *c* = 4000 N s/m, and *k* = 10,000 N/m. Assume that the system is driven at resonance by a 10-N force.

Solution:
Given: m = 100 kg, c = 4000 N s/m, k = 10000 N/m, F_o = 10 N,

$$\omega = \omega_n = \sqrt{\frac{k}{m}} = 10 \text{ rad/s}$$

$$\phi = \tan^{-1}\left[\frac{cw}{k - m\omega^2}\right] = \tan^{-1}\left[\frac{(40{,}000)}{(10{,}000 - 10{,}000)}\right] = 90° = \frac{\pi}{2} \text{ rad}$$



From the figure:

$$X = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (c\,\omega)^2}} = \frac{10}{\sqrt{(10{,}000 - 10{,}000)^2 + (40{,}000)^2}}$$

X = 0.00025 m

2.33 Use the graphical method to compute the phase shift for the system of Problem 2.32 if $\omega = \omega_n/2$ and again for the case $\omega = 2\omega_n$.

Solution:

From Problem 2.32 $\omega_n = 10$ rad/s

(a) $\omega = \dfrac{\omega_n}{2} = 5$ rad/s

$$X = \frac{10}{\sqrt{(10{,}000 - 2500)^2 + (20{,}000)^2}} = .000468 \text{ m}$$

$kX = (10{,}000)(.000468) = 4.68$ N

$c\omega X = (4000)(5)(.000468) = 9.36$ N

$m\omega^2 X = (100)(5)^2(.000468) = 1.17$ N

From the figure given in problem 2.32:

$$\phi = \tan^{-1}\left[\frac{9.36}{4.68 - 1.17}\right] = 69.4° = 1.21 \text{rad}$$

(b) $\omega = 2\omega_n = 20$ rad/s

$$X = \frac{10}{\sqrt{(10000 - 40000)^2 + (80000)^2}} = .000117 \text{ m}$$

$kX = (10000)(.000117) = 1.17$ N

$c\omega X = (4000)(20)(.000117) = 9.36$ N

$m\omega^2 X = (100)(20)^2(.000117) = 4.68$ N

From the figure:

$$\phi = \tan^{-1}\left[\frac{9.36}{1.17 - 4.68}\right] = -69.4° = -1.21 \text{rad}$$

2.34 A body of mass 100 kg is suspended by a spring of stiffness of 30 kN/m and dashpot of damping constant 1000 N s/m. Vibration is excited by a harmonic force of amplitude 80 N and a frequency of 3 Hz. Calculate the amplitude of the displacement for the vibration and the phase angle between the displacement and the excitation force using the graphical method.

Solution:

Given: m = 100kg, k = 30 kN/m, F_o = 80 N, c = 1000 Ns/m,

$\omega = 3(2\pi) = 18.85$ rad/s

$kX = 30000\,X$

$c\omega X = 18850\,X$

$m\omega^2 X = 35530\,X$

Following the figure given in problem 2.32:

$$\phi = \tan^{-1}\left[\frac{c\omega X}{\left(k - m\omega^2\right)X}\right]$$

$$\phi = \tan^{-1}\left[\frac{(18850)X}{(30000 - 35530)X}\right] = 106.4° = 1.86\,\text{rad}$$

Also from the figure, $$X = \frac{F_0}{\sqrt{\left(k - m\omega^2\right)^2 + \left(c\omega\right)^2}}$$

$$X = \frac{80}{\sqrt{(30000 - 35530)^2 + (18850)^2}} = 0.00407 \text{ m}$$

2.35 Calculate the real part of equation (2.55) to verify that it yields equation (2.36) and hence establish the equivalence of the exponential approach to solving the damped vibration problem.

Solution:

Equation (2.55) $$x_p(t) = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} e^{j(\omega t - \theta)}$$

where $\theta = \tan^{-1}\left[\dfrac{c\omega}{k - m\omega^2}\right]$

Using Euler's Rule: $$x_p(t) = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}[\cos(\omega t - \theta) + j\sin(\omega t - \theta)]$$

The real part is: $$x_p(t) = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}\cos(\omega t - \theta)$$

Rearranging: $$x_p(t) = \frac{F_o / m}{\sqrt{(\omega^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}\cos\left(\omega t - \tan^{-1}\left[\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right]\right)$$

which is Equation (2.36).

2.36 Referring to equation (2.56) and Appendix B, calculate the solution $x(t)$ by using a table of Laplace transform pairs and show that the solution obtained this way is equivalent to (2.36).

Solution: Taking the Laplace transform of the equation of motion is given in Equation (2.56): $$X_p = (ms^2 + cs + k)X(s) = \frac{F_o s}{s^2 + \omega^2}$$

Solving this expression algebraically for X yields

$$X(s) = \frac{F_0 s}{(ms^2 + cs + k)(s^2 + \omega^2)} = \frac{f_0 s}{(s^2 + 2\zeta\omega_n s + \omega^2)(s^2 + \omega^2)}$$

Using Laplace Transform pairs from the table, this last expression is changed into the time domain to get:

$$x(t) = \frac{f_0}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}} \cos(\omega t\text{-}\theta)$$