Problems and Solutions Section 2.4 (2.37 through 2.50)

2.37 A machine weighing 2000 N rests on a support as illustrated in Figure P2.37. The support deflects about 5 cm as a result of the weight of the machine. The floor under the support is somewhat flexible and moves, because of the motion of a nearby machine, harmonically near resonance (r =1) with an amplitude of 0.2 cm. Model the floor as base motion, and assume a damping ratio of ζ = 0.01, and calculate the transmitted force and the amplitude of the transmitted displacement.

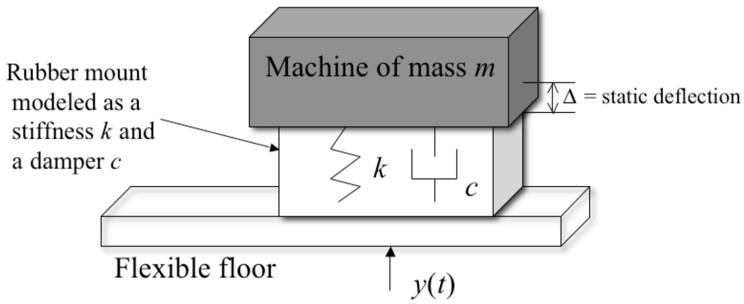


Figure P2.37

Solution:

Given: Y = 0.2 cm, ζ = 0.01, r = 1, mg = 2000N. The stiffness is computed from the static deflection and weight:

Deflection of 5 cm implies: $k = \dfrac{mg}{\Delta} = \dfrac{mg}{5\text{cm}} = \dfrac{2000}{0.05}$ = 40,000 N/m

Transmitted displacement from equation (2.70): $X = Y\left[\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$ = 10 cm

Transmitted force from equation (2.77): $F_T = kYr^2\left[\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$ = 4001N

2.38 Derive Equation (2.70) from (2.68) to see if the author has done it correctly.

Solution:

Equation (2.68) states:

$$x_p(t) = \omega_n Y\left[\frac{\omega_n^2+(2\zeta\omega_b)^2}{(\omega_n^2-\omega_b^2)^2+(2\zeta\omega_n\omega_b)^2}\right]^{1/2}\cos(\omega_b t-\theta_1-\theta_2)$$

The magnitude is: $X = \omega_n Y\left[\dfrac{\omega_n^2+(2\zeta\omega_b)^2}{(\omega_n^2-\omega_b^2)^2+(2\zeta\omega_n\omega_b)^2}\right]^{1/2}$

$$= \omega_n Y \left[\frac{(\omega_n^{-4})(\omega_n^2 + (2\zeta\omega_b)^2)}{(\omega_n^{-4})((\omega_n^2 - \omega_b^2)^2 + (2\zeta\omega_n\omega_b)^2)} \right]^{1/2}$$

$$= \omega_n Y \left[\frac{(\omega_n^{-2})(1 + (2\zeta r)^2)}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2} \Rightarrow$$

$$= \omega_n Y \frac{1}{\omega_n} \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2} \Rightarrow$$

$$X = Y \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2}$$

This is equation (2.71).

2.39 From the equation describing Figure 2.13, show that the point ($\sqrt{2}$, 1) corresponds to the value TR > 1 (i.e., for all $r < \sqrt{2}$, TR > 1).

Solution:

Equation (2.71) is $TR = \dfrac{X}{Y} = \left[\dfrac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2}$

Show $TR > 1$ for $r < \sqrt{2}$

$$TR = \frac{X}{Y} = \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2} > 1$$

$$\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} > 1$$

$$1 + (2\zeta r)^2 > (1 - r^2)^2 + (2\zeta r)^2$$

$$1 > (1 - r^2)^2$$

Take the real solution:

$$1 - r^2 < +1 \text{ or } 1 - r^2 < -1 \Rightarrow$$
$$-r^2 > -2 \Rightarrow r^2 < 2 \Rightarrow r < \sqrt{2}$$

2.40 Consider the base excitation problem for the configuration shown in Figure P2.40. In this case the base motion is a displacement transmitted through a dashpot or pure damping element. Derive an expression for the force transmitted to the support in steady state.

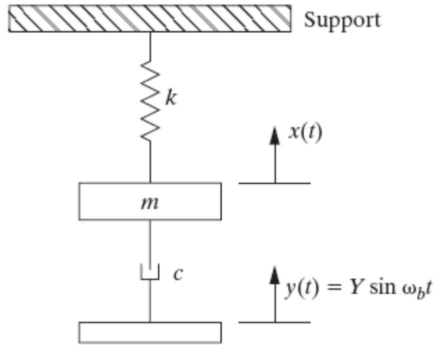


Figure P2.40

Solution: The entire force passes through the spring. Thus the support sees the force $F_T = kX$ where X is the magnitude of the displacement. From equation (2.65)

$$F_T = kX = \frac{2\zeta\omega_n\omega_b kY}{\sqrt{(\omega_n^2 - \omega_b^2)^2 + (2\zeta\omega_n\omega_b)^2}}$$

$$= \frac{2\zeta rkY}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

2.41 A very common example of base motion is the single-degree-of-freedom model of an automobile driving over a rough road. The road is modeled as providing a base motion displacement of $y(t) = (0.01)\sin(5.818t)$ m. The suspension provides an equivalent stiffness of $k = 4 \times 10^5$ N/m, a damping coefficient of $c = 40 \times 10^3$ kg/s and a mass of 1007 kg. Determine the amplitude of the absolute displacement of the automobile mass.

Solution:

From the problem statement we have (working in Mathcad)

$\omega b := 5.818$ $\quad k := 4 \cdot 10^5$ N/m $\quad c := 40 \cdot 10^3$ kg/s

$Y := 0.01$ m $\quad m := 1007$ kg

$\omega n := \sqrt{\frac{k}{m}}$ $\quad \zeta := \frac{c}{2 \cdot \sqrt{m \cdot k}}$ $\quad \omega n = 19.93$ $\quad \zeta = 0.997$ $\quad r := \frac{\omega b}{\omega n}$ $\quad r = 0.292$

still underdamped, but very high damping. From equation (2.70)

$$X := Y \cdot \sqrt{\frac{1 + (2 \cdot \zeta \cdot r)^2}{\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2}} \qquad X = 0.011 \text{ m}$$

2.42 A vibrating mass of 300 kg, mounted on a massless support by a spring of stiffness 40,000 N/m and a damper of unknown damping coefficient, is observed to vibrate with a 10-mm amplitude while the support vibration has a maximum amplitude of only 2.5 mm (at resonance). Calculate the damping constant and the amplitude of the force on the base.

Solution:

Given: m = 300 kg, k = 40,000 N/m, $\omega_b = \omega_n (r = 1)$, X = 10 mm, Y = 2.5 mm.
Find damping constant (Equation 2.71)

$$\frac{X}{Y} = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2} \Rightarrow \frac{10}{2.5} = \left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2} \Rightarrow$$

$$16 = \frac{1+4\zeta^2}{4\zeta^2} \Rightarrow \zeta^2 = \frac{1}{60} = \frac{c^2}{4km} \quad \text{or}$$

$$c = \sqrt{\frac{4(40{,}000)(300)}{60}} = 894.4 \text{ kg/s}$$

Amplitude of force on base: (equation (2.76))

$$F_T = kYr^2\left[\frac{1+(2\zeta r)^2}{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}\right]^{1/2} \Rightarrow$$

$$F_T = (40{,}000)(0.0025)(1)^2\left[\frac{1+4\left(\frac{1}{60}\right)}{4\left(\frac{1}{60}\right)}\right]^{1/2} \Rightarrow$$

$$F_T = 400 \text{ N}$$

2.43 Referring to Example 2.4.1, at what speed does car 1 experience resonance? At what speed does car 2 experience resonance? Calculate the maximum deflection of both cars at resonance.

Solution:

Given: m_1 = 1007 kg, m_2 =1585 kg, k = 4x10^5 N/m; c = 2,000 kg/s, Y = 0.01m

Velocity for resonance: (from Example 2.4.1)

$$\omega_b = 0.2909v \text{ } (v \text{ in km/h})$$

Car 1: $\omega_1 = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{4\times 10^4}{1007}} = \omega_b = 0.2909v_1$

$$v_1 = 21.7 \text{ km/h}$$

Car 2: $\omega_2 = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{4\times 10^4}{1585}} = \omega_b = 0.2909v_2$

$$v_2 = 17.3 \text{ km/h}$$

Maximum deflection: (Equation 2.71 with r = 1)

$$X = Y\left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2} \Rightarrow$$

Car 1: $\zeta_1 = \dfrac{c}{2\sqrt{km_1}} = \dfrac{2000}{2\sqrt{(4\times 10^5)(1007)}} = 0.158$

$$X_1 = (0.01)\left[\frac{1+4(0.158)^2}{4(0.158)^2}\right]^{1/2} = 0.033 \text{ m}$$

Car 2: $\zeta_2 = \dfrac{c}{2\sqrt{km_2}} = \dfrac{2000}{2\sqrt{(4\times 10^4)(1585)}} = 0.126$

$$X_2 = (0.01)\left[\frac{1+4(0.126)^2}{4(0.126)^2}\right]^{1/2} = 0.041 \text{ m}$$

2.44 For cars of Example 2.4.1, calculate the best choice of the damping coefficient so that the transmissibility is as small as possible by comparing the magnitude of ζ = 0.01, ζ = 0.1 and ζ = 0.2 for the case r = 2. What happens if the road "frequency" changes?

Solution:

From Equation 2.62, with r = 2, the displacement transmissibility is:

$$\frac{X}{Y} = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2} = \left[\frac{1+16\zeta^2}{9+16\zeta^2}\right]^{1/2}$$

For $\zeta = 0.01$, $\frac{X}{Y} = 0.334$

For $\zeta = 0.1$, $\frac{X}{Y} = 0.356$

For $\zeta = 0.2$, $\frac{X}{Y} = 0.412$

The best choice would be $\zeta = 0.01$.

If the road frequency increases, the lower damping ratio would still be the best choice. However, if the frequency decreases, a higher damping ratio would be better because it would approach resonance.

2.45 A system modeled by Figure 2.12**,** has a mass of 225 kg with a spring stiffness of 3.5 $\times 10^4$ N/m. Calculate the damping coefficient given that the system has a deflection (X) of 0.7 cm when driven at its natural frequency while the base amplitude (Y) is measured to be 0.3 cm.

Solution:

Given: m = 225 kg, k = 3.5x10^4 N/m, X = 0.7 cm, Y = 0.3 cm, $\omega = \omega_b$.

Base excitation: (Equation (2.71) with r = 1)

$$\frac{X}{Y} = \left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2} \Rightarrow \frac{0.7}{0.3} = \left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2} \Rightarrow$$

$$\zeta = 0.237 = \frac{c}{2\sqrt{km}}$$

$$c = (0.237)(2)[(3.5\text{x}10^4)(225)]^{1/2}$$

$\underline{c = 1331 \text{ kg/s}}$

2.46 Consider Example 2.4.1 for car 1 illustrated in Figure P2.46, if three passengers totaling 200 kg are riding in the car. Calculate the effect of the mass of the passengers on the deflection at 20, 80, 100, and 150 km/h. What is the effect of the added passenger mass on car 2?

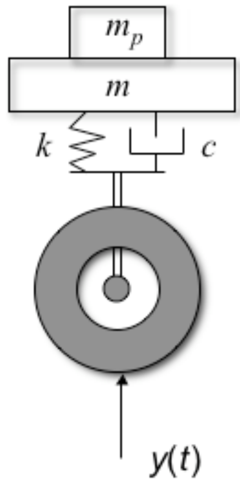


Figure P2.46 Model of a car suspension with the mass of the occupants, m_p, included.

Solution:

Add a mass of 200 kg to each car. From Example 2.4.1, the given values are:
m_1 = 1207 kg, m_2 = 1785 kg, k = 4x10^4 N/m; c = 2,000 kg/s, ω_b = 0.29v.

Car 1: $\omega_1 = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{4\times10^4}{1207}} = 5.76$ rad/s

$$\zeta_1 = \frac{c}{2\sqrt{km_1}} = \frac{2000}{2\sqrt{(4\times10^5)(1207)}} = 0.144$$

Car 2: $\omega_2 = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{4\times10^4}{1785}} = 4.73$ rad/s

$$\zeta_2 = \frac{c}{2\sqrt{km_2}} = \frac{2000}{2\sqrt{(4\times10^5)(1785)}} = 0.118$$

Using Equation (2.71): $X = Y\left[\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$ produces the following:

Speed (km/h)	ω_b (rad/s)	r_1	r_2	x_1 (cm)	x_2 (cm)
20	5.817	1.01	1.23	3.57	1.77
80	23.271	3.871	4.71	0.107	0.070
100	29.088	5.05	6.15	0.072	0.048
150	2.40	7.58	9.23	0.042	0.028

At lower speeds there is little effect from the passengers weight, but at higher speeds the added weight reduces the amplitude, particularly in the smaller car.

2.47 Consider Example 2.4.1. Choose values of *c* and *k* for the suspension system for car 2 (the sedan) such that the amplitude transmitted to the passenger compartment is as small as possible for the 1 cm bump at 50 km/h. Also calculate the deflection at 100 km/h for your values of *c* and *k*.

Solution:

For car 2, m = 1585 kg.
Also, ω_b = 0.2909(50) = 14.545 rad/s and Y = 0.01 m.
From equation (2.70),

$$X = Y\left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$$

From Figure 2.9, we can choose a value of r away from resonance and a low damping ratio. Choose r = 2.5 and ζ =0.05.

So, $r = 2.5 = \dfrac{\omega_b}{\omega} = \dfrac{14.545}{\sqrt{k/1585}}$

k = 53,650 N/m

$\zeta = 0.05 = \dfrac{c}{2\sqrt{km}}$

c = 922.2 kg/s

So, $$X = (0.01)\left[\frac{1+[2(0.05)(2.5)]^2}{\left(1-\left(2.5\right)^2\right)^2+[2(0.05)(2.5)]^2}\right]^{1/2} = 0.00196 \text{ m}$$

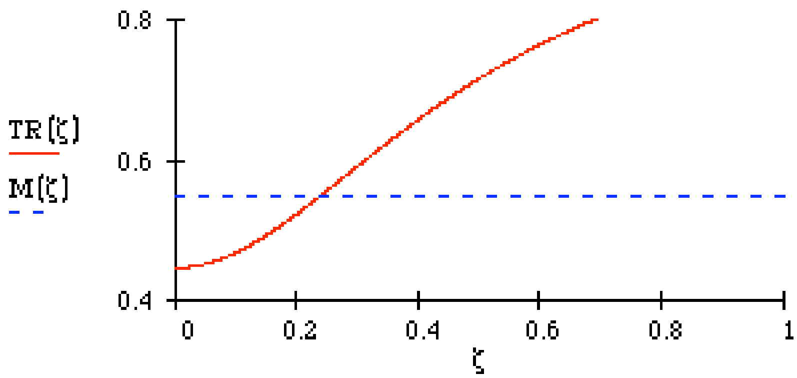
At 100 km/h, ω_b = 29.09 rad/s and $r = \dfrac{\omega_b}{\sqrt{k/m}} = 5.$

2.48 Consider the base motion problem of Figure 2.12. a) Compute the damping ratio needed to keep the displacement magnitude transmissibility less then 0.55 for a frequency ratio of r = 1.8. b) What is the value of the force transmissibility ratio for this system?

Solution: Working with equation (2.71), make a plot of TR versus ζ and use equation (2.77) to compute the value of the force transmissibility. The following Mathcad session illustrates the procedure.

$r := 1.8$

$$TR(\zeta) := \sqrt{\frac{1 + (2\cdot\zeta\cdot r)^2}{(1 - r^2)^2 + (2\cdot\zeta\cdot r)^2}} \qquad M(\zeta) := 0.55$$



$F(\zeta) := r^2\cdot TR(\zeta)$

$F(.2) = 1.697$

From the plot a value of ζ = 0.2 keeps the displacement transmissibility less then 0.55 as desired. The value of the force transmissibility is then 1.697. Precise values can be found by equating the above expression to 0.55.

2.49 Consider the effect of variable mass on an aircraft landing suspension system by modeling the landing gear as a moving base problem similar to that shown in Figure P2.46 for a car suspension. The mass of a regional jet is 13, 236 kg empty and its maximum takeoff mass is 21,523 kg. Compare the maximum deflection for a wheel motion of magnitude 0.50 m and frequency of 35 rad/s, for these two different masses. Take the damping ratio to be ζ = 0.1 and the stiffness to be 4.22×10^6 N/m.

Solution: Using a Mathcad worksheet the following calculations result:

$$Y := 0.5 \cdot m \quad k = 4.22 \times 10^{6} kg \cdot s^{-2} \quad mf := 21523kg \quad me := 13236 \cdot kg$$

From equation (2.70):

$$\omega b := 35 \cdot \frac{rad}{sec}$$

$$re := \frac{\omega b}{\sqrt{\frac{k}{me}}} \quad re = 1.96 \quad \zeta := 0.1$$

$$\sqrt{\frac{k}{mf}} = 14.002 s^{-1}$$

$$Xe := Y \cdot \sqrt{\frac{1 + (2 \cdot \zeta \cdot re)^2}{\left(1 - re^2\right)^2 + (2 \cdot \zeta \cdot re)^2}} \quad Xe = 0.187 m$$

$$rf := \frac{\omega b}{\sqrt{\frac{k}{mf}}} \quad rf = 2.5$$

$$Xf := Y \cdot \sqrt{\frac{1 + (2 \cdot \zeta \cdot rf)^2}{\left(1 - rf^2\right)^2 + (2 \cdot \zeta \cdot rf)^2}} \quad Xf = 0.106 m$$

Note that if the suspension stiffness were defined around the full case, when empty the plane would bounce with a larger amplitude then when full. Note Mathcad does not have a symbol for a Newton so the units on stiffness above are kg/sec^2 in order to allow Mathcad to compute the units.

2.50 Consider the simple model of a building subject to ground motion suggested in Figure P2.50. The building is modeled as a single degree of freedom spring-mass system where the building mass is lumped atop of two beams used to model the walls of the building in bending. Assume the ground motion is modeled as having amplitude of 0.1 m at a frequency of 7.5 rad/s. Approximate the building mass by 10^5 kg and the stiffness of each wall by 3.519×10^6 N/m. Compute the magnitude of the deflection of the top of the building.

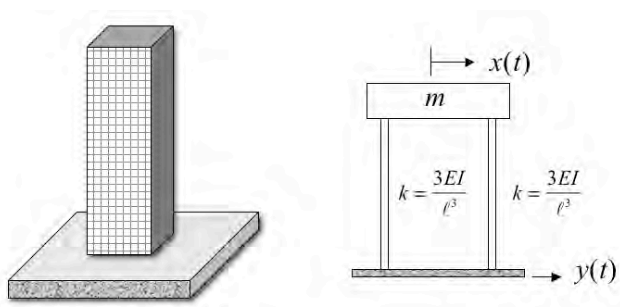


Figure P2.50 A simple model of a building subject to ground motion, such as an earthquake.

Solution: The equation of motion is

$$m\ddot{x}(t) + 2kx(t) = 0.1\cos 7.5t$$

The natural frequency and frequency ratio are

$$\omega_n = \sqrt{\frac{2k}{m}} = 8.389 \text{ rad/s} \quad \text{and} \quad r = \frac{\omega}{\omega_n} = \frac{7.5}{8.389} = 0.894$$

The amplitude of the steady state response is given by equation (2.70) with $\zeta = 0$ in this case:

$$X = Y\left|\frac{1}{1-r^2}\right| = 0.498 \text{ m}$$

Thus the earthquake will cause serious motion in the building and likely break.