Problems and Solutions Section 2.5 (2.51 through 2.58)

2.51 A lathe can be modeled as an electric motor mounted on a steel table. The table plus the motor have a mass of 50 kg. The rotating parts of the lathe have a mass of 5 kg at a distance 0.1 m from the center. The damping ratio of the system is measured to be ζ = 0.06 (viscous damping) and its natural frequency is 7.5 Hz. Calculate the amplitude of the steady-state displacement of the motor, assuming ω_r = 30 Hz.

Soltuion:

Given: m = 50 kg, $m_o = 5$, e = 0.1m, $\zeta = 0.06$, $\omega_n = 7.5$Hz

Let ω_r=30 Hz

So, $r = \dfrac{\omega_r}{\omega_n} = 4$

From Equation (2.84),

$$X = \frac{m_o e}{m}\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{(5)(0.1)}{50}\frac{4^2}{\sqrt{(1-4^2)^2+[2(0.06)(4)]^{-2}}}$$

X = 0.011m

X = 1.1 cm

2.52 The system of Figure 2.18 produces a forced oscillation of varying frequency. As the frequency is changed, it is noted that at resonance, the amplitude of the displacement is 10 mm. As the frequency is increased several decades past resonance the amplitude of the displacement remains fixed at 1 mm. Estimate the damping ratio for the system.

Solution: Equation (2.84) is

$$X = \frac{m_o e}{m}\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

At resonance, X = 10 mm = $\dfrac{m_o e}{m}\dfrac{1}{2\zeta}$

$$\frac{10m}{m_o e} = \frac{1}{2\zeta}$$

When r is very large, $\dfrac{Xm}{m_o e} = 1$ and X = 1 mm, so

$$\frac{m}{m_o e} = 1$$

Therefore, 10(1) = $\dfrac{1}{2\zeta}$

$$\zeta = 0.05$$

2.53 An electric motor (Figure P2.53) has an eccentric mass of 10 kg (10% of the total mass) and is set on two identical springs (k = 3200 /m). The motor runs at 1750 rpm, and the mass eccentricity is 100 mm from the center. The springs are mounted 250 mm apart with the motor shaft in the center. Neglect damping and determine the amplitude of the vertical vibration.

Solution:
Given m_0 = 10 kg, m= 100 kg, k = 2x3.2 N/mm, , e = 0.1 m

$$\omega_r = 1750\frac{\text{rev}}{\text{min}}\left(\frac{\text{min}}{60\,\text{sec}}\frac{2\pi\ \text{rad}}{\text{rev}}\right) = 183.26\frac{\text{rad}}{\text{s}}\ \text{rad/s}$$

Vertical vibration:

$$\omega_n = \sqrt{\frac{2(3.2)(1000)}{100}} = 8 \text{ rad/s}$$

$$r = \frac{\omega_r}{\omega_n} = \frac{183.3}{8} = 22.9$$

From equation (2.84)

$$X = e\frac{m_0}{m}\frac{r^2}{|1-r^2|} == 0.01 \text{ m}$$

2.54 Consider a system with rotating unbalance as illustrated in Figure P2.53. Suppose the deflection at 1750 rpm is measured to be 0.05 m and the damping ratio is measured to be ζ = 0.1. The out-of-balance mass is estimated to be 10%. Locate the unbalanced mass by computing e.

Solution: Given: X = 0.05 m, $\zeta = 0.1$, $m_e = 0.1m$, and from the solution to problem 2.53 the frequency ratio is calculated to be r = 22.9. Solving the rotating unbalance Equation (2.84) for e yields:

$$X = \frac{m_0 e}{m}\frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \Rightarrow e = \frac{mX}{m_0}\frac{\sqrt{(1-r^2)^2+(2\zeta r)^2}}{r^2} = \underline{0.499 \text{ m}}$$

This sort of calculation can be introduced to discuss the application of machinery diagnostics if time permits. Machinery diagnostics deals with determining the location and extend of damage from measurements of the response and input.

2.55 A fan of 45 kg has an unbalance that creates a harmonic force. A spring-damper system is designed to minimize the force transmitted to the base of the fan. A damper is used having a damping ratio of ζ = 0.2. Calculate the required spring stiffness so that only 10% of the force is transmitted to the ground when the fan is running at 10,000 rpm.

Solution: The equation of motion of the fan is

$$m\ddot{x} + c\dot{x} + kx = m_0 e\omega^2 \sin(\omega t + \theta)$$

The steady state solution as given by equation (2.84) is

$$x(t) = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \sin \omega t$$

where r is the standard frequency ratio. The force transmitted to the ground is

$$F(t) = kx + c\dot{x} = \frac{m_0 e}{m} \frac{kr^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \sin \omega t + \frac{m_0 e}{m} \frac{c\omega r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \cos \omega t$$

Taking the magnitude of this quantity, the magnitude of the force transmitted becomes

$$F_0 = \frac{m_0 e}{m} \frac{r^2 \sqrt{k^2 + c^2 \omega^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = m_0 e \omega \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

From equation (2.81) the magnitude of the force generated by the rotating mass F_r is

$$F_r = m_0 e \omega^2$$

The limitation stated in the problem is that $F_0 = 0.1F_r$, or

$$m_0 e \omega^2 \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = 0.1 m_0 e \omega^2$$

Setting $\zeta = 0.2$ and solving for r yields:

$$r^4 - 17.84 r^2 - 99 = 0$$

which yields only one positive solution for r^2, which is

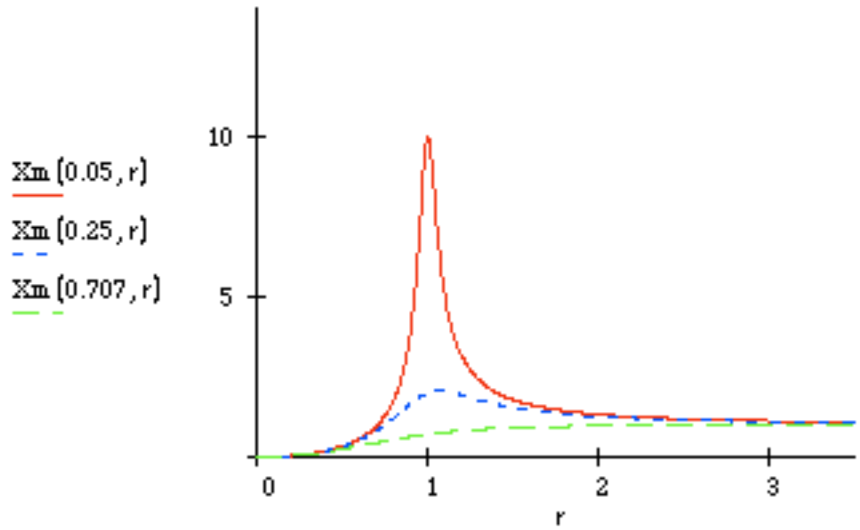
$$r^2 = 22.28 = \frac{\omega^2}{k/m} \Rightarrow \frac{k}{m} = \left(\frac{10000 \times 2\pi}{60}\right)^2 \frac{1}{22.28}$$

$$\Rightarrow k = 45 \left(\frac{10000 \times 2\pi}{60}\right)^2 \frac{1}{22.28} = 2.21 \times 10^6 \text{ N/m}$$

2.56 Plot the normalized displacement magnitude versus the frequency ratio for the out of balance problem (i.e., repeat Figure 2.20) for the case of ζ = 0.05.

Solution: Working in Mathcad using equation (2.84) yields:

$$Xm(\zeta, r) := \frac{r^2}{\sqrt{(1 - r^2)^2 + (2\cdot\zeta\cdot r)^2}}$$



2.57 Consider a typical unbalanced machine problem as given in Figure P2.57 with a machine mass of 120 kg, a mount stiffness of 800 kN/m and a damping value of 500 kg/s. The out of balance force is measured to be 374 N at a running speed of 3000 rev/min. a) Determine the amplitude of motion due to the out of balance. b) If the out of balance mass is estimated to be 1% of the total mass, estimate the value of the *e*.

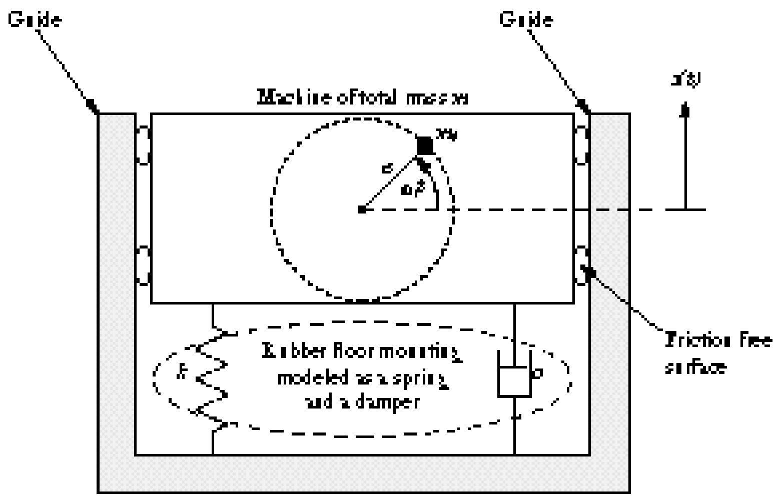


Figure P2.57 Typical unbalance machine problem.

Solution:

a) Using equation (2.84) with $m_0e = F_0/\omega_r^2$ yields:

$k := 800 \cdot 1000 \quad m := 120 \quad c := 500 \quad F0 := 374$

$\omega r := 100 \cdot \pi \qquad \omega n := \sqrt{\dfrac{k}{m}} \qquad \zeta := \dfrac{c}{2 \cdot \sqrt{k \cdot m}} \qquad k = 8 \cdot 10^5$

$r := \dfrac{\omega r}{\omega n} \qquad \omega n = 81.65 \qquad r = 3.848 \qquad \zeta = 0.026$

$$X := \frac{F0}{\omega r^2 \cdot m} \cdot \frac{r^2}{\sqrt{\left(1 - r^2\right)^2 + \left(2 \cdot \zeta \cdot r\right)^2}} \qquad X = 3.386 \cdot 10^{-5}$$

b) Use the fact that $F_0 = m_0 e \omega_r^2$ to get

$$e := \frac{F0}{\omega r^2 \cdot (0.01 \cdot m)} \qquad e = 3.158 \cdot 10^{-3}$$

in meters.

2.58 Plot the response of the mass in Problem 2.57 assuming zero initial conditions.

Solution: The steady state response is the particular solution given by equation (2.84) and is plotted here in Mathcad:

$m := 120 \qquad k := 120 \qquad c := 500 \qquad F0 := 374$

$$\omega n := \sqrt{\frac{k}{m}} \qquad \omega r := \frac{3000 \cdot 2 \cdot \pi}{60} \qquad r := \frac{\omega r}{\omega n} \qquad \zeta := \frac{c}{2 \cdot \sqrt{m \cdot k}}$$

$$X := \frac{F0}{\omega r^2 \cdot m} \cdot \frac{r^2}{\sqrt{\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2}} \qquad \theta := \operatorname{atan}\left(\frac{2 \cdot \zeta \cdot r}{1 - r^2}\right)$$

$$x(t) := X \cdot \sin(\omega r \cdot t - \theta)$$