Problems and Solutions Section 2.6 (2.59 through 2.62)

2.59 Calculate damping and stiffness coefficients for the accelerometer of Figure 2.23 with moving mass of 0.04 kg such that the accelerometer is able to measure vibration between 0 and 50 Hz within 5%. (*Hint*: For an accelerometer it is desirable for $Z/\omega_b^2 Y =$ constant.)

Solution: Use equation (2.90):

Given: $m = 0.04$ kg with error < 5%

$0.2f = 50\text{ Hz} \rightarrow f = 250\text{ Hz} \rightarrow \omega = 2\pi f = 1570.8\text{ rad/s}$

Thus, $k = m\omega^2 = 98{,}696\text{ N/m}$

When $r = .2$, $0.95 < \dfrac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} < 1.05$ (± 5% error)

This becomes $0.8317 + 0.1444\zeta^2 < 1 < 1.016 + 0.1764\zeta^2$

Therefore, $\zeta = 0.7 = \dfrac{c}{2\sqrt{km}}$

$$c = 2(.7)\sqrt{(98696)(.04)}$$

$$c = 87.956\text{ Ns/m}$$

2.60 The damping constant for a particular accelerometer of the type illustrated in Figure 2.23 is 50 N s/m. It is desired to design the accelerometer (i.e., choose *m* and *k*) for a maximum error of 3% over the frequency range 0 to 75 Hz.

Solution: Given $0.2f = 75\text{ Hz} \rightarrow f = 375\text{ Hz} \rightarrow \omega_n = 2\pi f = 2356.2\text{ rad/s}$. Using equation (2.93) when $r = 0.2$:

$$0.97 < \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} < 1.03 \quad (\pm 3\%\text{ error})$$

This becomes $0.8671 + 0.1505\zeta^2 < 1 < 0.9777 + 0.1697\zeta^2$

Therefore, $0.3622 < \zeta < 0.9395$

Choose $\zeta = 0.7 = \dfrac{c}{2m\omega} = \dfrac{50}{2m(2356.2)}$

$$m = 0.015\text{ kg}$$

$$k = m\omega_n^2 = 8.326 \times 10^4\text{ N/m}$$

2.61 The accelerometer of Figure 2.23 has a natural frequency of 120 kHz and a damping ratio of 0.2. Calculate the error in measurement of a sinusoidal vibration at 60 kHz.

Solution:

Given: ω = 120 kHz, $\zeta = .2$, ω_b = 60 kHz

So, $$\frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{1}{\sqrt{(1-.5^2)^2+(2(.2)(.5))^2}} = 1.288 > 1$$

The error is $\frac{1.288-1}{1} \times 100\%$ = 28.8%

2.62 Design an accelerometer (i.e., choose *m*, *c* and *k*) configured as in Figure 2.23 with very small mass that will be accurate to 1% over the frequency range 0 to 50 Hz.

Solution:

Given: error < 1% , $0.2f$ = 50 Hz $\rightarrow$ f = 250 Hz $\rightarrow$ $\omega = 2\pi f$ = 1570.8 rad/s

When r =0.2, $$0.99 < \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} < 1.01 \qquad (\pm\ 1\%\ \text{error})$$

This becomes $0.9032 + 0.1568\zeta^2 < 1 < 0.9401 + 0.1632\zeta^2$

Therefore, $0.6057 < \zeta < 0.7854$

Choose m = 0.01 kg , then $k = m\omega^2$ = 24,674 N/m

Thus $\zeta = 0.7 = \frac{c}{2\sqrt{km}}$ implies that: c = 21.99 Ns/m