Problems and Solutions Section 2.7 (2.63 through 2.79)

2.63 Consider a spring-mass sliding along a surface providing Coulomb friction, with stiffness 1.2×10^4 N/m and mass 10 kg, driven harmonically by a force of 50 N at 10 Hz. Calculate the approximate amplitude of steady-state motion assuming that both the mass and the surface that it slides on, are made of lubricated steel.

Solution: Given: m = 10 kg, k = 1.2x10^4 N/m, F_o = 50 N, ω =10(2π) = 20π rad/s

$$\omega = \sqrt{\frac{k}{m}} = 34.64 \text{ rad/s}$$

for lubricated steel, μ = 0.07

From Equation (2.109) $X = \dfrac{F_o}{k}\dfrac{\sqrt{1-\left[\dfrac{4\mu mg}{\pi(F_o)}\right]^2}}{\left|(1-r^2)\right|}$

$$X = \frac{50}{1.2\times 10^4}\frac{\sqrt{1-\left[\dfrac{4(.07)(10)(9.81)}{\pi(50)}\right]^2}}{\left|(1-\left(\dfrac{20\pi}{34.64}\right)^2)\right|}$$

$$X = 1.79\times 10^{-3}\text{ m}$$

2.64 A spring-mass system with Coulomb damping of 10 kg, stiffness of 2000 N/m, and coefficient of friction of 0.1 is driven harmonically at 10 Hz. The amplitude at steady state is 5 cm. Calculate the magnitude of the driving force.

Solution:
Given: m = 10 kg, k = 2000 N/m, μ = 0.1, ω=10(2π) = 10(2π) = 20π rad/s,

$$\omega_n = \sqrt{\frac{k}{m}} = 14.14 \text{ rad/s}, \; X = 5 \text{ cm}$$

Equation (2.108) $X = \dfrac{\dfrac{F_0}{k}}{\sqrt{(1-r^2)^2 + \left[\dfrac{4\mu mg}{\pi kX}\right]^2}} \Rightarrow F_0 = Xk\sqrt{(1-r^2)^2 + \left[\dfrac{4\mu mg}{\pi kX}\right]^2}$

$$F_0 = (0.05)(2000)\sqrt{\left(1-\left[\frac{20\pi}{14.14}\right]^2\right)^2 + \left(\frac{4(0.1)(10)(9.81)}{\pi(2000)(.05)}\right)^2} = 1874 \text{ N}$$

2.65 A system of mass 10 kg and stiffness 1.5×10^4 N/m is subject to Coulomb damping. If the mass is driven harmonically by a 90-N force at 25 Hz, determine the equivalent viscous damping coefficient if the coefficient of friction is 0.1.

Solution:

Given: m = 10 kg, k = 1.5x10^4 N/m, F_0 = 90 N, ω = 25(2π) = 50π rad/s,

$$\omega_n = \sqrt{\frac{k}{m}} = 38.73 \text{ rad/s}, \ \mu = 0.1$$

Steady-state Amplitude using Equation (2.109) is

$$X = \frac{F_0}{k}\frac{\sqrt{1-\left[\frac{4\mu mg}{\pi(F_o)}\right]^2}}{\left|(1-r^2)\right|} = \frac{90}{1.5\times 10^4}\frac{\sqrt{1-\left[\frac{4(0.1)(10)(9.81)}{\pi(90)}\right]^2}}{\left|1-\left(\frac{50\pi}{38.73}\right)^2\right|} = 3.85\times 10^{-4} \text{ m}$$

From equation (2.105), the equivalent Viscous Damping Coefficient becomes:

$$c_{eq} = \frac{4\mu mg}{\pi\omega X} = \frac{4(0.1)(10)(9.81)}{\pi(50\pi)(3.85\times 10^{-4})} = 206.7 \text{ Ns/m}$$

2.66 a. Plot the free response of the system of Problem 2.65 to initial conditions of $x(0) = 0$ and $\dot{x}(0) = |F_0/m| = 9$ m/s using the solution in Section 1.10**.**

b. Use the equivalent viscous damping coefficient calculated in Problem 2.65 and plot the free response of the "equivalent" viscously damped system to the same initial conditions.

Solution: See Problem 2.65

(a) $x(0) = 0$ and $\dot{x}(0) = \dfrac{F_o}{m} = 9$ m/s

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.5x10^4}{10}} = 38.73 \text{ rad/s}$$

From section 1.10:

$$m\ddot{x} + kx = \mu mg \text{ for } \dot{x} < 0$$
$$m\ddot{x} + kx = -\mu mg \text{ for } \dot{x} > 0$$

Let $F_d = \mu mg = (0.1)(10)(9.81) = 9.81$ N

To start, $\dot{x}(0) = \omega_n B_1 = 9$

Therefore, $A_1 = \dfrac{F_d}{k}$ and $B_1 = \dfrac{9}{\omega_n}$

So, $x(t) = \dfrac{F_d}{k}\cos\omega_n t + \dfrac{9}{\omega}\sin\omega_n t - \dfrac{F_d}{k}$

This will continue until $\dot{x} = 0$, which occurs at time t_1:

$$x(t) = A_2\cos\omega_n t + B_2\sin\omega_n t + \frac{F_d}{k}$$
$$\dot{x}(t) = -\omega_n A_2\sin\omega_n t + \omega_n B_2\cos\omega_n t$$
$$x(t_1) = A_2\cos\omega_n t_1 + B_2\sin\omega_n t_1 + \frac{F_d}{k}$$
$$\dot{x}(t_1) = 0 = -\omega_n A_2\sin\omega_n t_1 + \omega_n B_2\cos\omega_n t_1$$

Therefore, $A_2 = \left(x(t_1) - F_d/k\right)\cos\omega_n t_1$ and $B_2 = \left(x(t_1) - F_d/k\right)\sin\omega_n t_1$

So, $x(t) = \left[\left(x(t_1) - F_d/k\right)\cos\omega_n t_1\right]\cos\omega_n t + \left[\left(x(t_1) - F_d/k\right)\sin\omega_n t_1\right]\sin\omega_n t + \dfrac{F_d}{k}$

Again, when $\dot{x} = 0$ at time t_2, the motion will reverse:

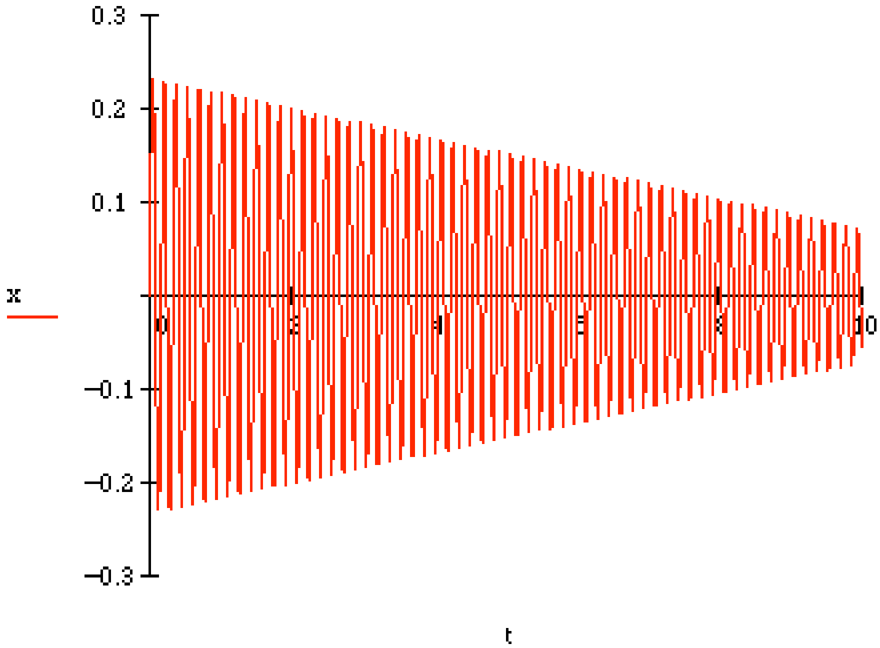
$$x(t) = A_3\cos\omega_n t + B_3\sin\omega_n t - \frac{F_d}{k}$$
$$\dot{x}(t) = -\omega_n A_3\sin\omega_n t + \omega_n B_3\cos\omega_n t$$
$$x(t_2) = A_3\cos\omega_n t_2 + B_3\sin\omega_n t_2 - \frac{F_d}{k}$$

$$\dot{x}(t_2) = 0 = -\omega_n A_3 \sin\omega_n t_2 + \omega_n B_3 \cos\omega_n t_2$$

Therefore, $A_3 = \left(x(t_2) + F_d / k\right)\cos\omega_n t_2$ and $B_3 = \left(x(t_2) - F_d / k\right)\sin\omega_n t_2$

So, $x(t) = \left[\left(x(t_2) + F_d / k\right)\cos\omega_n t_2\right]\cos\omega_n t + \left[\left(x(t_2) + F_d / k\right)\sin\omega_n t_2\right]\sin\omega_n t - \dfrac{F_d}{k}$

This continues until $\dot{x} = 0$ and $kx < \mu mg = 9.81$ N



(b) From Problem 2.65, $c_{eq} = 206.7$ kg/s

The equivalently damped system would be:

$$m\ddot{x} + c_{eq}\dot{x} + kx = 0$$

Also, $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{1.5x10^4}{10}} = 38.73$ rad/s

$$\zeta = \frac{c_{eq}}{2\sqrt{km}} = \frac{206.7}{2\sqrt{(1.5x10^4)(10)}} = 0.2668$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 37.33 \text{ rad/s}$$

The solution would be found from Equation 1.36:

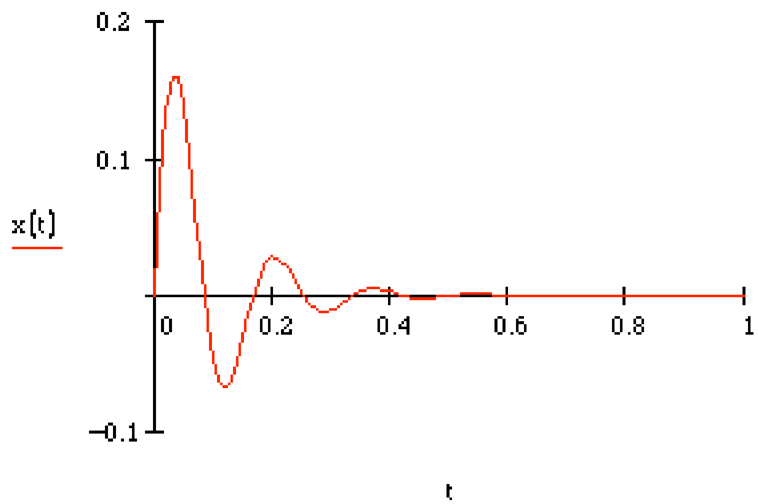
$$x(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$
$$\dot{x}(t) = -\zeta\omega_n Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi) + \omega_d Ae^{-\zeta\omega_n t}\cos(\omega_d t + \phi)$$
$$x(0) = A\sin\phi = 0$$
$$\dot{x}(0) = -\zeta\omega_n A\sin\phi + \omega_d A\cos\phi = 9$$

Therefore, $A = \dfrac{9}{\omega_d} = 0.2411\text{m}$ and $\phi = 0$ rad

So, $x(t) = 0.2411 e^{-10.335t}\sin(37.33t)$

$t := 0, 0.01 .. 1$

$x(t) := 0.2411 \cdot e^{-10.35 \cdot t} \cdot \sin(37.33 \cdot t)$



2.67 Referring to the system of Example 2.7.1, calculate how large the magnitude of the driving force must be to sustain motion if the steel is lubricated. How large must this magnitude be if the lubrication is removed?

Solution:

From Example 2.7.1 $m = 10$ kg, $k = 1.5 \times 10^4$ N/m, $F_o = 90$ N,
$\omega = 25(2\pi) = 50\pi$ rad/s

Lubricated Steel $\mu = 0.07$

Unlubricated Steel $\mu = 0.3$

Lubricated: $F_o > \dfrac{4\mu mg}{\pi} = \dfrac{4(0.07)(10)(9.81)}{\pi}$

$F_o = 8.74$ N

Unlubricated: $F_o > \dfrac{4\mu mg}{\pi} = \dfrac{4(0.3)(10)(9.81)}{\pi}$

$F_o = 37.5$ N

2.68 Calculate the phase shift between the driving force and the response for the system of Problem 2.67 using the equivalent viscous damping approximation.

Solution:

From Problem 2.67: $m = 10$ kg, $k = 1.5\times 10^4$ N/m, $F_o = 90$ N,

$$\omega = 25(2\pi) = 157.1 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{k}{m}} = 38.73 \text{ rad/s}$$

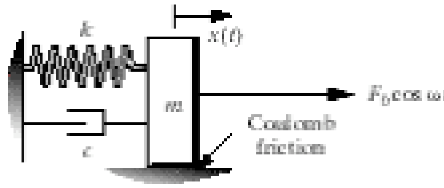
From Equation (2.111), and since r>1

$$\theta = \tan^{-1}\left[\frac{-4\mu mg}{\pi F_0\sqrt{1-\left(\frac{4\mu mg}{\pi F_o}\right)^2}}\right]$$

Since in Problem 2.67, $\pi F_o = 4\mu mg$, this reduces to

$$\theta = \tan^{-1}\left[\frac{-1}{0}\right] = \frac{-\pi}{2} \text{ rad} = -90^\circ$$

2.69 Derive the equation of vibration for the system of Figure P2.69 assuming that a viscous dashpot of damping constant c is connected in parallel to the spring. Calculate the energy loss and determine the magnitude and phase relationships for the forced response of the equivalent viscous system.



Solution: Sum of the forces in Figure P2.69

$$m\ddot{x} = -kx - c\dot{x} - \mu mg \operatorname{sgn}(\dot{x})$$

$$m\ddot{x} + c\dot{x} + \mu mg \operatorname{sgn}\ (\dot{x}) + kx = 0$$

Assume the mass is moving to the left $(\dot{x}(0) = 0, x(0) = x_0)$

$$m\ddot{x} - c\dot{x} + \mu mg + kx = 0$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} - \mu g + \omega_n^2 x = 0$$

The solution of the form:

$$x(t) = ae^{rt} + \frac{\mu g}{\omega_n{}^2}$$

Substituting:

$$ar^2e^{rt} + 2\zeta\omega_n are^{rt} - \mu g + \omega_n{}^2 ae^{rt} + \mu g = 0$$

$$r^2 + 2\zeta\omega_n r + \omega_n{}^2 = 0$$

$$r = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n{}^2 - 4\omega_n{}^2}}{2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

So, $$x(t) = a_1 e^{(-\zeta\omega_n + \omega_n\sqrt{\zeta^2-1})t} + a_1 e^{(-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1})t} + \frac{\mu g}{\omega_n^2}$$

$$x(t) = e^{-\zeta\omega_n t}(a_1 e^{-\zeta\omega_d t} + a_2 e^{-\zeta\omega_d t}) + \frac{\mu g}{\omega_n{}^2}$$

$$x(t) = Xe^{-\zeta\omega_n t}\sin(\omega_d t + \theta) + \frac{\mu g}{\omega_n{}^2}$$

Initial conditions

$$x(0) = X\sin(\theta) + \frac{\mu g}{\omega_n{}^2} = x_o$$

$$\dot{x}(0) = X(-\zeta\omega_n)(\sin\theta) + X\omega_d\cos\theta = 0$$

$$-X\zeta\omega_n\sin\theta + X\omega_d\cos\theta = 0$$

$$\tan\theta = \frac{\omega_d}{\zeta\omega_n} \Rightarrow \theta = \tan^{-1}\left[\frac{\omega_d}{\zeta\omega_n}\right]$$

$$X = \frac{\left(x_o - \frac{\mu g}{\omega_n{}^2}\right)\sqrt{\omega_d{}^2 + (\zeta\omega_n)^2}}{\omega_d}$$

$$x(t) = \frac{\left(x(0) - \frac{\mu g}{\omega_n{}^2}\right)\sqrt{\omega_d{}^2 + (\zeta\omega_n)^2}}{\omega_d} e^{-\zeta\omega_n t} \sin\left(\omega_d t + \tan^{-1}\left[\frac{\omega_d}{\zeta\omega_n}\right]\right) + \frac{\mu g}{\omega_n{}^2} \ (1)$$

This will occur until $\dot{x}(t) = 0$:

$$\dot{x}(t) = X(-\zeta\omega_n)e^{-\zeta\omega_n t}\sin(\omega_d t + \theta) + A_0 e^{-\zeta\omega_n t}\omega_d \cos(\omega_d t + \theta) = 0$$

$$-\zeta\omega_n \sin(\omega_d t + \theta) + \omega_d \cos(\omega_d t + \theta) = 0$$

$$\Rightarrow \tan(\omega_d t + \theta) = \frac{\omega_d}{\zeta\omega_n}$$

$$t = \frac{\pi}{\omega_d}$$

So Equation (1) is valid from $0 \le t \le \frac{\pi}{\omega_d}$

For motion to the right

Initial conditions (From Equation (1)):

$$x\left(\frac{\pi}{\omega_d}\right) = Xe^{-\zeta\omega_n\left(\frac{\pi}{\omega_d}\right)}\cos\theta + \frac{\mu g}{\omega_n{}^2} = \frac{\left(x(0) - \frac{\mu g}{\omega_n{}^2}\right)\zeta\omega_n}{\omega_d} e^{-\zeta\omega_n\left(\frac{\pi}{\omega_d}\right)} + \frac{\mu g}{\omega_n{}^2}$$

$$\dot{x}\left(\frac{\pi}{\omega_d}\right) = 0$$

$$x(t) = A_1 e - ^{-\zeta\omega_n t} \sin(\omega_d t + \theta_1) - \frac{\mu g}{\omega_n{}^2}$$

$$x(0) = A_1 \sin\theta_1 - \frac{\mu g}{\omega_n{}^2} = \frac{\left(x(0) - \frac{\mu g}{\omega_n{}^2}\right)\zeta\omega}{\omega_d} e^{-\zeta\omega_n\left(\frac{\pi}{\omega_d}\right)} + \frac{\mu g}{\omega_n{}^2}$$

$$\dot{x}(0) = A_1(-\zeta\omega_n)\sin\theta_1 + X\omega_d \cos\theta_1 = 0$$

Solution: $x(t) = A_1 e^{-\zeta\omega_n t}\sin(\omega_d t + \theta_1) - \frac{\mu g}{\omega_n^2}$

$$A_1 = \frac{\sqrt{\omega_d^2 + (\zeta\omega_n)^2}}{\omega_d}\left[\frac{\left(x(0) - \frac{\mu g}{\omega_n{}^2}\right)\zeta\omega_n}{\omega_d} e^{-\zeta\omega_n\left(\frac{\pi}{\omega_d}\right)} + \frac{\mu g}{\omega_n{}^2}\right]$$

$$\theta = \tan^{-1}\left[\frac{\omega_d}{\zeta\omega_n}\right]$$

Forced Case:

$$m\ddot{x} - c\dot{x} + \mu mg\,\mathrm{sgn}(\dot{x}) + kx = F_o\cos(\omega t)$$

Approximate Steady-state Response:

$$x_{ss}(t) = X\sin(\omega t - \theta)$$

Energy Dissipated per Cycle:

$$\Delta E = \int F_d dx = \int_{2\pi}^{\frac{2\pi}{\omega}}\left[c\dot{x}\frac{dx}{dt} + \mu mg\,\mathrm{sgn}\,\dot{x}\frac{dx}{dt}\right]dt$$

$$= \int_{2\pi}^{\frac{2\pi}{\omega}}(c\dot{x}^2 dt) + \mu mg\int_{2\pi}^{\frac{2\pi}{\omega}}\mathrm{sgn}(\dot{x})\dot{x}dt$$

$$\Delta E = \pi c\omega X^2 + 4\mu mgX$$

This results in an equivalent viscously damped system:

$$\ddot{x} + 2(\zeta + \zeta_{eq})\omega_n\dot{x} + \omega_n{}^2 x = F_o\cos\omega t$$

where $\zeta_{eq} = \dfrac{2\mu g}{\pi\omega_n\omega X}$

The magnitude is:

$$X = \frac{\frac{F_0}{k}}{\sqrt{(1-r)^2 + (2(\zeta + \zeta_{eq})r)^2}}$$

Solving for X:

$$X = \frac{\left(\frac{8\mu gcr^2}{\pi k\omega}\right) + \sqrt{\left(\frac{8\mu gcr^2}{\pi k\omega}\right) - 2\left[(1-r^2)^2 + \frac{c^2r^2}{km}\right]\left[\left(\frac{4\mu gr}{\pi\omega_n\omega}\right)^2 - \left(\frac{F_0}{k}\right)^2\right]}}{4\left[(1-r^2)^2 + \frac{c^2r^2}{km}\right]}$$

The phase is:

$$\theta = \tan^{-1}\left[\frac{2(\zeta + \zeta_{eq})r}{1-r^2}\right] = \tan^{-1}\left[\frac{2\zeta r + \frac{4\mu gr}{\pi\omega_n\omega X}}{1-r^2}\right]$$

2.70 A system of unknown damping mechanism is driven harmonically at 10 Hz with an adjustable magnitude. The magnitude is changed, and the energy lost per cycle and amplitudes are measured for five different magnitudes. The measured quantities are:

$\Delta E(J)$	0.25	0.45	0.8	1.16	3.0
$X\ (M)$	0.01	0.02	0.04	0.08	0.15

Is the damping viscous or Coulomb?

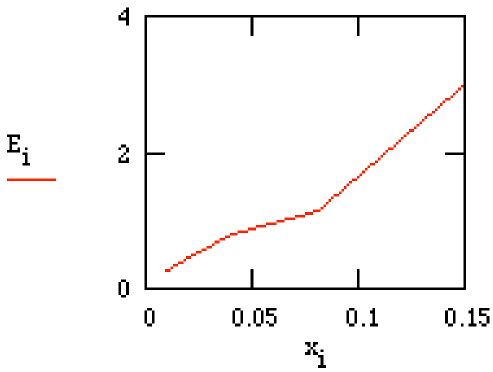
Solution:

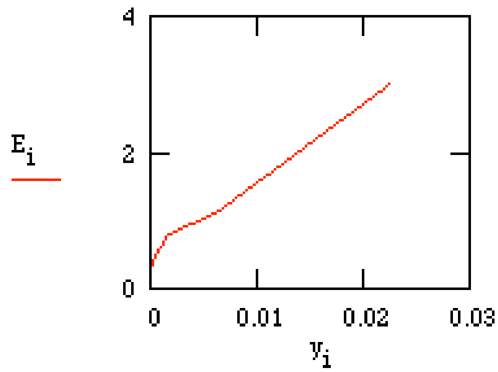
For viscous damping, $\Delta E = \pi c \omega X^2$

For Coulomb damping, $\Delta E = 4\mu m g X$

$$x := \begin{bmatrix} 0.01 \\ 0.02 \\ 0.04 \\ 0.08 \\ 0.15 \end{bmatrix} \qquad i := 0, 1 .. 4 \qquad y_i := (x_i)^2$$

$$E := \begin{bmatrix} 0.25 \\ 0.45 \\ 0.8 \\ 1.16 \\ 3 \end{bmatrix}$$





For the data given, a plot of ΔE vs X^2 yields a curve, while ΔE vs X yields a straight line. Therefore, the damping is likely Coulomb in nature

2.71 Calculate the equivalent loss factor for a system with Coulomb damping.

Solution:

Loss Factor: $$\eta = \frac{\Delta E}{2\pi U_{\max}}$$

For Coulomb damping: $$\Delta E = 4\mu mgX$$

$$U_{\max} = \frac{1}{2}kX^2$$

$$\eta = \frac{4\mu mgX}{2\pi\left(\frac{1}{2}kX^2\right)} = \frac{4\mu mg}{\pi kX}$$

Substituting for X (from Equation 2.109):

$$\eta = \frac{4\mu mg}{\pi F_o}\frac{\left|1-r^2\right|}{\sqrt{1-\left(\frac{4\mu mg}{\pi F_o}\right)^2}}$$

2.72 A spring-mass system (m = 10 kg, $k = 4\times10^3$ N/m) vibrates horizontally on a surface with coefficient of friction μ = 0.15. When excited harmonically at 5 Hz, the steady-state displacement of the mass is 5 cm. Calculate the amplitude of the harmonic force applied.

Solution: Given: m = 10 kg, $k = 4\times10^3$ N/m, μ = 0.15, X = 5 cm = 0.05 m,

$$\omega = 5(2\pi) = 10\pi \text{ rad/s},\ \omega_n = \sqrt{\frac{k}{m}} = 20 \text{ rad/s}$$

Equation (2.109)

$$X = \frac{\frac{F_0}{k}}{\sqrt{(1-r^2)^2 + \left(\frac{4\mu mg}{\pi kX}\right)^2}} \Rightarrow$$

$$F_o = kX\sqrt{(1-r^2)^2 + \left(\frac{4\mu mg}{\pi kX}\right)^2} = (0.05)(4\times10^3)\sqrt{\left(1-\left(\frac{10\pi}{20}\right)^2\right)^2 + \left(\frac{4(0.15)(10)(9.81)}{\pi(4x10^3)(0.05)}\right)^2}$$

$$F_o = 294 \text{ N}$$

2.73 Calculate the displacement for a system with air damping using the equivalent viscous damping method.

Solution:

The equivalent viscous damping for air is given by Equation (2.131):

$$c_{eq} = \frac{8}{3\pi}\alpha\omega X$$

From Equation 2.31:

$$X = \frac{F_o}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}} = \frac{F_o}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(\dfrac{c_{eq}}{m}\omega_n\right)^2}}$$

$$X = \frac{F_o}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(\dfrac{8}{3\pi m}\alpha\omega X\right)^2}} = \frac{F_o m}{k\sqrt{(1-r^2)^2 + \left(\dfrac{8}{3\pi m}\alpha r^2 X\right)^2}}$$

Solving for X and taking the real solution:

$$X = \frac{\sqrt{-\dfrac{1}{2}(1-r^2)^2 + \dfrac{1}{2}\sqrt{(1-r^2)^2 + \left(\dfrac{16F_o\alpha r^2}{3\pi km}\right)^2}}}{\left(\dfrac{8\alpha r^2}{3\pi m}\right)}$$

2.74 Calculate the semimajor and semiminor axis of the ellipse of equation (2.119). Then calculate the area of the ellipse. Use c = 10 kg/s, ω = 2 rad/s and X = 0.01 m.

Solution: The equation of an ellipse usually appears when the plot of the ellipse is oriented along with the x axis along the principle axis of the ellipse. Equation (2.1109) is the equation of an ellipse rotated about the origin. If k is known, the angle of rotation can be computed from formulas given in analytical geometry. However, we know from the energy calculation that the stiffness does not effect the amount of energy dissipated. Thus only the orientation of the ellipse is effected by the stiffness, not its area or axis. Thus we can use this fact to answer the question. First re-write equation (2.119) with k = 0 to get:

$$F^2 + c^2\omega^2 x^2 = c^2\omega^2 X^2$$

$$\Rightarrow \left(\frac{F}{c\omega X}\right)^2 + \left(\frac{x}{X}\right)^2 = 1$$

This is the equation of an ellipse with major axis a and minor axis b given by

$$a = X = 0.01 \text{ m}, \quad \text{and} \quad b = c\omega X = 0.2 \text{ kg m/s}^2$$

The area, and hence energy lost per cycle through the damper then becomes

$$\pi c\omega_n X^2 = (3.14159)(10)(2)(.0001) = 0.006283 \text{ Joules.}$$

Alternately, realized that Equation 2.119 is that of ellipse rotated by an angle θ defined by tan2θ = -2k/($c^2\omega_n^2 + k^2 - 1$). Then match the ellipse to standard form, read off the major and minor axis (say a and b) and calculate the area from πab. See the following web site for an elipse http://mathworld.wolfram.com/Ellipse.html

2.75 The area of a force deflection curve of Figure P2.28 is measured to be 2.5 N- m, and the maximum deflection is measured to be 8 mm. From the "slope" of the ellipse the stiffness is estimated to be 5×10^4 N/m. Calculate the hysteretic damping coefficient. What is the equivalent viscous damping if the system is driven at 10 Hz?

Solution:

Given: Area = 2.5 N • m , k = 5x10^4 N/m, X = 8 mm, $\omega = 10(2\pi) = 20\pi$ rad/s

Hysteric Damping Coefficient:

$$\Delta E = \text{Area} = \pi k \beta X^2$$

$$2.5 = \pi(5\times 10^4)\beta(0.008)^2$$

$$\beta = 0.249$$

Equivalent Viscous Damping:

$$c_{eq} = \frac{k\beta}{\omega} = \frac{(5\times 10^4)(0.249)}{20\pi}$$

$$c_{eq} = 198 \text{ kg/s}$$

2.76 The area of the hysteresis loop of a hysterically damped system is measured to be 5 $\text{N} \bullet \text{m}$ and the maximum deflection is measured to be 1 cm. Calculate the equivalent viscous damping coefficient for a 20-Hz driving force. Plot c_{eq} versus ω for $2\pi \le \omega \le 100\pi$ rad/s.

Solution:

Given: Area = $5\,\text{N} \bullet \text{m}$, X = 1 cm, $\omega = 20(2\pi) = 40\pi$ rad/s

Hysteric Damping Coefficient:

$$\Delta E = \text{Area} = \pi k \beta X^2$$

$$5 = \pi k \beta (0.01)^2$$

$$k\beta = 15{,}915 \text{ N/m}$$

Equivalent Viscous Damping:

$$c_{eq} = \frac{k\beta}{\omega} = \frac{15915}{40\pi}$$

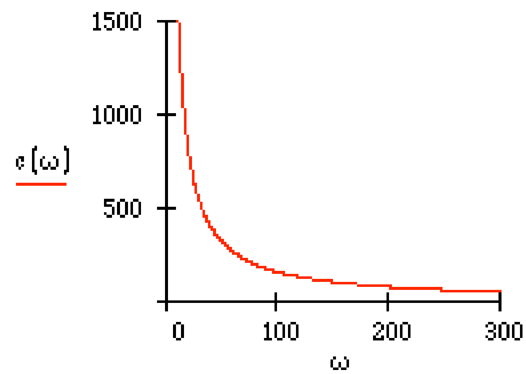
$$c_{eq} = 126.65 \text{ kg/s}$$

To plot, rearrange so that

$$\pi c_{eq} \omega X^2 = \Delta E$$

$$c_{eq} = \frac{\Delta E}{\pi \omega X^2} = \frac{5}{\pi \omega (.01)^2} = \frac{50{,}000}{\pi \omega}$$

$$c(\omega) := \frac{50000}{\pi \cdot \omega}$$



2.77 Calculate the nonconservative energy of a system subject to both viscous and hysteretic damping.

Solution:

$$\Delta E = \Delta E_{hys} + \Delta E_{visc}$$
$$\Delta E = \pi c\omega X^2 + k\pi\beta X^2$$
$$\Delta E = (c\omega + k\beta)\pi X^2$$

2.78 Derive a formula for equivalent viscous damping for the damping force of the form, F_d = $c(\dot{x})^n$ where n is an integer.

Solution:

Given: $F_d = c(\dot{x})^n$

Assume the steady-state response $x = X\sin\omega t$.

The energy lost per cycle is given by Equation (2.99) as:

$$\Delta E = \oint F_d dx = \int_0^{\frac{2\pi}{\omega}} c(\dot{x})^n \dot{x}dt = c\int_0^{\frac{2\pi}{\omega}} (\dot{x})^{n+1}dt$$

Substituting for $\dot{x}$:

$$\Delta E = \int_0^{\frac{2\pi}{\omega}} \left[\omega^{n+1}X^{n+1}\cos^{n+1}(\omega t)\right]dt$$

Let $u = \omega t$:

$$\Delta E = cX^{n+1}\omega^n \int_0^{2\pi}\left(\cos^{n+1}u\right)du$$

Equating this to Equation 2.91 yields:

$$\pi c_{eq}\omega X^2 = cX^{n+1}\omega^n \int_0^{2\pi}(\cos^{n+1}u)du$$

$$c_{eq} = \frac{cX^{n-1}\omega^{n-1}}{\pi}\int_0^{2\pi}(\cos^{n+1}u)du$$

2.79 Using the equivalent viscous damping formulation, determine an expression for the steady-state amplitude under harmonic excitation for a system with both Coulomb and viscous damping present.

Solution:

$$\Delta E = \Delta E_{visc} + \Delta E_{coul}$$
$$\Delta E = \pi c\omega X^2 + 4umgX$$

Equate to Equivalent Viscously Damped System

$$\pi c_{eq}\omega X^2 = \pi c\omega X^2 + 4\mu mg$$
$$c_{eq} = \frac{\pi c\omega X + 4\mu mg}{\pi\omega X} = c + \frac{4\mu mg}{\pi\omega X} = 2\zeta_{eq}\omega_n m$$
$$\zeta_{eq} = \zeta + \frac{2\mu g}{\pi\omega\omega_n X}$$

Amplitude:

$$X = \frac{\frac{F_o}{k}}{\sqrt{(1-r^2)^2 + (2\zeta_{eq} r)^2}} = \frac{\frac{F_o}{k}}{\sqrt{(1-r^2)^2 + \left(2\zeta r + \frac{4\mu mg}{\pi kX}\right)^2}}$$

Solving for X:

$$X = \frac{-\left(\frac{8\mu gcr^2}{\pi k\omega}\right) + \sqrt{\left(\frac{8\mu gcr^2}{\pi k\alpha\omega}\right)^2 - 4\left[(1-r^2)^2 + \frac{c^2\ r^2}{km}\right]\left[\left(\frac{4\mu gr}{\pi\omega_n\omega}\right)^2 - \left(\frac{F_o}{k}\right)^2\right]}}{2\left[(1-r^2)^2 + \frac{c^2r^2}{km}\right]}$$