Problems and Solutions Section 2.8 (2.80 through 2.86)

2.80*. Numerically integrate and plot the response of an underdamped system determined by m = 100 kg, k = 20,000 N/m, and c = 200 kg/s, subject to the initial conditions of x_0 = 0.01 m and v_0 = 0.1 m/s, and the applied force $F(t)$ = 150cos5t. Then plot the exact response as computed by equation (2.33). Compare the plot of the exact solution to the numerical simulation.

Solution: The solution is presented in Matlab:

First the m file containing the state equation to integrate is set up and saved as ftp2_72.m

```
function xdot=f(t, x)
xdot=[-(200/100)*x(1)-(20000/100)*x(2)+(150/100)*cos(5*t); x(1)];
% xdot=[x(1)'; x(2)']=[-2*zeta*wn*x(1)-wn^2*x(2)+fo*cos(w*t) ; x(1)]
% which is a state space form of
% x" + 2*zeta*wn*x' + (wn^2)*x = fo*cos(w*t)    (fo=Fo/m)

clear all;
```

Then the following m file is created and run:

```
%---- numerical simulation ---
x0=[0.1; 0.01];          %[xdot(0); x(0)]
tspan=[0 10];
[t,x]=ode45('fp2_72',tspan,x0);
plot(t, x(:,2), '.');
hold on;

%--- exact solution ----
t=0: .002: 10;
m=100; k=20000; c=200; Fo=150 ; w=5
wn=sqrt(k/m); zeta=c/(2*wn*m); fo=Fo/m; wd=wn*sqrt(1-zeta^2)
x0=0.01; v0= 0.1;
xe= exp(-zeta*wn*t) .* ( (x0-fo*(wn^2-w^2)/((wn^2-w^2)^2 ...
 +(2*zeta*wn*w)^2))*cos(wd*t) ...
 + (zeta*wn/wd*( x0-fo*(wn^2-w^2)/((wn^2-w^2)^2+(2*zeta*wn*w)^2)) ...
 - 2*zeta*wn*w^2*fo/(wd*((wd^2-w^2)^2  ...
 + (2*zeta*wn*w)^2))+v0/wd)*sin(wd*t) ) ...
 + fo/((wn^2-w^2)^2+(2*zeta*wn*w)^2)*((wn^2-w^2)*cos(w*t) ...
 + 2*zeta*wn*w*sin(w*t))

plot(t, xe, 'w');
hold off;
```

This produces the following plot:

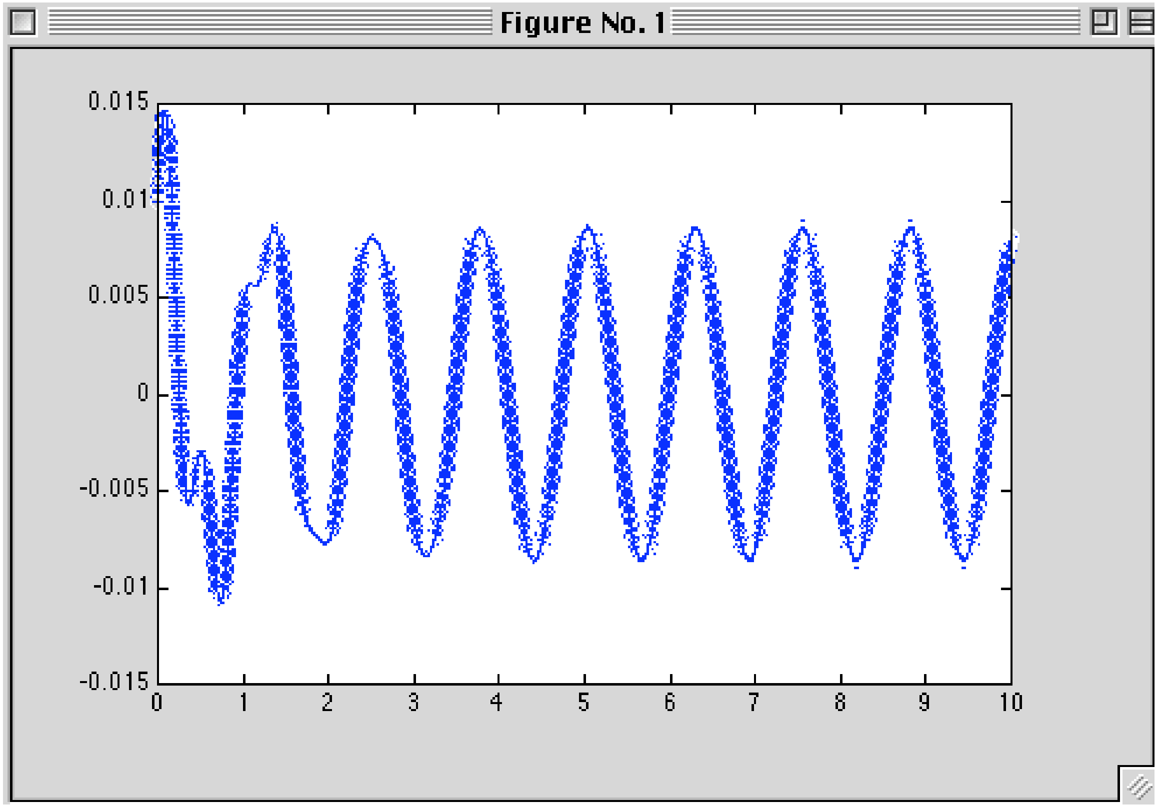


Figure No. 1
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
0
1
2
3
4
5
6
7
8
9
10

2.81*. Numerically integrate and plot the response of an underdamped system determined by m = 150 kg, and k = 4000 N/m subject to the initial conditions of x_0 = 0.01 m and v_0 = 0.1 m/s, and the applied force $F(t)$ = 15cos10t , for various values of the damping coefficient. Use this “program” to determine a value of damping that causes the transient term to die out with in 3 seconds. Try to find the smallest such value of damping remembering that added damping is usually expensive.

Solution: The solution is given by the following Mathcad session. A value of c = 350 kg/s corresponding to ζ = 0.226 gives the desired result.

2.73

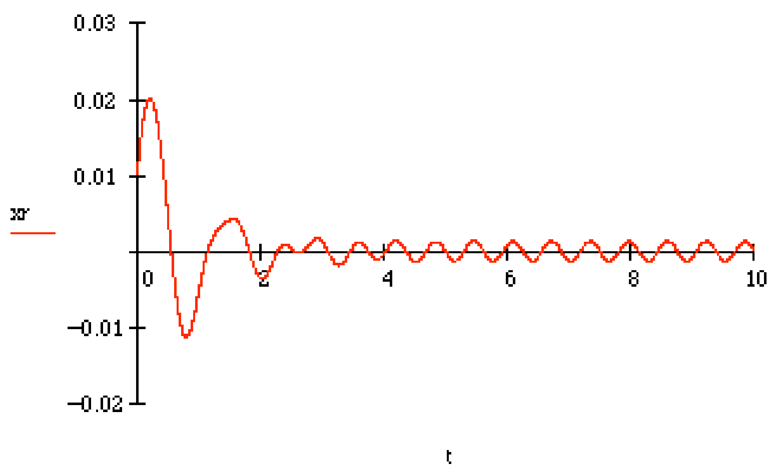
$m := 150 \qquad k := 4000 \qquad c := 350$

$\omega n := \sqrt{\dfrac{k}{m}} \qquad \zeta := \dfrac{c}{2\cdot\sqrt{k\cdot m}} \qquad F0 := 15 \qquad \omega := 10$

$$x := \begin{bmatrix} 0.01 \\ 0.1 \end{bmatrix} \qquad D(t,x) := \begin{bmatrix} x_1 \\ \left(-2\cdot\zeta\cdot\omega n\cdot x_1 - \omega n^2\cdot x_0\right) + \dfrac{F0}{m}\cdot\sin(\omega\cdot t) \end{bmatrix}$$

$Z := \text{rkfixed}(x, 0, 10, 4000, D)$

$t := Z^{<0>} \qquad xr := Z^{<1>} \qquad \zeta = 0.226 \qquad \omega n = 5.164$



2.82*. Solve Problem 2.7 by numerically integrating rather than using analytical expressions.

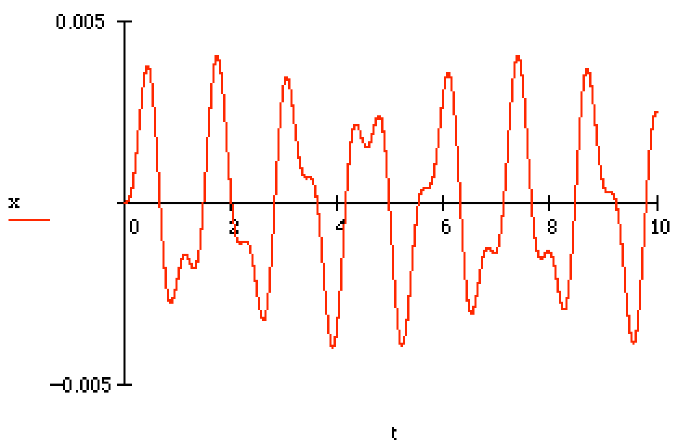
Solution: The following session in Mathcad illustrates the solution:
a) zero initial conditions

Prob 2.74

$x0 := 0.0 \quad v0 := 0 \quad m := 100 \quad k := 2000 \quad F0 := 10 \quad \omega := 10 \quad f0 := \frac{F0}{m}$

$\omega n := \sqrt{\frac{k}{m}} \quad \omega n = 4.472 \quad X := \begin{bmatrix} x0 \\ v0 \end{bmatrix} \quad D(t, X) := \begin{bmatrix} X_1 \\ -\omega n^2 \cdot X_0 + f0 \cdot \sin(\omega \cdot t) \end{bmatrix}$

$Z := \text{rkfixed}(X, 0, 10, 4000, D) \quad x := Z^{\langle 1 \rangle} \quad t := Z^{\langle 0 \rangle}$



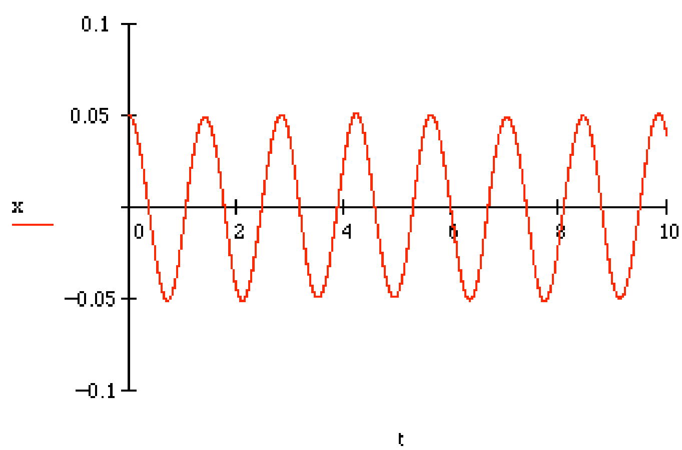
b) Using and initial condition of $x(0) = 0.05$ m.   Note the difference in the response.

Prob 2.74

$x0 := 0.05 \quad v0 := 0 \quad m := 100 \quad k := 2000 \quad F0 := 10 \quad \omega := 10 \quad f0 := \frac{F0}{m}$

$\omega n := \sqrt{\frac{k}{m}} \quad \omega n = 4.472 \quad X := \begin{bmatrix} x0 \\ v0 \end{bmatrix} \quad D(t, X) := \begin{bmatrix} X_1 \\ -\omega n^2 \cdot X_0 + f0 \cdot \sin(\omega \cdot t) \end{bmatrix}$

$Z := \mathrm{rkfixed}(X, 0, 10, 4000, D) \quad x := Z^{<1>} \quad t := Z^{<0>}$



2.83*. Numerically simulate the response of the system of Problem 2.30.

Solution: From problem 2.30, the equation of motion is

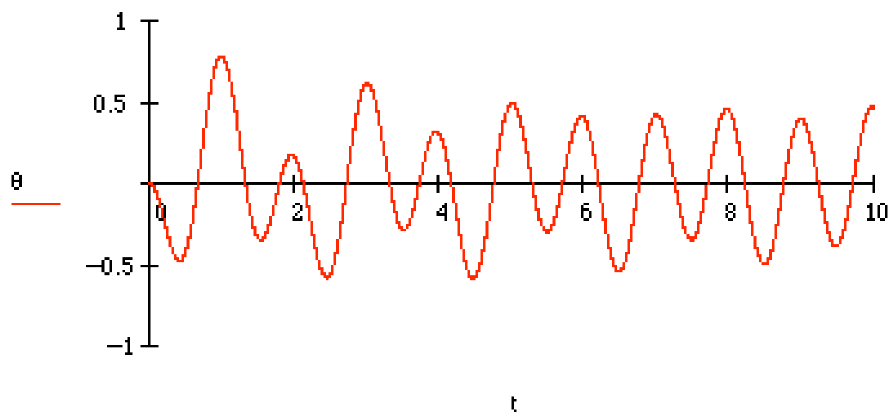
$$9a^2 m\ddot{\theta} + 4a^2 c\cos\theta\dot{\theta} + a^2 k\sin\theta = -3aF(t)$$

where k = 2000 kg, c = 25 kg/s , m = 25 kg , $F(t) = 50\cos 2\pi t$, a = 0.05 m
Placing the equation of motion in first order form and numerically integrating using Mathcad yields

$k := 2000$ $c := 25$ $m := 25$ $a := 0.05$ $f0 := \frac{-50}{3\cdot a\cdot m}$

$$X := \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad D(t,X) := \begin{bmatrix} X_1 \\ \frac{-4\cdot c}{9\cdot m}\cdot X_1\cdot\cos\left(X_0\right) - \frac{k}{9\cdot m}\cdot\sin\left(X_0\right) + f0\cdot\cos\left(2\cdot\pi\cdot t\right) \end{bmatrix}$$

$Z := \mathrm{rkfixed}(X, 0, 10, 4000, D)$ $t := Z^{\langle 0\rangle}$ $\theta := Z^{\langle 1\rangle}$



2.84*. Numerically integrate the system of Example 2.8.1 for the following sets of initial conditions:

a) x_0 = 0.0 m and v_0 = 0.1 m/s

b) x_0 = 0.01 m and v_0 = 0.0 m/s

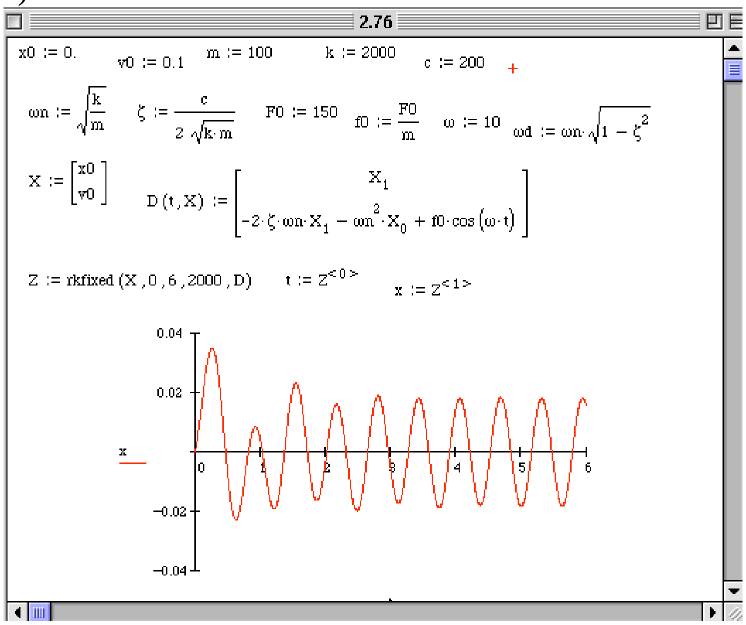
c) x_0 = 0.05 m and v_0 = 0.0 m/s

d) x_0 = 0.0 m and v_0 = 0.5 m/s

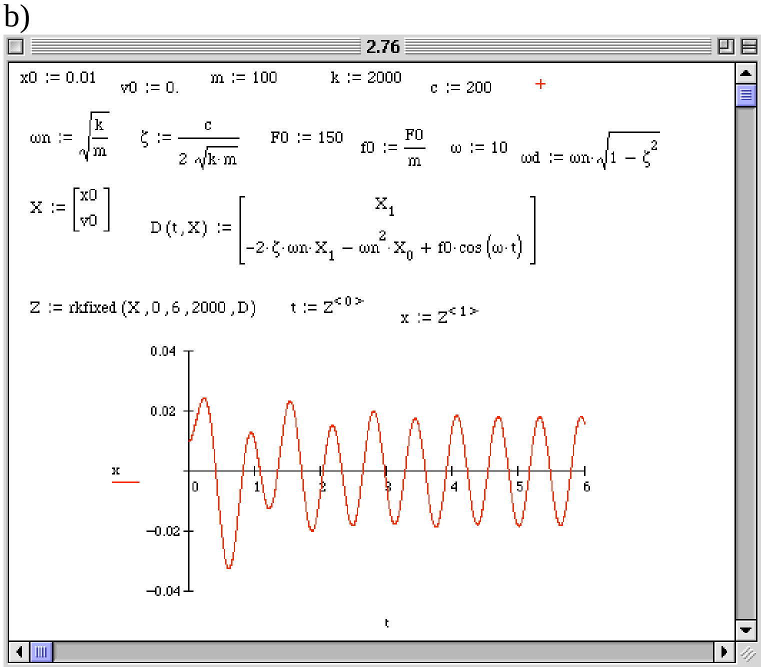
Plot these responses on the same graph and note the effects of the initial conditions on the transient part of the response.

Solution: The following are the solutions in Mathcad. Of course the other codes and Toolbox will yield the same results.

a)



b)



c)

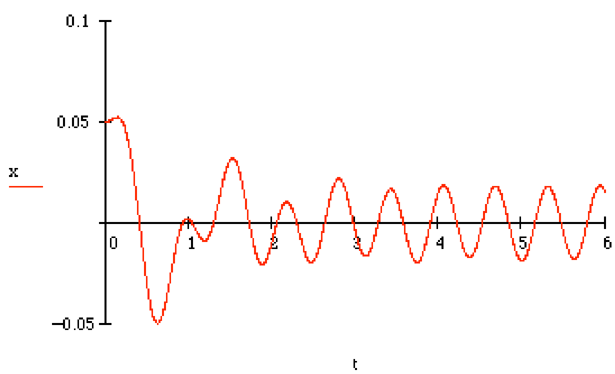
2.76

$x0 := 0.05 \quad v0 := 0. \quad m := 100 \quad k := 2000 \quad c := 200$

$\omega n := \sqrt{\frac{k}{m}} \quad \zeta := \frac{c}{2\sqrt{k \cdot m}} \quad F0 := 150 \quad f0 := \frac{F0}{m} \quad \omega := 10 \quad \omega d := \omega n \cdot \sqrt{1 - \zeta^2}$

$X := \begin{bmatrix} x0 \\ v0 \end{bmatrix} \quad D(t, X) := \begin{bmatrix} X_1 \\ -2 \cdot \zeta \cdot \omega n \cdot X_1 - \omega n^2 \cdot X_0 + f0 \cdot \cos(\omega \cdot t) \end{bmatrix}$

$Z := \mathrm{rkfixed}(X, 0, 6, 2000, D) \quad t := Z^{\langle 0 \rangle} \quad x := Z^{\langle 1 \rangle}$



d)

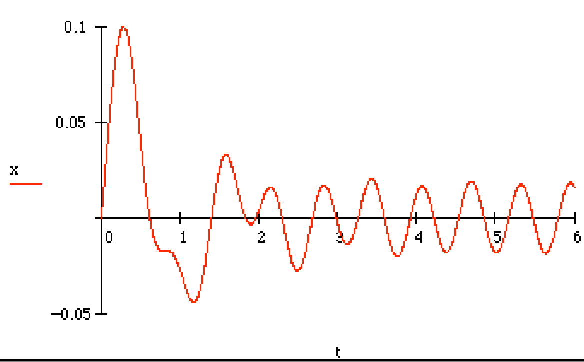
2.76

$x0 := 0.0 \quad v0 := 0.5 \quad m := 100 \quad k := 2000 \quad c := 200$

$\omega n := \sqrt{\frac{k}{m}} \quad \zeta := \frac{c}{2\sqrt{k \cdot m}} \quad F0 := 150 \quad f0 := \frac{F0}{m} \quad \omega := 10 \quad \omega d := \omega n \cdot \sqrt{1 - \zeta^2}$

$X := \begin{bmatrix} x0 \\ v0 \end{bmatrix} \quad D(t, X) := \begin{bmatrix} X_1 \\ -2 \cdot \zeta \cdot \omega n \cdot X_1 - \omega n^2 \cdot X_0 + f0 \cdot \cos(\omega \cdot t) \end{bmatrix}$

$Z := \mathrm{rkfixed}(X, 0, 6, 2000, D) \quad t := Z^{\langle 0 \rangle} \quad x := Z^{\langle 1 \rangle}$



Note the profound effect on the transient, but of course no effect on the steady state.

2.85*. A DVD drive is mounted on a chassis and is modeled as a single degree-degree-of-freedom spring, mass and damper. During normal operation, the drive (having a mass of 0.4 kg) is subject to a harmonic force of 1 N at 10 rad/s. Because of material considerations and static deflection, the stiffness is fixed at 500 N/m and the natural damping in the system is 10 kg/s. The DVD player starts and stops during its normal operation providing initial conditions to the module of x_0 = 0.001 m and v_0 = 0.5 m/s. The DVD drive must not have an amplitude of vibration larger then 0.008 m even during the transient stage. First compute the response by numerical simulation to see if the constraint is satisfied. If the constraint is not satisfied, find the smallest value of damping that will keep the deflection less then 0.008 m.

Solution: The solution is given by the following Mathcad session:

2.77

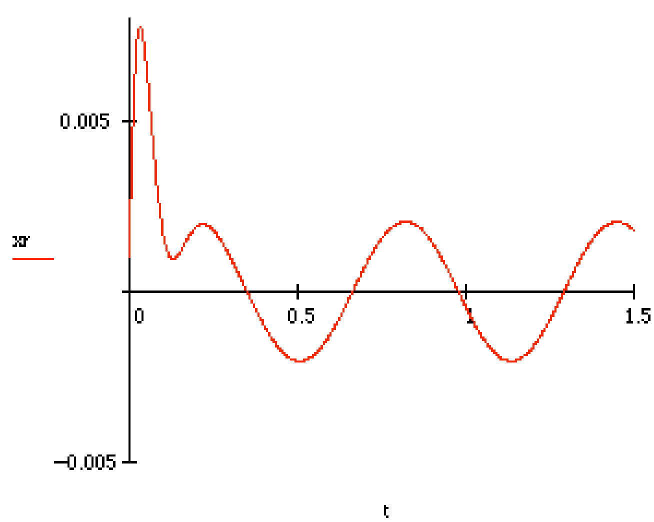
$m := 0.4 \qquad k := 500 \qquad c := 17$

$\omega n := \sqrt{\frac{k}{m}} \qquad \zeta := \frac{c}{2\cdot\sqrt{k\cdot m}} \qquad F0 := 1 \qquad \omega := 10$

$$x := \begin{bmatrix} 0.001 \\ 0.5 \end{bmatrix} \qquad D(t,x) := \begin{bmatrix} x_1 \\ \left(-2\cdot\zeta\cdot\omega n\cdot x_1 - \omega n^2\cdot x_0\right) + \frac{F0}{m}\cdot\sin(\omega\cdot t) \end{bmatrix}$$

$Z := \mathrm{rkfixed}(x, 0, 1.5, 1000, D)$

$t := Z^{\langle 0 \rangle} \qquad xr := Z^{\langle 1 \rangle}$



This yields *c* =17 kg/s as a solution.

2.86 Use a plotting routine to examine the base motion problem of Figure 2.12 by plotting the particular solution (for an undamped system) for the three cases k = 1500 N/m, and k = 700 N/m. Also note the values of the three frequency ratios and the corresponding amplitude of vibration of each case compared to the input. Use the following values: ω_b = 4.4 rad/s, m = 100 kg, and Y = 0.05 m.

Solution; The following Mathcad worksheet shows the plotting:

$m := 100 \quad k1 := 1500 \quad \omega 1 := \sqrt{\frac{k1}{m}}$

$k2 := 700$

$k3 := 2500 \quad \omega 2 := \sqrt{\frac{k2}{m}} \quad \omega 3 := \sqrt{\frac{k3}{m}} \quad \omega 1 = 3.873 \quad \omega b := 4.4$

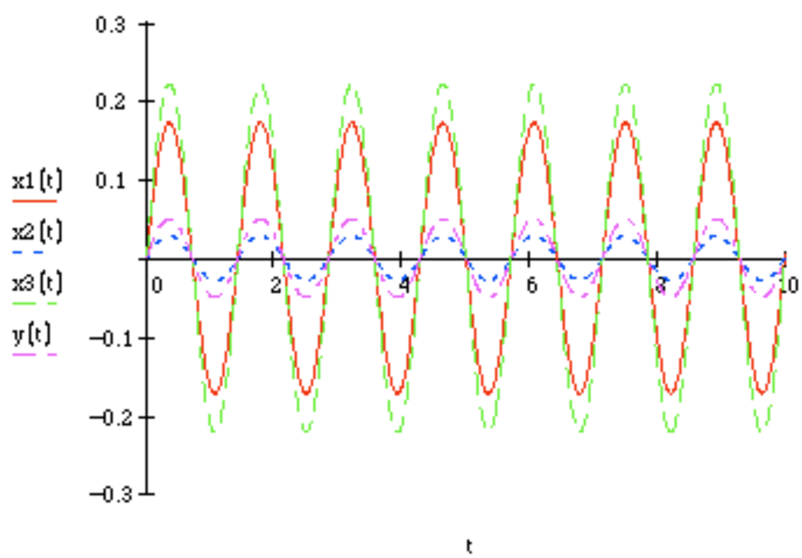
$r1 := \frac{\omega b}{\omega 1} \quad r2 := \frac{\omega b}{\omega 2} \quad r3 := \frac{\omega b}{\omega 3} \quad \omega 2 = 2.646 \quad \omega 3 = 5$

$r1 = 1.136 \quad r2 = 1.663 \quad r3 = 0.88 \quad Y := 0.05$

$y(t) := Y \cdot \sin(\omega b \cdot t)$ then from equation 2.68 and 2.70 with zero damping:

$$x1(t) := Y \cdot \frac{1}{\left|1 - r1^2\right|} \cdot \cos\left(\omega b \cdot t - \frac{\pi}{2}\right) \qquad x2(t) := Y \cdot \frac{1}{\left|1 - r2^2\right|} \cdot \cos\left(\omega b \cdot t - \frac{\pi}{2}\right)$$

$$x3(t) := Y \cdot \frac{1}{\left|1 - r3^2\right|} \cdot \cos\left(\omega b \cdot t - \frac{\pi}{2}\right)$$



Note that $k2$, the softest system (smallest k) has the smallest amplitude, smaller than the amplitude of the input as predicted by the magnitude plots in section 2.3. Thus when $r > \sqrt{2}$, the amplitude is the smallest.