

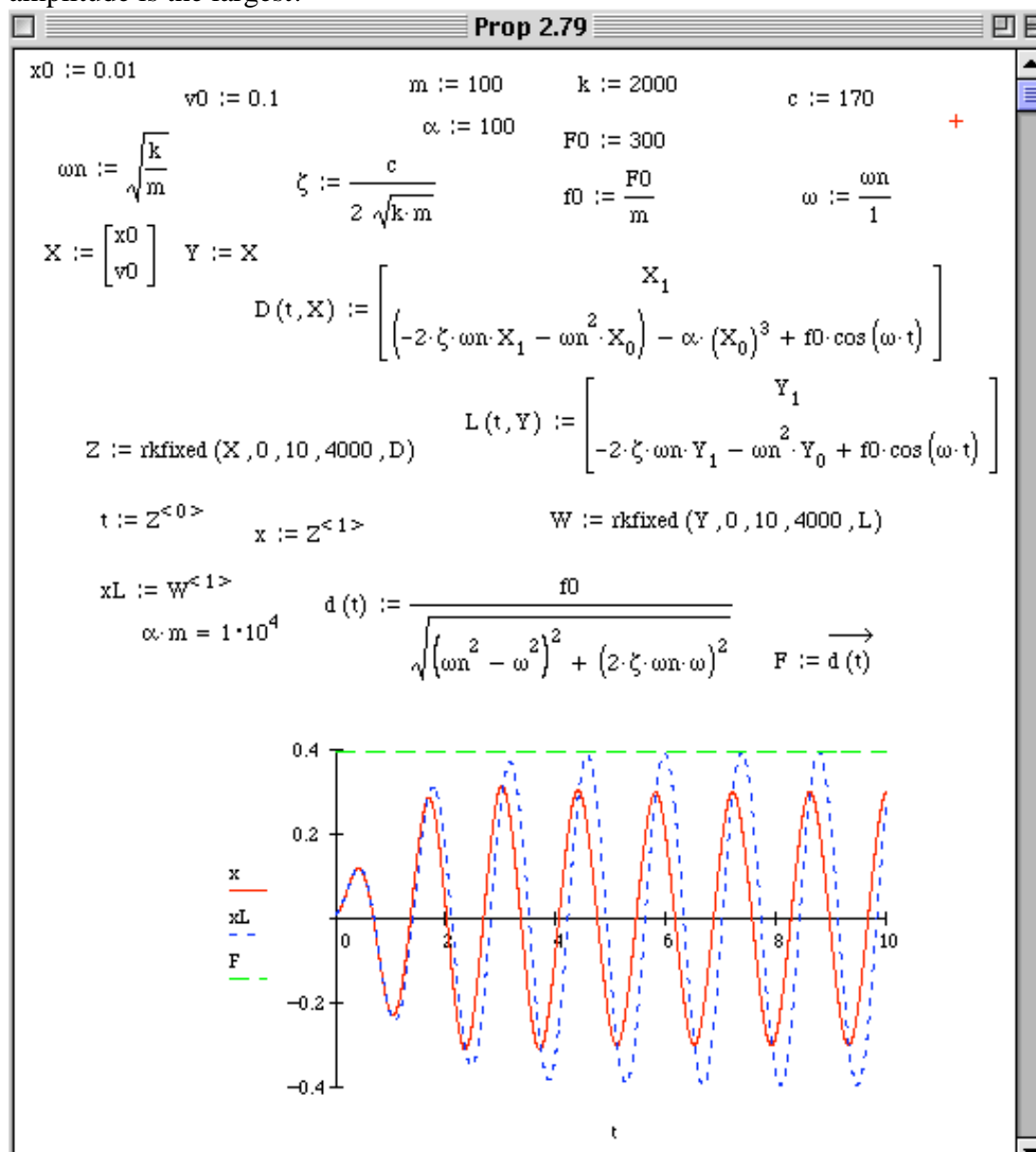


2.88*. Compute the response of the system in Figure 2.34 for the case that the damping is linear viscous and the spring is a nonlinear hard spring of the form

$$k(x) = kx + k_1x^3$$

and the system is subject to a harmonic excitation of 300 N at a frequency equal to the natural frequency ($\omega = \omega_n$) and initial conditions of $x_0 = 0.01$ m and $v_0 = 0.1$ m/s. The system has a mass of 100 kg, a damping coefficient of 170 kg/s and a linear stiffness coefficient of 2000 N/m. The value of k_1 is taken to be 10000 N/m³. Compute the solution and compare it to the linear solution ($k_1 = 0$). Which system has the largest magnitude?

Solution: The Mathcad solution appears below. Note that in this case the linear amplitude is the largest!

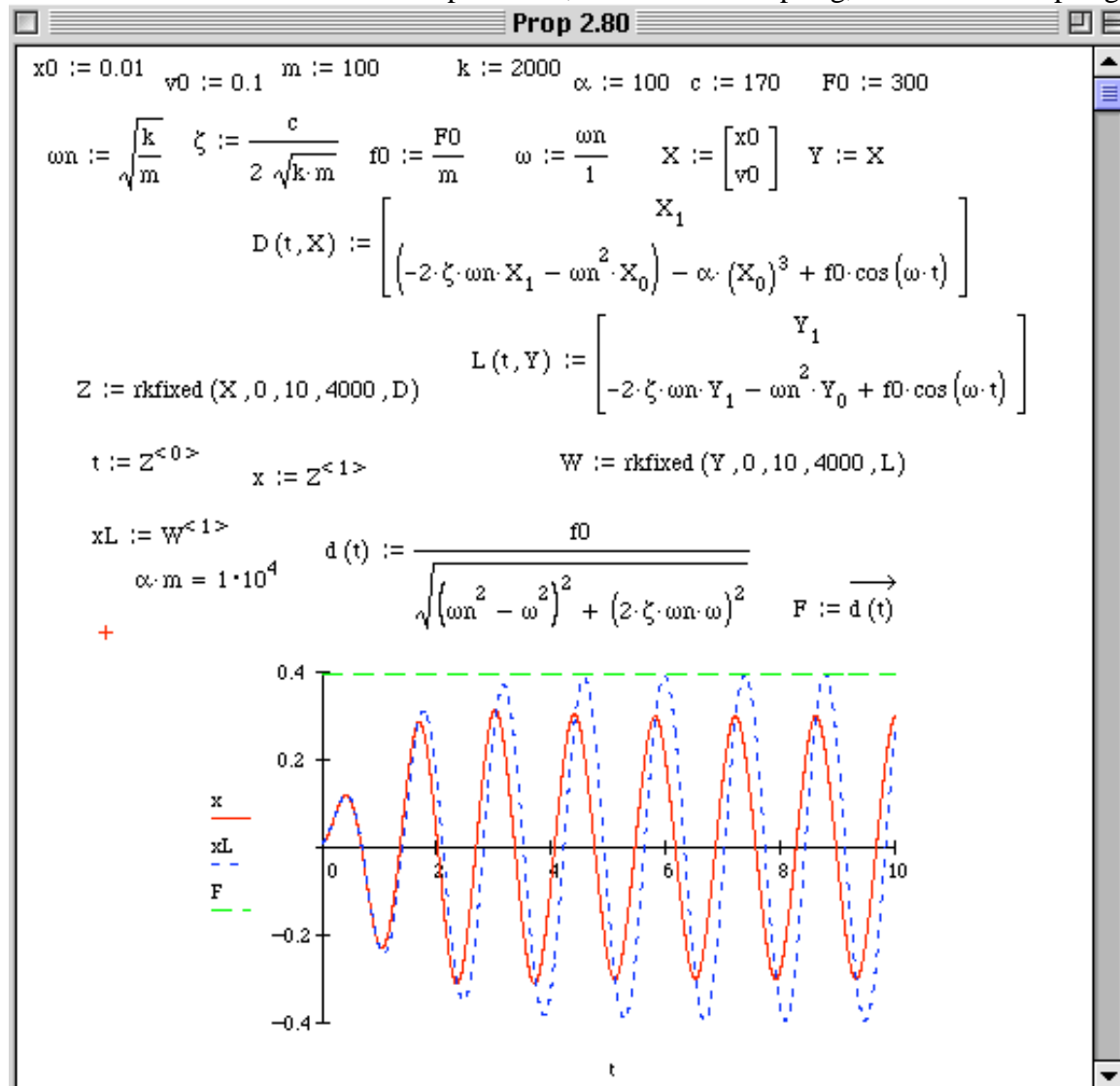


2.89*. Compute the response of the system in Figure 2.34 for the case that the damping is linear viscous and the spring is a nonlinear soft spring of the form

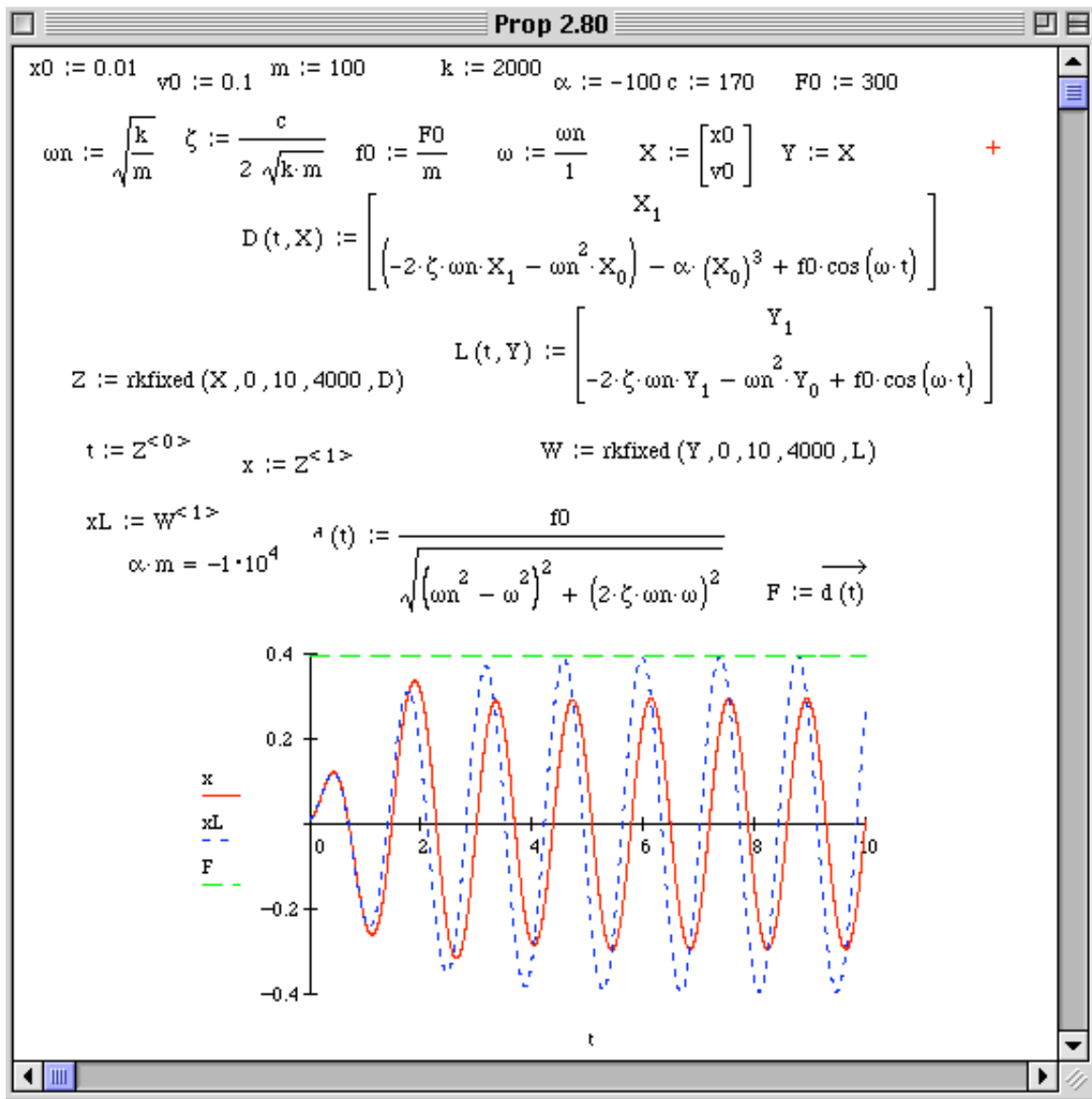
$$k(x) = kx - k_1 x^3$$

and the system is subject to a harmonic excitation of 300 N at a frequency equal to the natural frequency ($\omega = \omega_n$) and initial conditions of $x_0 = 0.01$ m and $v_0 = 0.1$ m/s. The system has a mass of 100 kg, a damping coefficient of 15 kg/s and a linear stiffness coefficient of 2000 N/m. The value of k_1 is taken to be 100 N/m³. Compute the solution and compare it to the hard spring solution ($k(x) = kx + k_1 x^3$).

Solution: The Mathcad solution is presented, first for a hard spring, then for a soft spring



Next consider the result for the soft spring and note that the nonlinear response is higher in the transient than the linear case (opposite of the hardening spring), but nearly the same in steady state as the hardening spring.

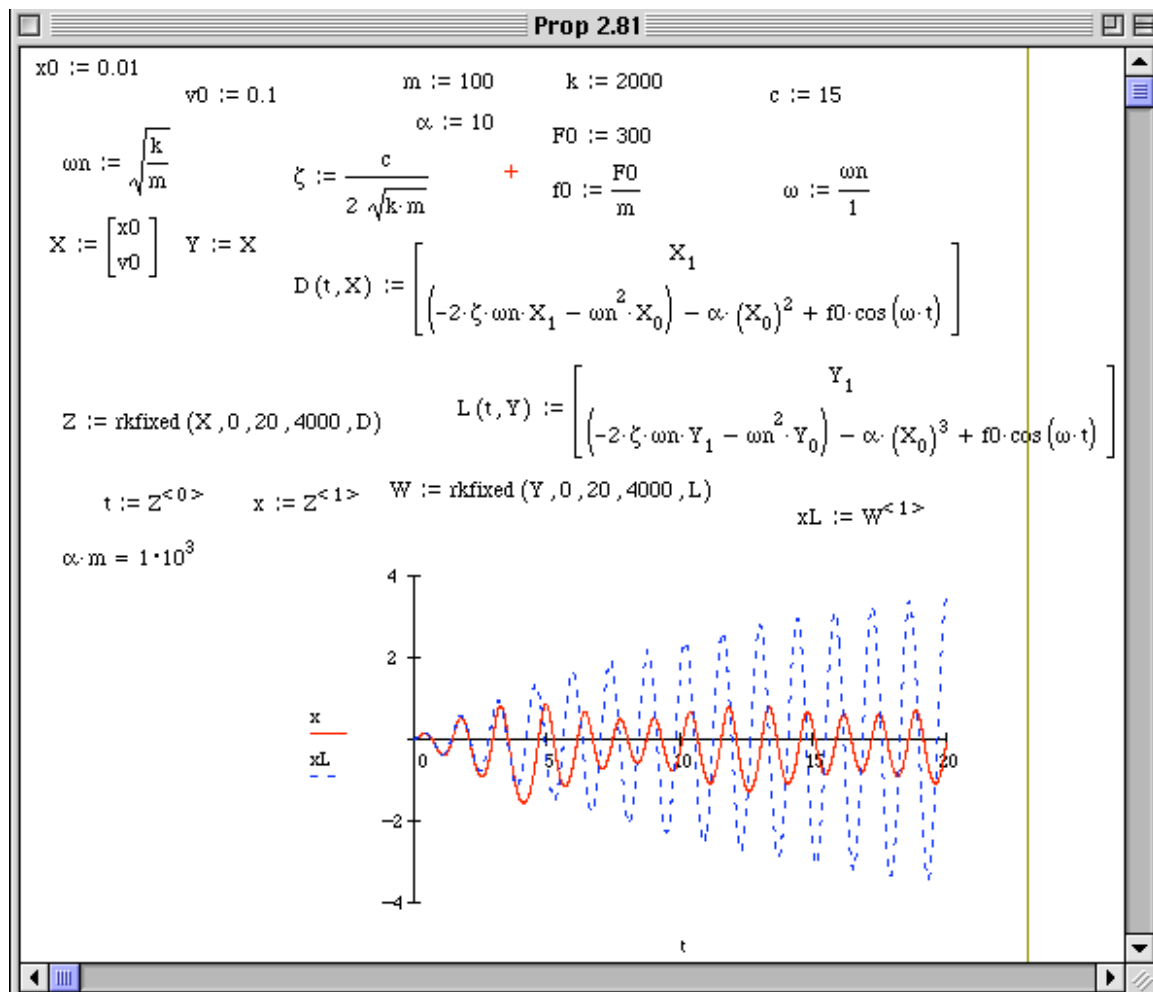


2.90*. Compute the response of the system in Figure 2.34 for the case that the damping is linear viscous and the spring is a nonlinear soft spring of the form

$$k(x) = kx - k_1x^3$$

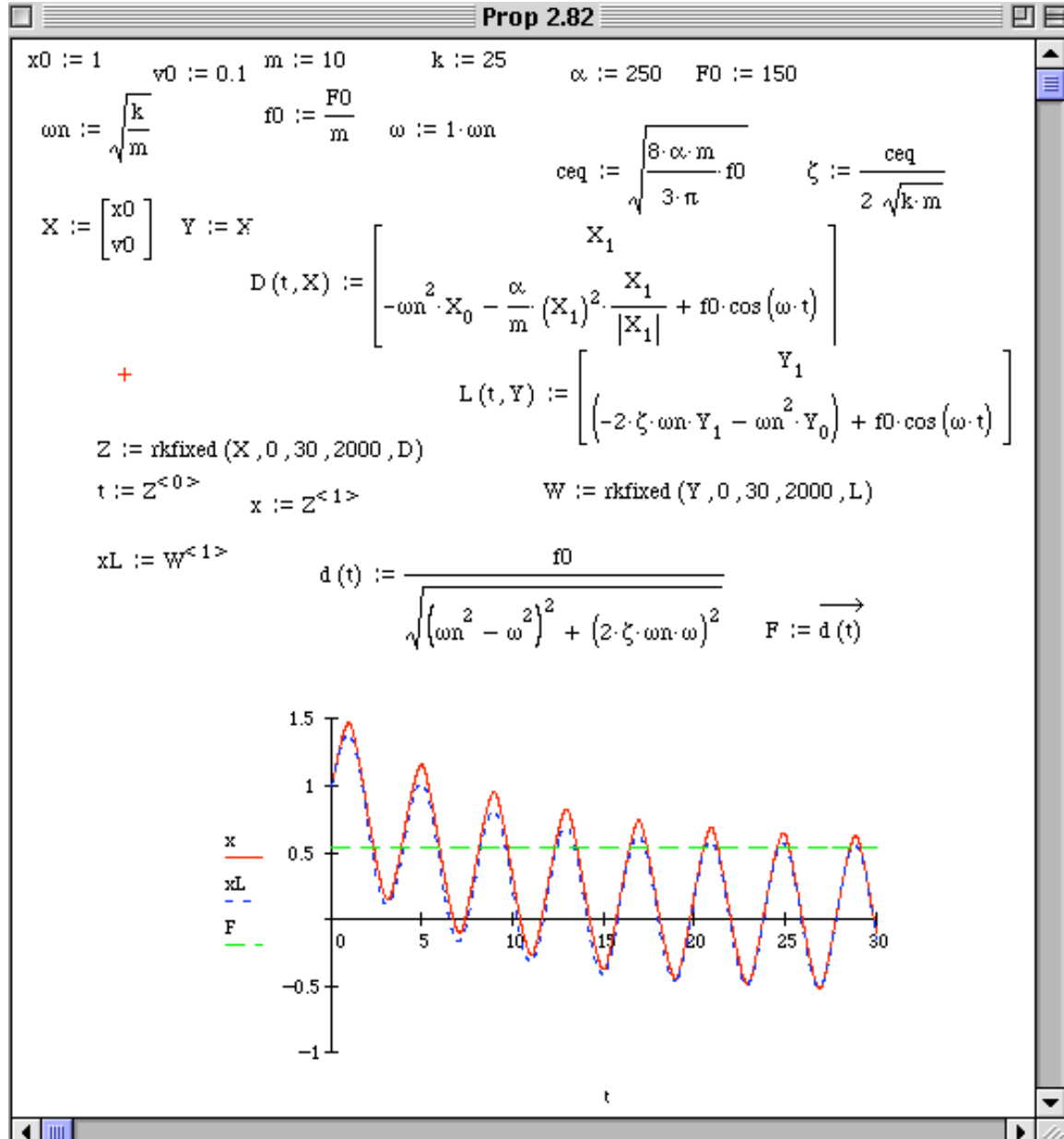
and the system is subject to a harmonic excitation of 300 N at a frequency equal to the natural frequency ($\omega = \omega_n$) and initial conditions of $x_0 = 0.01$ m and $v_0 = 0.1$ m/s. The system has a mass of 100 kg, a damping coefficient of 15 kg/s and a linear stiffness coefficient of 2000 N/m. The value of k_1 is taken to be 1000 N/m³. Compute the solution and compare it to the quadratic soft spring ($k(x) = kx + k_1x^2$).

Solution: The response to both the hardening and softening spring are given in the following Mathcad sessions. In each case the linear response is also shown for comparison. With the soft spring, the response is more variable, whereas the hardening spring seems to reach steady state.



2.91*. Compare the forced response of a system with velocity squared damping as defined in equation (2.129) using numerical simulation of the nonlinear equation to that of the response of the linear system obtained using equivalent viscous damping as defined by equation (2.131). Use as initial conditions, $x_0 = 0.01$ m and $v_0 = 0.1$ m/s with a mass of 10 kg, stiffness of 25 N/m, applied force of $150 \cos(\omega_n t)$ and drag coefficient of $\alpha = 250$.

Solution:



2.92*. Compare the forced response of a system with structural damping (see table 2.2) using numerical simulation of the nonlinear equation to that of the response of the linear system obtained using equivalent viscous damping as defined in Table 2.2. Use as initial conditions, $x_0 = 0.01$ m and $v_0 = 0.1$ m/s with a mass of 10 kg, stiffness of 25 N/m, applied force of $150 \cos(\omega_n t)$ and solid damping coefficient of $b = 25$.

Solution: The solution is presented here in Mathcad

