Problems and Solutions Section 3.10 (3.65 through 3.71)

3.65*. Compute the response of the system in Figure 3.26 for the case that the damping is linear viscous and the spring is a nonlinear soft spring of the form

$$k(x) = kx - k_1 x^3$$

and the system is subject to a excitation of the form (t_1 = 1.5 and t_2 = 1.6)

$$F(t) = 1500\left[\Phi(t - t_1) - \Phi(t - t_2)\right] \text{ N}$$

and initial conditions of x_0 = 0.01 m and v_0 = 1.0 m/s. The system has a mass of 100 kg, a damping coefficient of 30 kg/s and a linear stiffness coefficient of 2000 N/m. The value of k_1 is taken to be 300 N/m^3. Compute the solution and compare it to the linear solution (k_1 = 0). Which system has the largest magnitude? Compare your solution to that of Example 3.10.1.

Solution: The solution in Mathcad is

$x0 := 0.01 \quad v0 := 1 \qquad m := 100 \qquad k := 2000 \qquad k1 := 300 \qquad c := 30$

$\omega n := \sqrt{\frac{k}{m}} \qquad \zeta := \frac{c}{2\sqrt{k \cdot m}} \qquad \alpha := \frac{k1}{m} \qquad F0 := 1500 \qquad t1 := 1.5 \qquad t2 := 1.6$

$f0 := \frac{F0}{m} \qquad \zeta = 0.034 \qquad \alpha = 3$

$X := \begin{pmatrix} x0 \\ v0 \end{pmatrix} \quad Y := X$

$f(t) := f0 \cdot \Phi(t - t1) - f0 \cdot \Phi(t - t2)$

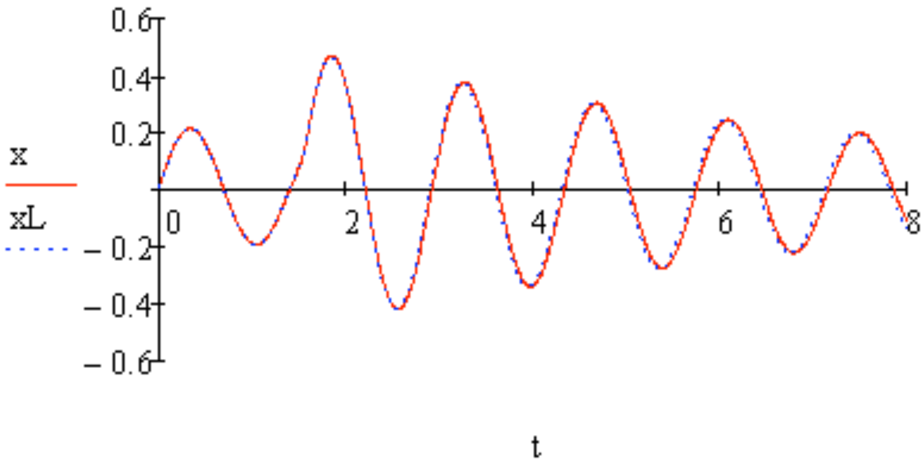
$$D(t, X) := \begin{bmatrix} X_1 \\ -2 \cdot \zeta \cdot \omega n \cdot X_1 - \omega n^2 \cdot X_0 + \left[\alpha \cdot \left(X_0\right)^3 + f(t)\right] \end{bmatrix}$$

$$L(t, Y) := \begin{bmatrix} Y_1 \\ \left(-2 \cdot \zeta \cdot \omega n \cdot Y_1 - \omega n^2 \cdot Y_0\right) + f(t) \end{bmatrix}$$

$Z := \text{rkfixed}(X, 0, 10, 2000, D)$

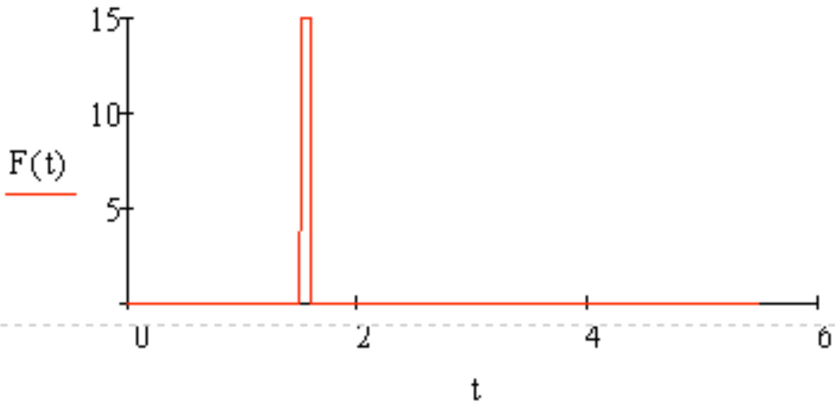
$t := Z^{\langle 0 \rangle} \qquad x := Z^{\langle 1 \rangle} \qquad W := \text{rkfixed}(Y, 0, 10, 2000, L)$

$xL := W^{\langle 1 \rangle}$



$t := 0, 0.001 .. 5.5$

$F(t) := f0 \cdot \Phi(t - t1) - f0 \cdot \Phi(t - t2)$



Note that for this load the load, which is more like an impulse, the linear and nonlinear responses are similar, whereas in Example 3.10.1 the applied load is a "wider" impulse and the linear and nonlinear responses differ quite a bit.

3.66*. Compute the response of the system in Figure 3.26 for the case that the damping is linear viscous and the spring is a nonlinear soft spring of the form

$$k(x) = kx - k_1 x^3$$

and the system is subject to a excitation of the form (t_1 = 1.5 and t_2 = 1.6)

$$F(t) = 1500\left[\Phi(t - t_1) - \Phi(t - t_2)\right] \text{ N}$$

and initial conditions of x_0 = 0.01 m and v_0 = 1.0 m/s. The system has a mass of 100 kg, a damping coefficient of 30 kg/s and a linear stiffness coefficient of 2000 N/m. The value of k_1 is taken to be 300 N/m^3. Compute the solution and compare it to the linear solution (k_1 = 0). How different are the linear and nonlinear responses? Repeat this for t_2 = 2. What can you say regarding the effect of the time length of the pulse?

Solution: The solution in Mathcad for the case t_2 = 1.6 is

Prob 3.59

$x0 := 0.01$ $\quad v0 := 1$ $\quad m := 100$ $\quad k := 2000$ $\quad c := 30$

$\alpha := 3$ $\quad F0 := 1500$ $\quad t1 := 1.5$ $\quad t2 := 1.6$

$$\omega n := \sqrt{\frac{k}{m}} \qquad \zeta := \frac{c}{2\sqrt{k \cdot m}} \qquad f0 := \frac{F0}{m} \qquad \zeta = 0.034$$

$$X := \begin{bmatrix} x0 \\ v0 \end{bmatrix} \qquad Y := X$$

$$f(t) := f0 \cdot \Phi(t - t1) - f0 \cdot \Phi(t - t2)$$

$$D(t, X) := \begin{bmatrix} X_1 \\ -2 \cdot \zeta \cdot \omega n \cdot X_1 - \omega n^2 \cdot X_0 + \left[\alpha \cdot (X_0)^3 + f(t)\right] \end{bmatrix}$$

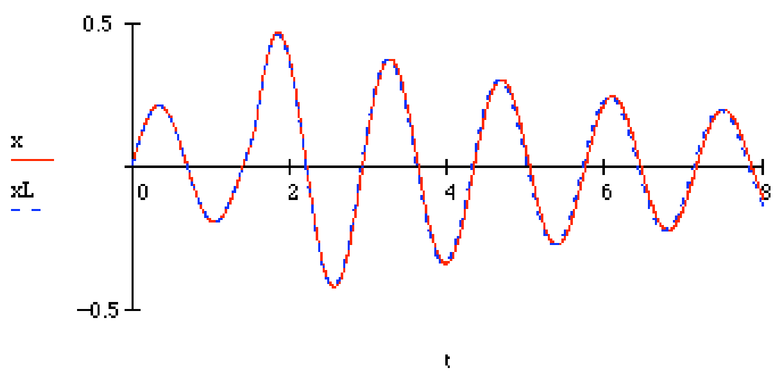
$$L(t, Y) := \begin{bmatrix} Y_1 \\ \left(-2 \cdot \zeta \cdot \omega n \cdot Y_1 - \omega n^2 \cdot Y_0\right) + f(t) \end{bmatrix}$$

$Z := \mathrm{rkfixed}(X, 0, 10, 2000, D)$

$t := Z^{\langle 0 \rangle} \qquad x := Z^{\langle 1 \rangle}$

$W := \mathrm{rkfixed}(Y, 0, 10, 2000, L)$

$xL := W^{\langle 1 \rangle}$



Note in this case the linear response is almost the same as the nonlinear response. Next changing the time of the pulse input to $t_2 = 2$ yields the following:

Prob 3.59

$x0 := 0.01$ $\quad v0 := 1$ $\quad m := 100$ $\quad k := 2000$ $\quad c := 30$

$\alpha := 3$ $\quad F0 := 1500$ $\quad t1 := 1.5$ $\quad t2 := 2$

$$\omega n := \sqrt{\frac{k}{m}} \qquad \zeta := \frac{c}{2\sqrt{k \cdot m}} \qquad f0 := \frac{F0}{m} \qquad \zeta = 0.034$$

$$X := \begin{bmatrix} x0 \\ v0 \end{bmatrix} \qquad Y := X$$

$$f(t) := f0 \cdot \Phi(t - t1) - f0 \cdot \Phi(t - t2)$$

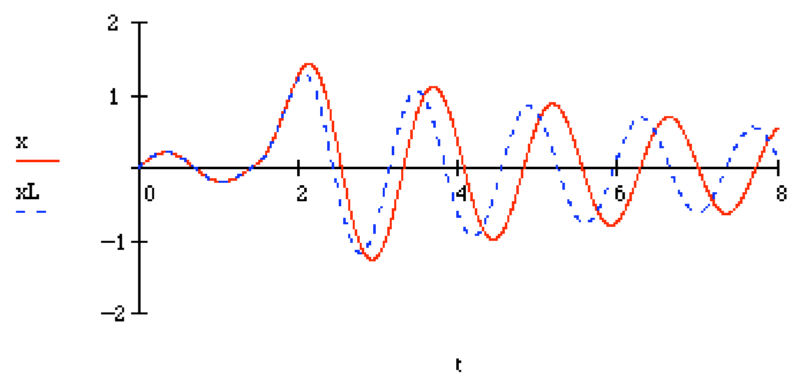
$$D(t, X) := \begin{bmatrix} X_1 \\ -2 \cdot \zeta \cdot \omega n \cdot X_1 - \omega n^2 \cdot X_0 + \left[\alpha \cdot (X_0)^3 + f(t)\right] \end{bmatrix}$$

$$L(t, Y) := \begin{bmatrix} Y_1 \\ \left(-2 \cdot \zeta \cdot \omega n \cdot Y_1 - \omega n^2 \cdot Y_0\right) + f(t) \end{bmatrix}$$

$Z := \mathrm{rkfixed}(X, 0, 10, 2000, D)$

$t := Z^{\langle 0 \rangle}$ $\quad x := Z^{\langle 1 \rangle}$ $\quad W := \mathrm{rkfixed}(Y, 0, 10, 2000, L)$

$xL := W^{\langle 1 \rangle}$



Note that as the step input last for a longer time, the response of the linear and the nonlinear becomes much different.

3.67*. Compute the response of the system in Figure 3.26 for the case that the damping is linear viscous and the spring stiffness is of the form

$$k(x) = kx - k_1x^2$$

and the system is subject to a excitation of the form (t_1 = 1.5 and t_2 = 2.5)

$$F(t) = 1500\left[\Phi(t - t_1) - \Phi(t - t_2)\right] \text{ N}$$

initial conditions of x_0 = 0.01 m and v_0 = 1 m/s. The system has a mass of 100 kg, a damping coefficient of 30 kg/s and a linear stiffness coefficient of 2000 N/m. The value of k_1 is taken to be 450 N/m^3. Which system has the largest magnitude?

Solution: The solution is computed in Mathcad as follows:

Prob 3.60

$x0 := 0.01$ $\quad v0 := 1$ $\quad m := 100$ $\quad k := 2000$ $\quad c := 30$

$\omega n := \sqrt{\frac{k}{m}}$ $\quad \zeta := \frac{c}{2\sqrt{k \cdot m}}$ $\quad \alpha := 4.5$ $\quad F0 := 1500$ $\quad t1 := 1.5$ $\quad t2 := 2.5$

$f0 := \frac{F0}{m}$ $\quad \zeta = 0.034$

$X := \begin{bmatrix} x0 \\ v0 \end{bmatrix}$ $\quad Y := X$

$f(t) := f0 \cdot \Phi(t - t1) - f0 \cdot \Phi(t - t2)$

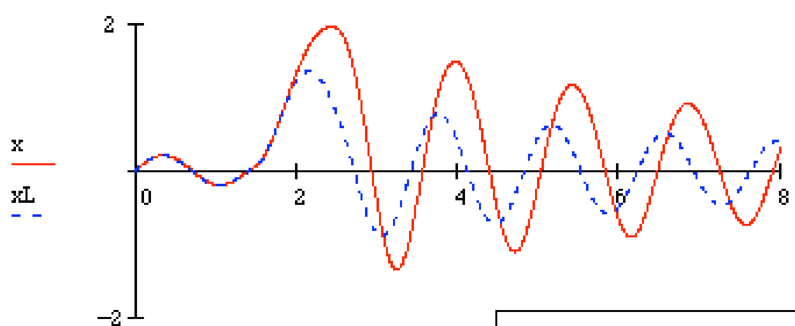
$$D(t, X) := \begin{bmatrix} X_1 \\ -2 \cdot \zeta \cdot \omega n \cdot X_1 - \omega n^2 \cdot X_0 + \left[\alpha \cdot (X_0)^2 + f(t)\right] \end{bmatrix}$$

$$L(t, Y) := \begin{bmatrix} Y_1 \\ \left(-2 \cdot \zeta \cdot \omega n \cdot Y_1 - \omega n^2 \cdot Y_0\right) + f(t) \end{bmatrix}$$

$Z := \mathrm{rkfixed}(X, 0, 10, 2000, D)$

$t := Z^{\langle 0 \rangle}$ $\quad x := Z^{\langle 1 \rangle}$ $\quad W := \mathrm{rkfixed}(Y, 0, 10, 2000, L)$

$xL := W^{\langle 1 \rangle}$



Note that the linear response under predicts the actual response

3.68*. Compute the response of the system in Figure 3.26 for the case that the damping is linear viscous and the spring stiffness is of the form

$$k(x) = kx + k_1 x^2$$

and the system is subject to a excitation of the form (t_1 = 1.5 and t_2 = 2.5)

$$F(t) = 1500\left[\Phi(t-t_1) - \Phi(t-t_2)\right] \text{ N}$$

initial conditions of x_0 = 0.01 m and v_0 = 1 m/s. The system has a mass of 100 kg, a damping coefficient of 30 kg/s and a linear stiffness coefficient of 2000 N/m. The value of k_1 is taken to be 450 N/m^3. Which system has the largest magnitude?

Solution: The solution is calculated in Mathcad as follows:

$x0 := 0.01 \quad m := 100 \quad k := 2000$

$\omega n := \sqrt{\dfrac{k}{m}} \quad v0 := 1 \quad \alpha := 4.5 \quad F0 := 1500 \quad t1 := 1.5 \quad c := 30$

$X := \begin{pmatrix} x0 \\ v0 \end{pmatrix} \quad Y := X \quad \zeta := \dfrac{c}{2\sqrt{k \cdot m}} \quad f0 := \dfrac{F0}{m} \quad t2 := 2.5 \quad \zeta = 0.034$

$f(t) := f0 \cdot \Phi(t - t1) - f0 \cdot \Phi(t - t2)$

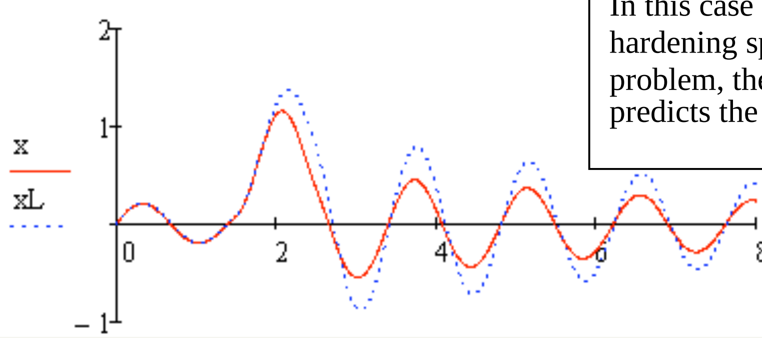
$$D(t,X) := \begin{bmatrix} X_1 \\ -2 \cdot \zeta \cdot \omega n \cdot X_1 - \omega n^2 \cdot X_0 + \left[-\alpha \cdot (X_0)^2 + f(t)\right] \end{bmatrix}$$

$Z := \text{rkfixed}(X, 0, 10, 2000, D)$

$$L(t,Y) := \begin{bmatrix} Y_1 \\ \left(-2 \cdot \zeta \cdot \omega n \cdot Y_1 - \omega n^2 \cdot Y_0\right) + f(t) \end{bmatrix}$$

$t := Z^{\langle 0 \rangle} \quad x := Z^{\langle 1 \rangle}$

$xL := W^{\langle 1 \rangle} \quad W := \text{rkfixed}(Y, 0, 10, 2000, L)$



In this case (compared to the hardening spring of the previous problem, the linear response over predicts the time history.

3.69*. Compute the response of the system in Figure 3.26 for the case that the damping is linear viscous and the spring stiffness is of the form

$$k(x) = kx - k_1 x^2$$

and the system is subject to a excitation of the form (t_1 = 1.5 and t_2 = 2.5)

$$F(t) = 150[\Phi(t - t_1) - \Phi(t - t_2)] \text{ N}$$

initial conditions of x_0 = 0.01 m and v_0 = 1 m/s. The system has a mass of 100 kg, a damping coefficient of 30 kg/s and a linear stiffness coefficient of 2000 N/m. The value of k_1 is taken to be 5500 N/m^3. Which system has the largest magnitude transient? Which has the largest magnitude in steady state?

Solution: The solution in Mathcad is given below. Note that the linear system response is less than that of the nonlinear system, and hence underestimates the actual response.

Prob 3.62

$x0 := 0.01$ $\quad v0 := 1$ $\quad m := 100$ $\quad k := 2000$ $\quad c := 30$

$\omega n := \sqrt{\dfrac{k}{m}}$ $\quad \zeta := \dfrac{c}{2\sqrt{k \cdot m}}$ $\quad \alpha := 55$ $\quad F0 := 150$ $\quad t1 := 1.5$ $\quad t2 := 2.5$

$f0 := \dfrac{F0}{m}$ $\quad \zeta = 0.034$

$X := \begin{bmatrix} x0 \\ v0 \end{bmatrix}$ $\quad Y := X$

$f(t) := f0 \cdot \Phi(t - t1) - f0 \cdot \Phi(t - t2)$

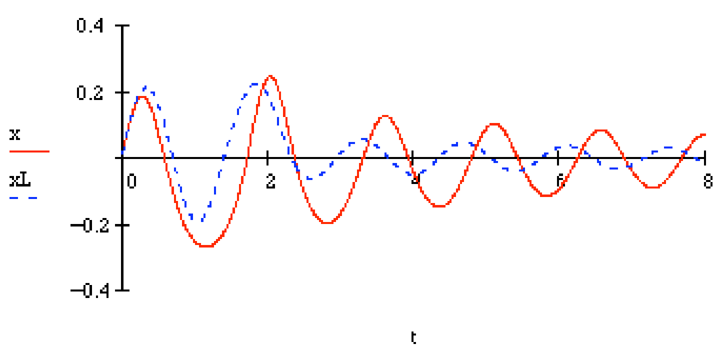
$$D(t, X) := \begin{bmatrix} X_1 \\ -2 \cdot \zeta \cdot \omega n \cdot X_1 - \omega n^2 \cdot X_0 + \left[-\alpha \cdot (X_0)^2 + f(t)\right] \end{bmatrix}$$

$$L(t, Y) := \begin{bmatrix} Y_1 \\ \left(-2 \cdot \zeta \cdot \omega n \cdot Y_1 - \omega n^2 \cdot Y_0\right) + f(t) \end{bmatrix}$$

$Z := \text{rkfixed}(X, 0, 10, 2000, D)$

$t := Z^{\langle 0 \rangle}$ $\quad x := Z^{\langle 1 \rangle}$ $\quad W := \text{rkfixed}(Y, 0, 10, 2000, L)$

$xL := W^{\langle 1 \rangle}$



3.70*. Compare the forced response of a system with velocity squared damping as defined in equation (2.129) using numerical simulation of the nonlinear equation to that of the response of the linear system obtained using equivalent viscous damping as defined by equation (2.131). Use as initial conditions, $x_0 = 0.01$ m and $v_0 = 0.1$ m/s with a mass of 10 kg, stiffness of 25 N/m, applied force of the form ($t_1 = 1.5$ and $t_2 = 2.5$)

$$F(t) = 15\left[\Phi(t-t_1) - \Phi(t-t_2)\right] \text{ N}$$

and drag coefficient of $\alpha = 25$.

Solution: The solution calculated in Mathcad is given in the follow:

$x0 := 0.01 \quad v0 := 0.1 \quad m := 10 \quad k := 25 \quad \alpha := 25 \quad F0 := 15$

$\omega n := \sqrt{\dfrac{k}{m}} \qquad f0 := \dfrac{F0}{m}$

$t1 := 1.5 \qquad t2 := 2.5 \qquad ceq := \sqrt{\dfrac{8\cdot\alpha\cdot m}{3\cdot\pi}\cdot f0}$

$\zeta := \dfrac{ceq}{2\sqrt{k\cdot m}} \qquad f(t) := f0\cdot\Phi(t-t1) - f0\cdot\Phi(t-t2)$

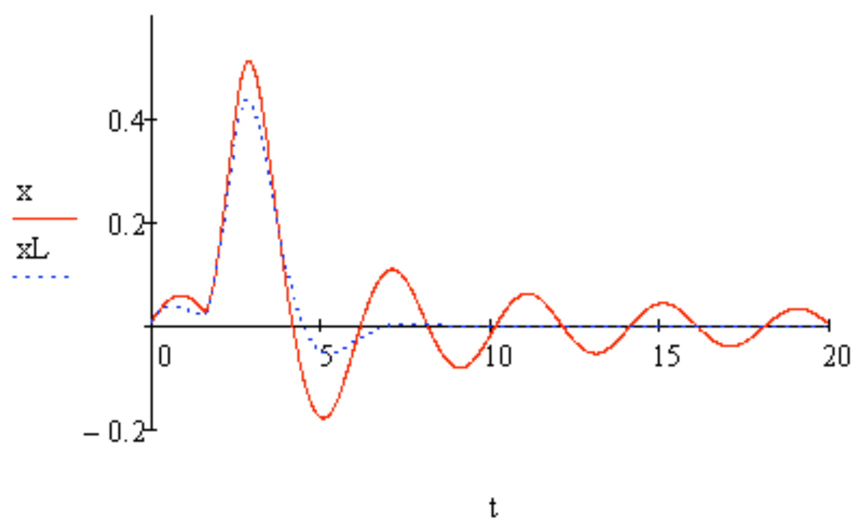
$$X := \begin{pmatrix} x0 \\ v0 \end{pmatrix} \qquad D(t,X) := \begin{bmatrix} X_1 \\ -\omega n^2\cdot X_0 - \dfrac{\alpha}{m}\cdot\left(X_1\right)^2\cdot\dfrac{X_1}{\left|X_1\right|} + f(t) \end{bmatrix} \qquad Y := X$$

$\zeta = 0.564$

$$D1(t,Y) := \begin{bmatrix} Y_1 \\ \left(-2\cdot\zeta\cdot\omega n\cdot Y_1 - \omega n^2\cdot Y_0\right) + f(t) \end{bmatrix}$$

$Z := \text{rkfixed}(X, 0, 20, 2000, D)$

$t := Z^{\langle 0 \rangle} \qquad x := Z^{\langle 1 \rangle} \qquad xL := W^{\langle 1 \rangle} \qquad W := \text{rkfixed}(Y, 0, 20, 2000, D1)$



Note that the linear solution is very different from the nonlinear solution and dies out more rapidly.

3.71*. Compare the forced response of a system with structural damping (see table 2.2) using numerical simulation of the nonlinear equation to that of the response of the linear system obtained using equivalent viscous damping as defined in Table 2.2. Use the initial conditions, $x_0 = 0.01$ m and $v_0 = 0.1$ m/s with a mass of 10 kg, stiffness of 25 N/m, applied force of the form ($t_1 = 1.5$ and $t_2 = 2.5$)

$$F(t) = 15\left[\Phi(t - t_1) - \Phi(t - t_2)\right] \text{ N}$$

and solid damping coefficient of $b = 8$. Does the equivalent viscous damping linearization, over estimate the response or under estimate it?

Solution: The solution is calculated in Mathcad as follows. Note that the linear solution is an over estimate of the nonlinear response in this case.

Prob 3.64

$x0 := 0.01 \quad v0 := 0.1 \quad m := 10 \quad k := 25 \quad b := 8 \quad F0 := 15$

$\omega n := \sqrt{\frac{k}{m}} \qquad f0 := \frac{F0}{m} \qquad ceq := \frac{2 \cdot b}{\pi \cdot \omega n}$

$t1 := .5 \qquad t2 := 2.5$

$\zeta := \frac{ceq}{2\sqrt{k \cdot m}} \qquad f(t) := f0 \cdot \Phi(t - t1) - f0 \cdot \Phi(t - t2)$

$X := \begin{bmatrix} x0 \\ v0 \end{bmatrix} \qquad Y := X$

$$D(t, X) := \begin{bmatrix} X_1 \\ -\omega n^2 \cdot X_0 - \frac{b}{m} \cdot |X_0| \cdot \frac{X_1}{|X_1|} + f(t) \end{bmatrix}$$

$$L(t, Y) := \begin{bmatrix} Y_1 \\ \left(-2 \cdot \zeta \cdot \omega n \cdot Y_1 - \omega n^2 \cdot Y_0\right) + f(t) \end{bmatrix}$$

$Z := \mathrm{rkfixed}(X, 0, 20, 2000, D)$

$t := Z^{\langle 0 \rangle} \qquad x := Z^{\langle 1 \rangle} \qquad xL := W^{\langle 1 \rangle} \qquad W := \mathrm{rkfixed}(Y, 0, 20, 2000, L)$