Problems and Solutions from Section 3.9 (3.57-3.64)

3.57*. Numerically integrate and plot the response of an underdamped system determined by m = 100 kg, k = 1000 N/m, and c = 20 kg/s, subject to the initial conditions of $x_0 = 0$ and $v_0 = 0$, and the applied force F(t) = 30Φ(t -1). Then plot the exact response as computed by equation (3.17). Compare the plot of the exact solution to the numerical simulation.

Solution: First the solution is presented in Mathcad:

Prob 3.50

$m := 100 \quad k := 1000 \quad \omega n := \sqrt{\frac{k}{m}} \quad c := 20$

$F0 := 30 \quad \omega n = 3.162 \quad \zeta := \frac{c}{2\cdot\sqrt{k\cdot m}} \quad t0 := 1 \quad \zeta = 0.032$

$\theta := \operatorname{atan}\left[\frac{\zeta}{1-\zeta^2}\right] \quad \frac{F0}{k} = 0.03 \quad \frac{F0}{k\cdot\sqrt{1-\zeta^2}} = 0.03002 \quad \zeta\cdot\omega n = 0.1$

$\omega d := \omega n\cdot\sqrt{1-\zeta^2} \quad \omega d = 3.1607 \quad \theta = 0.032 \quad m := \left(\frac{k}{\omega n^2}\right)$

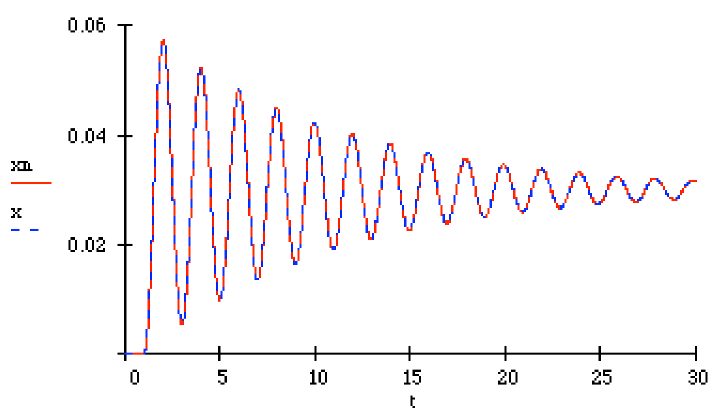
$$xa(t) := \left[\left[\frac{F0}{k} - \frac{F0}{k\cdot\sqrt{1-\zeta^2}}\cdot e^{-\zeta\cdot\omega n\cdot(t-t0)}\cdot\cos\left[\omega d\cdot(t-t0)-\theta\right]\right]\right]\cdot\Phi(t-t0)$$

$X := \begin{bmatrix}0\\0\end{bmatrix}$

$$D(t,X) := \begin{bmatrix} X_1 \\ -2\cdot\zeta\cdot\omega n\cdot X_1 - \omega n^2\cdot X_0 + \frac{F0}{m}\cdot\Phi(t-t0)\end{bmatrix}$$

$Z := \text{rkfixed}(X, 0, 30, 2000, D)$

$t := Z^{\langle 0\rangle} \quad x := \overrightarrow{xa(t)} \quad xn := Z^{\langle 1\rangle}$



The Matlab code to provide similar plots is given next:

```
%Numerical Solutions
%Problem #57
clc
clear
close all
%Numerical Solution
x0=[0;0];
tspan=[0 15];

[t,x]=ode45('prob57a',tspan,x0);

figure(1)
plot(t,x(:,1));
title('Problem #57');
xlabel('Time, sec.');
ylabel('Displacement, m');
hold on

%Analytical Solution
m=100;
c=20;
k=1000;
F=30;
w=sqrt(k/m);
d=c/(2*w*m);
wd=w*sqrt(1-d^2);
to=1;
phi=atan(d/sqrt(1-d^2));

%for t<to
t=linspace(0,1,3);
x=0.*t;
plot(t,x,'*');

%for t>=to
t=linspace(1,15);
x=F/k-F/(k*sqrt(1-d^2)).*exp(-d.*w.*(t-to)).*cos(wd.*(t-to)-phi);
plot(t,x,'*');
legend('Numerical', 'Analytical')

%M-file for Prob #50

function dx=prob(t,x);
[rows, cols]=size(x);dx=zeros(rows, cols);
m=100;
c=20;
k=1000;
F=30;

if t<1
   dx==0;
   else
      dx(1)=x(2);
   dx(2)=-c/m*x(2) - k/m*x(1) + F/m;
end
```

3.58*. Numerically integrate and plot the response of an underdamped system determined by m = 150 kg, and k = 4000 N/m subject to the initial conditions of x_0 = 0.01 m and v_0 = 0.1 m/s, and the applied force $F(t)$ = F(t) = 15Φ(t -1), for various values of the damping coefficient. Use this "program" to determine a value of damping that causes the transient term to die out with in 3 seconds. Try to find the smallest such value of damping remembering that added damping is usually expensive.

Solution: First the solution is given in Mathcad followed by the equivalent Matlab code.

$m := 150 \quad k := 4000 \quad \omega n := \sqrt{\frac{k}{m}} \quad c := 710 \quad x0 := 0.01 \quad v0 := 0.1$

$F0 := 15 \quad \omega n = \quad \zeta := \frac{c}{2\cdot\sqrt{k\cdot m}} \quad t0 := 1 \quad \zeta =$

$\theta := \operatorname{atan}\left(\frac{\zeta}{1-\zeta^2}\right) \quad \theta = \quad \omega d := \omega n\cdot\sqrt{1-\zeta^2} \quad A := \sqrt{\frac{(v0+\zeta\cdot\omega n\cdot x0)^2+(x0\cdot\omega d)^2}{\omega d^2}}$

$\omega d = \quad \phi := \operatorname{atan}\left(\frac{x0\cdot\omega d}{v0+\zeta\cdot\omega n\cdot x0}\right) \quad xh(t) := A\cdot e^{-\zeta\cdot\omega n\cdot t}\cdot\sin(\omega d\cdot t+\phi)$

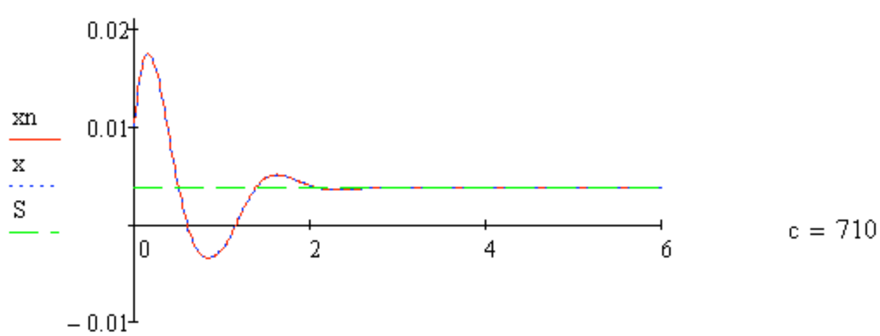
$$xa(t) := \left[\left[\frac{F0}{k}-\frac{F0}{k\cdot\sqrt{1-\zeta^2}}\cdot e^{-\zeta\cdot\omega n\cdot(t-t0)}\cdot\cos[\omega d\cdot(t-t0)-\theta]\right]\right]\cdot\Phi(t-t0)+xh(t)$$

$$xa(t) := \left[\left[\frac{F0}{k}-\frac{F0}{k\cdot\sqrt{1-\zeta^2}}\cdot e^{-\zeta\cdot\omega n\cdot(t-t0)}\cdot\cos[\omega d\cdot(t-t0)-\theta]\right]\right]\cdot\Phi(t-t0)+xh(t)$$

$$X := \begin{pmatrix} x0 \\ v0 \end{pmatrix} \quad D(t,X) := \begin{pmatrix} X_1 \\ -2\cdot\zeta\cdot\omega n\cdot X_1-\omega n^2\cdot X_0+\frac{F0}{m}\cdot\Phi(t-t0) \end{pmatrix} \quad F(t) := \frac{F0}{k}$$

$Z := \text{rkfixed}(X,0,30,2000,D)$

$t := Z^{\langle 0\rangle} \quad x := \overrightarrow{xa(t)} \quad xn := Z^{\langle 1\rangle} \quad S := \overrightarrow{F(t)}$



A value of c = 710 kg/s will do the job.

```
%Vibrations
%Numerical Solutions
%Problem #51

clc
clear
close all

%Numerical Solution

x0=[0.01;0];
tspan=[0 15];

[t,x]=ode45('prob51a',tspan,x0);

figure(1)
plot(t,x(:,1));
title('Problem #51');
xlabel('Time, sec.');
ylabel('Displacement, m');
hold on

%Analytical Solution

m=150;
c=0;
k=4000;
F=15;
w=sqrt(k/m);
d=c/(2*w*m);
wd=w*sqrt(1-d^2);
to=1;
phi=atan(d/sqrt(1-d^2));



%for t<to
t=linspace(0,1,10);
x0=0.01;
v0=0;
A=sqrt(v0^2+(x0*w)^2)/w;
theta=pi/2;
x=A.*sin(w.*t + theta);
plot(t,x,'*')

%for t>=to
t=linspace(1,15);
x2=F/k-F/(k*sqrt(1-d^2)).*exp(-d.*w.*(t-to)).*cos(wd.*(t-to)-phi);
x1=A.*sin(w.*t + theta);

x=x1+x2;
plot(t,x,'*');
legend('Numerical', 'Analytical')
%Clay
%Vibrations
%Solutions
```

```
%M-file for Prob #51

function dx=prob(t,x);
[rows, cols]=size(x);dx=zeros(rows, cols);
m=150;
c=0;
k=4000;
F=15;


if t<1
   dx(1)=x(2);
   dx(2)=-c/m*x(2)- k/m*x(1);
   else
      dx(1)=x(2);
   dx(2)=-c/m*x(2) - k/m*x(1)+ F/m;
end
```

3.59*. Solve Example 3.3.2, Figure 3.9 by numerically integrating rather then using analytical expressions, and plot the response.

Solution: Both Mathcad and Matlab solutions follow:

Prob 3.52

$x0 := 0.01 \quad v0 := 3 \quad \omega b := 3 \quad m := 1 \quad c := 10 \quad t0 := 0$

$k := 1000 \quad \omega n := \sqrt{\frac{k}{m}} \quad \zeta := \frac{c}{2 \cdot \sqrt{k \cdot m}} \quad \omega n = 31.623 \quad Y := 0.05$

$\omega d := \omega n \cdot \sqrt{1 - \zeta^2} \quad \omega d = 31.22499 \quad \theta := 1.53$

$A := 0.09341 \quad \phi := 0.1074$

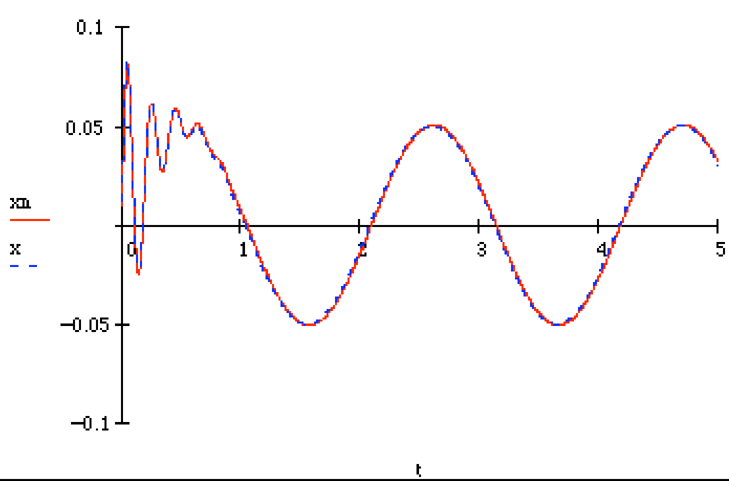
$xa(t) := A \cdot e^{-\zeta \cdot \omega n \cdot t} \sin(\omega d \cdot t + \phi) + 0.05 \cdot \cos(3 \cdot t - \theta)$

$X := \begin{bmatrix} x0 \\ v0 \end{bmatrix}$

$$D(t, X) := \begin{bmatrix} X_1 \\ -2 \cdot \zeta \cdot \omega n \cdot X_1 - \omega n^2 \cdot X_0 + \frac{c}{m} \cdot Y \cdot \omega b \cdot \cos(\omega b \cdot t) + \frac{k}{m} \cdot Y \cdot \sin(\omega b \cdot t) \end{bmatrix}$$

$Z := \text{rkfixed}(X, 0, 6, 2000, D)$

$t := Z^{<0>} \quad x := \overrightarrow{xa(t)} \quad xn := Z^{<1>}$




```
%Numerical Solutions
%Problem #53
clc
clear
close all
%Numerical Solution
```

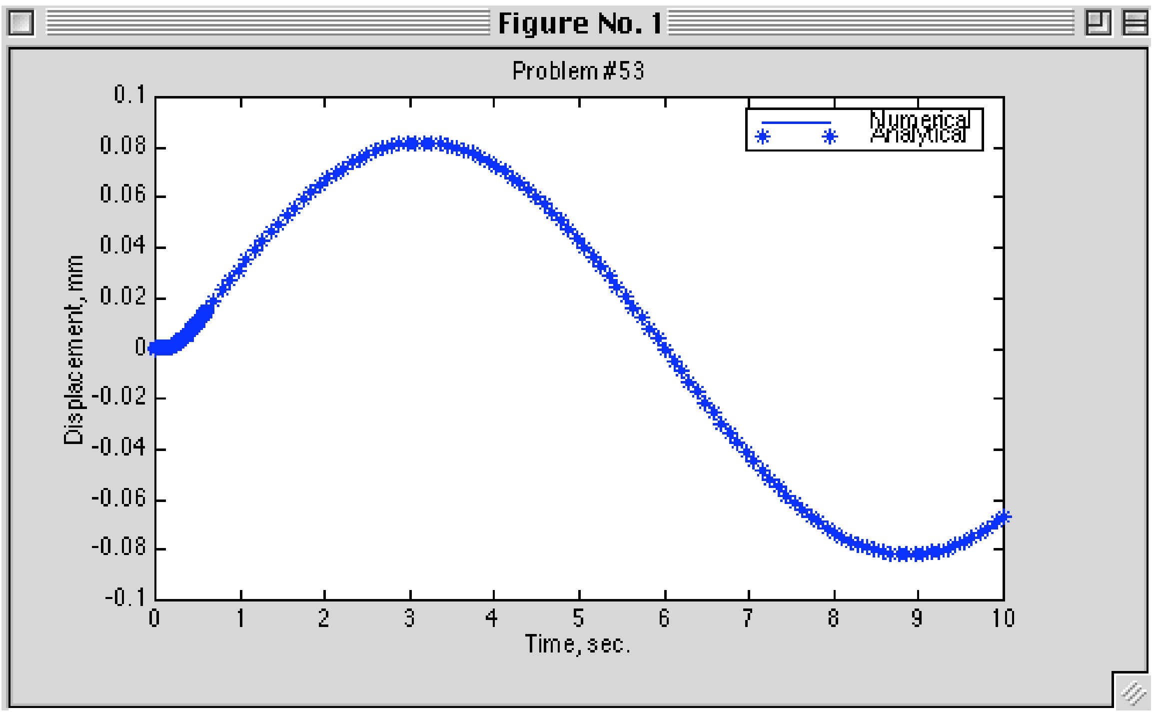
```
x0=[0;0];
tspan=[0 10];

[t,x]=ode45('prob53a',tspan,x0);

figure(1)
plot(t,x(:,1));
title('Problem #53');
xlabel('Time, sec.');
ylabel('Displacement, mm');
hold on

%Analytical Solution
t1=0.2;
t2=0.6;

%for t<to
t=linspace(0,t1);
x=2.5*t-4.56.*sin(0.548.*t);
plot(t,x,'*');

%for t1<t<t2
t=linspace(t1,t2);
x=0.75 - 1.25.*t + 6.84.*sin(0.548*(t-t1))- 4.56.*sin(0.548.*t);
plot(t,x,'*');

%for t2<t
t=linspace(t2,10);
x=6.84.*sin(0.548.*(t-t1))-2.28.*sin(0.548.*(t-t2))-
4.56.*sin(0.548.*t);
plot(t,x,'*');
legend('Numerical', 'Analytical')
%Clay
%Vibrations
%Solutions
%Clay
%Vibrations
%Solutions

%M-file for Prob #52

function dx=prob(t,x);
[rows, cols]=size(x);dx=zeros(rows, cols);
m=1;
c=10;
k=1000;
Y=0.05;
wb=3;

a=c*Y*wb;
b=k*Y;
alpha=atan(b/a);
AB=sqrt(a^2+b^2)/m;

dx(1)=x(2);
dx(2)=-c/m*x(2)- k/m*x(1)+ a/m*cos(wb*t) + b/m*sin(wb*t);
```

3.60*. Numerically simulate the response of the system of Problem 3.21 and plot the response.

Solution: The solution in Matlab is

```
%Clay
%Vibrations
%Numerical Solutions
%Problem #53

clc
clear
close all

%Numerical Solution

x0=[0;0];
tspan=[0 10];

[t,x]=ode45('prob53a',tspan,x0);

figure(1)
plot(t,x(:,1));
title('Problem #53');
xlabel('Time, sec.');
ylabel('Displacement, mm');
hold on

%Analytical Solution

t1=0.2;
t2=0.6;

%for t<to
t=linspace(0,t1);
x=2.5*t-4.56.*sin(0.548.*t);
plot(t,x,'*');

%for t1<t<t2
t=linspace(t1,t2);
x=0.75 - 1.25.*t + 6.84.*sin(0.548*(t-t1))- 4.56.*sin(0.548.*t);
plot(t,x,'*');

%for t2<t
t=linspace(t2,10);
x=6.84.*sin(0.548.*(t-t1))-2.28.*sin(0.548.*(t-t2))-
4.56.*sin(0.548.*t);
plot(t,x,'*');
legend('Numerical', 'Analytical')

%Clay
%Vibrations
%Solutions
```

```
%M-file for Prob #53

function dx=prob(t,x);
[rows, cols]=size(x);dx=zeros(rows, cols);

m=5000;
k=1.5e3;
ymax=0.5;
F=k*ymax;
t1=0.2;
t2=0.6;

if t<t1
   dx(1)=x(2);
   dx(2)= - k/m*x(1)+ F/m*(t/t1);
elseif t<t2 & t>t1
      dx(1)=x(2);
   dx(2)= - k/m*x(1)+ F/(2*t1*m)*(t2-t);
else
   dx(1)=x(2);
   dx(2)= - k/m*x(1);
end
```

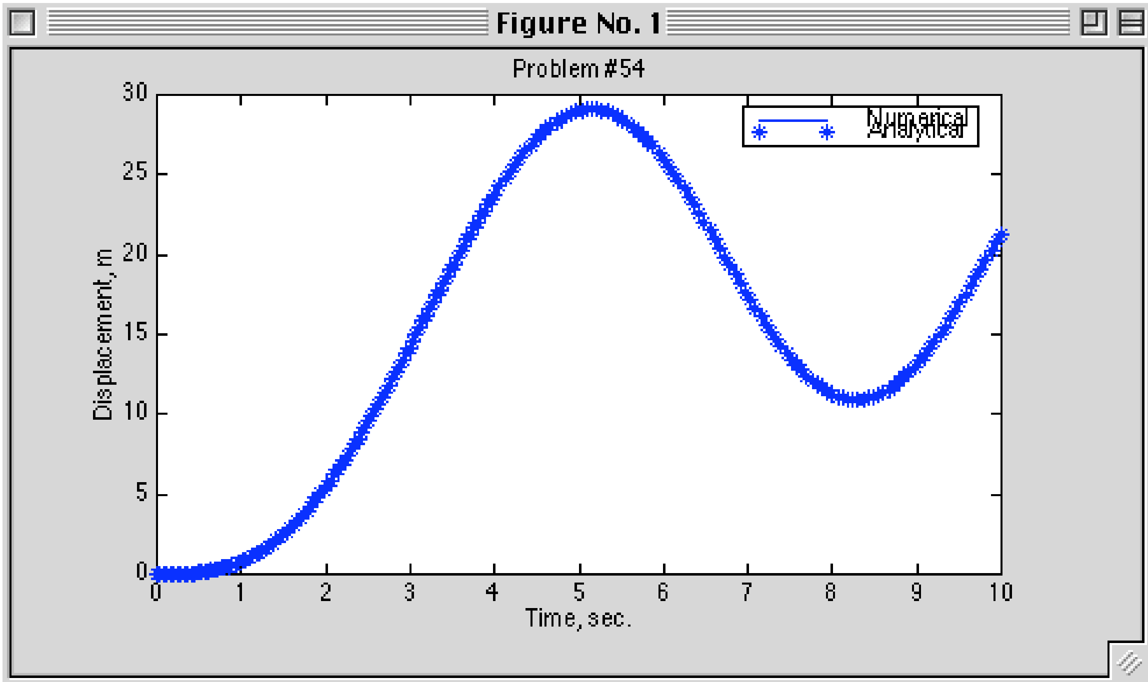


3.61*. Numerically simulate the response of the system of Problem 3.18 and plot the response.

Solution: The solution in Matlab is

```
%Clay
%Vibrations
%Numerical Solutions
%Problem #54

clc
clear
close all

%Numerical Solution

x0=[0;0];
tspan=[0 10];

[t,x]=ode45('prob54a',tspan,x0);

figure(1)
plot(t,x(:,1));
title('Problem #54');
xlabel('Time, sec.');
ylabel('Displacement, m');
hold on

%Analytical Solution

to=4;

%for t<to
t=linspace(0,to);
x=5*(t-sin(t));
plot(t,x,'*');

%for t>=to
t=linspace(to,10);
x=5*(sin(t-to)-sin(t))+20;

plot(t,x,'*');
legend('Numerical', 'Analytical')
%Clay
%Vibrations
%Solutions

%M-file for Prob #54


function dx=prob(t,x);
[rows, cols]=size(x);dx=zeros(rows, cols);

m=1;
k=1;
F=20;
```

```
to=4;

if t<to
   dx(1)=x(2);
   dx(2)= - k/m*x(1)+ F/m*(t/to);
else
      dx(1)=x(2);
   dx(2)= - k/m*x(1)+ F/m;
end
```

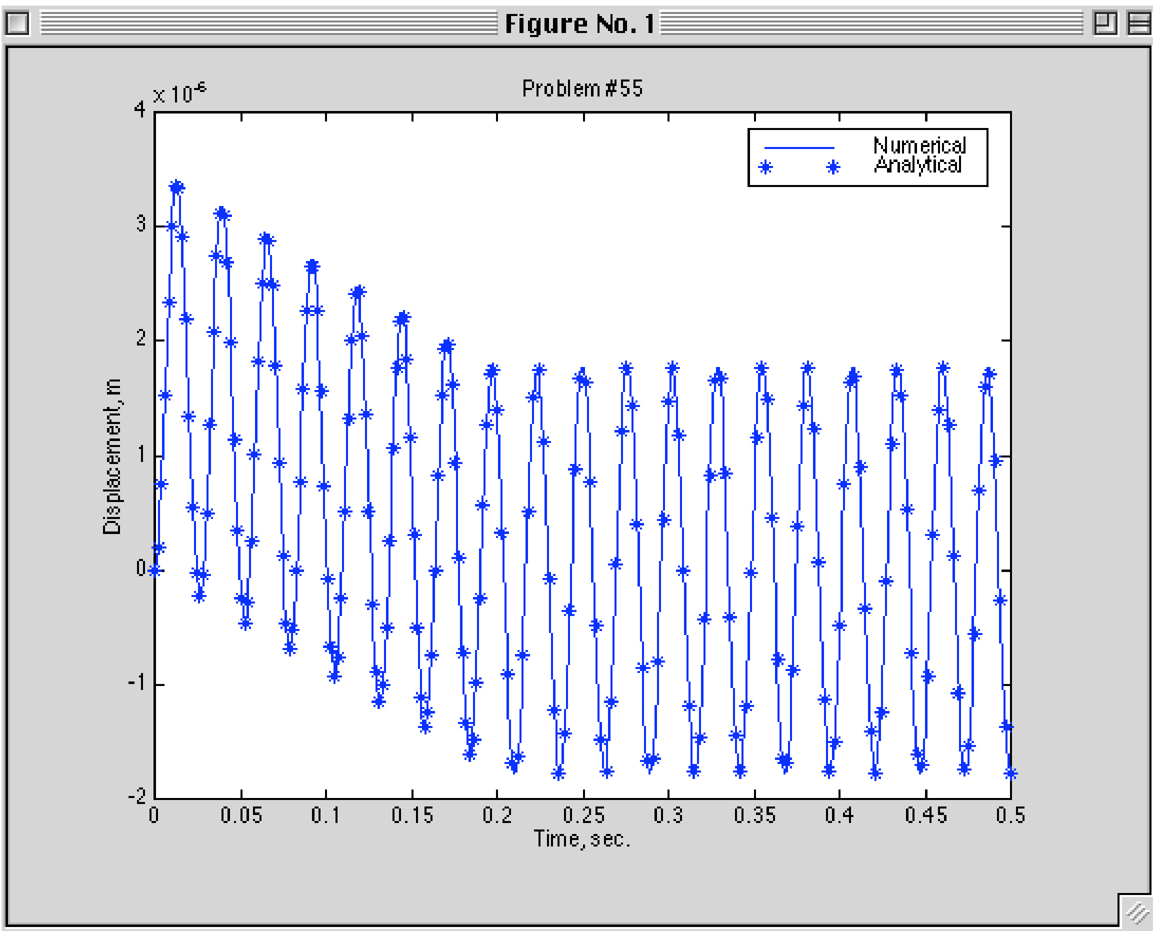


3.62*. Numerically simulate the response of the system of Problem 3.19 for a 2 meter concrete wall with cross section 0.03 m^2 and mass modeled as lumped at the end of 1000 kg. Use F_0 = 100 N, and plot the response for the case t_0 =0.25 s.

Solution The solution in Matlab is:

```
%Numerical Solutions
%Problem #3.62

clc
clear
close all

%Numerical Solution

x0=[0;0];
tspan=[0 0.5];

[t,x]=ode45('prob55a',tspan,x0);

figure(1)
plot(t,x(:,1));
title('Problem #55');
xlabel('Time, sec.');
ylabel('Displacement, m');
hold on

%Analytical Solution

m=1000;
E=3.8e9;
A=0.03;
l=2;
k=E*A/l;
F=100;
w=sqrt(k/m);
to=0.25;


%for t<to
t=linspace(0,to);
x=F/k*(1-cos(w*t))+ F/(to*k)*(1/w*sin(w*t)-t);
plot(t,x,'*');

%for t>=to
t=linspace(to,0.5);
x=-F/k*cos(w*t)- F/(w*k*to)*(sin(w*(t-to))-sin(w*t));
plot(t,x,'*');
legend('Numerical', 'Analytical')
%Clay
%Vibrations
%Solutions

%M-file for Prob #3.62
```

```
function dx=prob(t,x);
[rows, cols]=size(x);dx=zeros(rows, cols);

m=1000;
E=3.8e9;
A=0.03;
l=2;
k=E*A/l;
F=100;
w=sqrt(k/m);
to=0.25;


if t<to
   dx(1)=x(2);
   dx(2)= - k/m*x(1) + F/m*(1-t/to);
   else
      dx(1)=x(2);
   dx(2)= - k/m*x(1);
end
```



3.63*. Numerically simulate the response of the system of Problem 3.20 and plot the response.

Solution The solution in Matlab is:

```
%Clay
%Vibrations
%Numerical Solutions
%Problem #56

clc
clear
close all

%Numerical Solution

x0=[0;0];
tspan=[0 2];

[t,x]=ode45('prob56a',tspan,x0);

figure(1)
plot(t,x(:,1));
title('Problem #56');
xlabel('Time, sec.');
ylabel('Displacement, m');
hold on

%Analytical Solution

t=linspace(0,2);
x=0.5*t-0.05*sin(10*t);
plot(t,x,'*');
legend('Numerical', 'Analytical')


%Clay
%Vibrations
%Solutions

%M-file for Prob #56


function dx=prob(t,x);
[rows, cols]=size(x);dx=zeros(rows, cols);

m=1;
k=100;
F=50;


 dx(1)=x(2);
 dx(2)= - k/m*x(1) + F/m*(t);
```

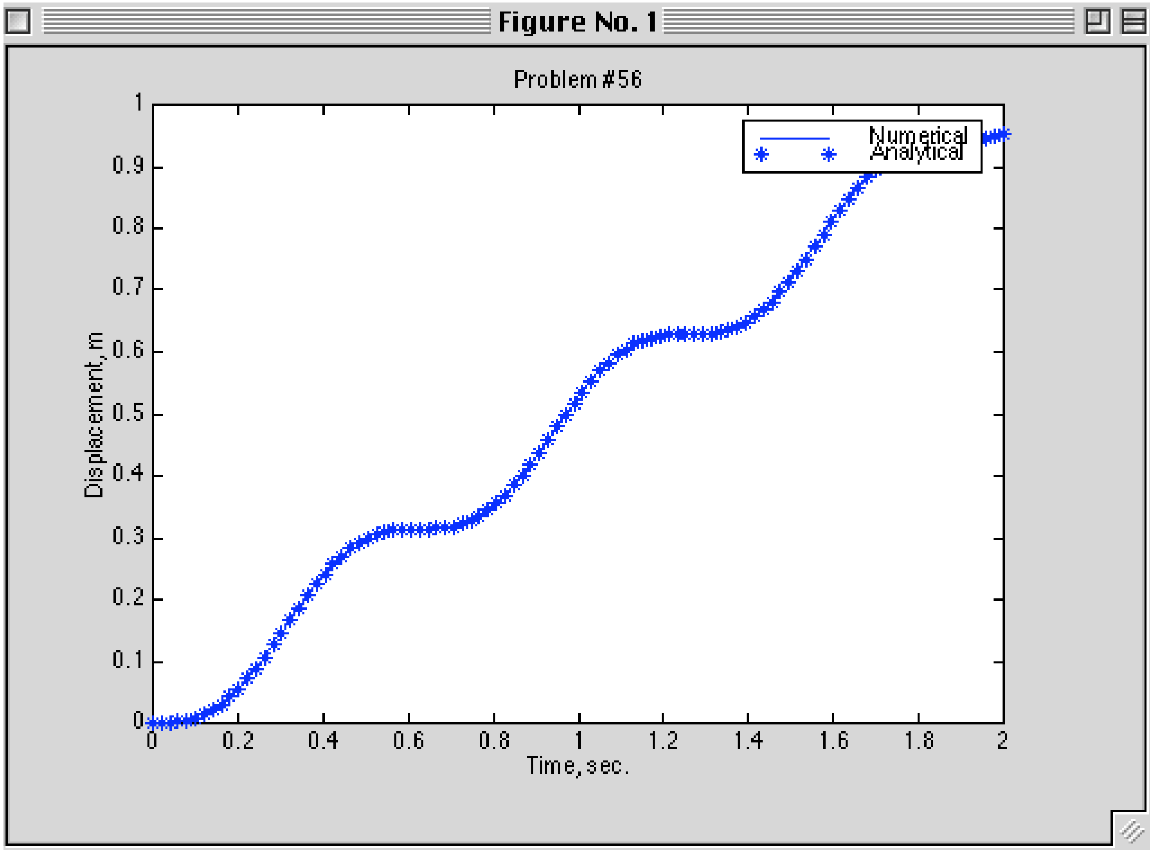


Figure No. 1
Problem #56
Numerical
Analytical
Displacement, m
Time, sec.
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

3.64*. Compute and plot the response of the system of following system using numerical integration:

$$10\ddot{x}(t) + 20\dot{x}(t) + 1500x(t) = 20\sin 25t + 10\sin 15t + 20\sin 2t$$

with initial conditions of x_0 = 0.01 m and v_0 = 1.0 m/s.

Solution The solution in Matlab is:

```
%Clay
%Vibrations
%Numerical Solutions
%Problem #57

clc
clear
close all

%Numerical Solution

x0=[0.01;1];
tspan=[0 5];

[t,x]=ode45('prob57a',tspan,x0);

figure(1)
plot(t,x(:,1));
title('Problem #57');
xlabel('Time, sec.');
ylabel('Displacement, m');
hold on

%Analytical Solution

m=10;
c=20;
k=1500;
w=sqrt(k/m);
d=c/(2*w*m);
wd=w*sqrt(1-d^2);


Y1=0.00419;
ph1=3.04;
wb1=25;

Y2=0.01238;
ph2=2.77;
wb2=15;

Y3=0.01369;
ph3=0.0268;
wb3=2;

A=0.1047;
phi=0.1465;
```

```
x=A.*exp(-d*w.*t).*sin(wd*t+phi)+ Y1.*sin(wb1*t-ph1) + Y2*sin(wb2*t-
ph2) + Y3*sin(wb3*t-ph3);

plot(t,x,'*')
legend('Numerical', 'Analytical')
%Clay
%Vibrations
%Solutions

%M-file for Prob #57


function dx=prob(t,x);
[rows, cols]=size(x);dx=zeros(rows, cols);

m=10;
c=20;
k=1500;


dx(1)=x(2);
dx(2)= -c/m*x(2) - k/m*x(1) + 20/m*sin(25*t) + 10/m*sin(15*t) +
20/m*sin(2*t) ;
```