Problems and Solutions Section 5.1 (5.1 through 5.5)

5.1 Using the nomograph of Figure 5.1, determine the frequency range of vibration for which a machine oscillation remains at a satisfactory level under rms acceleration of 1*g*.

Solution:

An rms acceleration of 1 *g* is about 9.81 m/s^2. From Figure 5.1, a satisfactory level would occur at frequencies above 650 Hz.

5.2 Using the nomograph of Figure 5.1, determine the frequency range of vibration for which a structure's rms acceleration will not cause wall damage if vibrating with an rms displacement of 1 mm or less.

Solution:

From Figure 5.1, an rms displacement of 1 mm (1000 μm) would not cause wall damage at frequencies below 3.2 Hz.

5.3 What natural frequency must a hand drill have if its vibration must be limited to a minimum rms displacement of 10 μm and rms acceleration of 0.1 m/s^2? What rms velocity will the drill have?

Solution:

From Figure 5.1, the natural frequency would be about 15.8 Hz or 99.6 rad/s. The rms velocity would be 1 mm/s.

5.4 A machine of mass 500 kg is mounted on a support of stiffness 197,392,000 N/m. Is the vibration of this machine acceptable (Figure 5.1) for an rms amplitude of 10 μm? If not, suggest a way to make it acceptable.

Solution:

The frequency is $\omega_n = \sqrt{\dfrac{k}{m}} =$ 628.3 rad/s = 100 Hz.

For an rms displacement of 10 μm the vibration is unsatisfactory. To make the vibration satisfactory, the frequency should be reduced to 31.6 Hz. This can be accomplished by reducing the stiffness and/or increasing the mass of the machine.

5.5 Using the expression for the amplitude of the displacement, velocity and acceleration of an undamped single-degree-of-freedom system, calculate the velocity and acceleration amplitude of a system with a maximum displacement of 10 cm and a natural frequency of 10 Hz. If this corresponds to the vibration of the wall of a building under a wind load, is it an acceptable level?

Solution:

The velocity amplitude is

$$\left|v(t)\right| = A\omega_n = \left(0.1\ \mathrm{m}\right)\left(\frac{10}{2\pi}\right) = 0.159\ \mathrm{m/s}$$

The acceleration amplitude is

$$\left|a\left(t\right)\right| = A\omega_n^2 = \left(0.1\ \mathrm{m}\right)\left(\frac{10}{2\pi}\right)^2 = 0.253\ \mathrm{m/s^2}$$

The rms displacement is $\dfrac{A}{\sqrt{2}} = \dfrac{0.1}{\sqrt{2}} = 0.0707\ \mathrm{m} = 70{,}700\ \mu\mathrm{m}$ (from equation (1.21)). At 10 Hz and $70{,}700\ \mu\mathrm{m}$, this could be destructive to a building.

Problems and Solutions Section 5.1 (5.6 through 5.26)

5.6 A 100-kg machine is supported on an isolator of stiffness 700×10^3 N/m. The machine causes a vertical disturbance force of 350 N at a revolution of 3000 rpm. The damping ratio of the isolator is $\zeta = 0.2$. Calculate (a) the amplitude of motion caused by the unbalanced force, (b) the transmissibility ratio, and (c) the magnitude of the force transmitted to ground through the isolator.

Solution:

(a) From Window 5.2, the amplitude at steady-state is

$$X = \frac{F_o / m}{\left[\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2\right]^{1/2}}$$

Since $\omega_n = \sqrt{\dfrac{k}{m}}$ = 83.67 rad/s and $\omega = 3000\left(\dfrac{2\pi}{60}\right)$ = 314.2 rad/s,

(b) From equation (5.7), the transmissibility ratio is

$$\frac{F_T}{F_0} = \sqrt{\frac{1+\left(2\zeta r\right)^2}{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$

Since $r = \dfrac{\omega}{\omega_n}$ = 3.755, this becomes

$$\frac{F_T}{F_0} = 0.1368$$

(c) The magnitude is

$$F_T = \left(\frac{F_T}{F_0}\right)F_0 = \left(0.1368\right)\left(350\right) = \left(47.9\right)$$

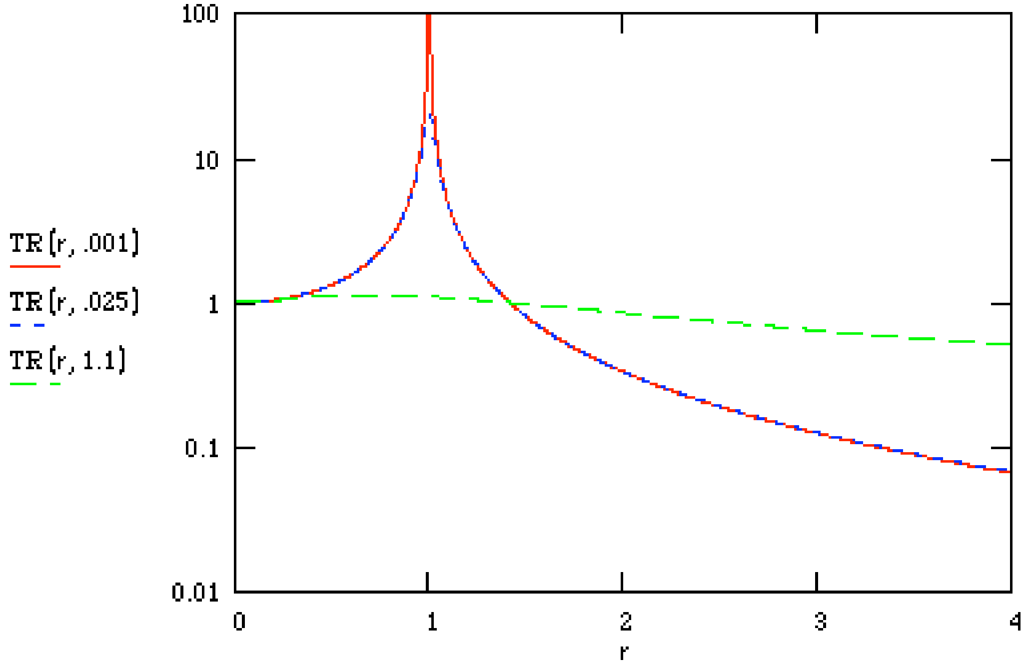
5.7 Plot the T.R. of Problem 5.6 for the cases ζ = 0.001, ζ = 0.025, and ζ = 1.1.

Solution:

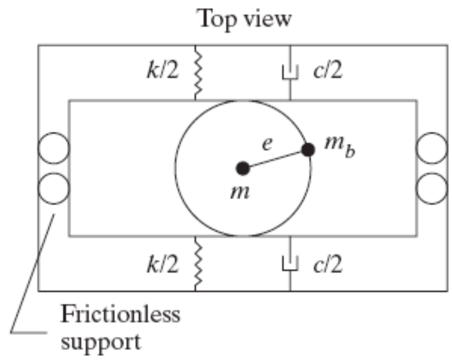
$$\text{T.R.} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

A plot of this is given for ζ = 0.001, ζ = 0.025, and ζ = 1.1. The plot is given here from Mathcad:

$$TR(r,\zeta) := \sqrt{\frac{1 + (2\cdot\zeta\cdot r)^2}{(1 - r^2)^2 + (2\cdot\zeta\cdot r)^2}}$$



5.8 A simplified model of a washing machine is illustrated in Figure P5.8. A bundle of wet clothes forms a mass of 10 kg (m_b) in the machine and causes a rotating unbalance. The rotating mass is 20 kg (including m_b) and the diameter of the washer basket ($2e$) is 50 cm. Assume that the spin cycle rotates at 300 rpm. Let k be 1000 N/m and ζ = 0.01. Calculate the force transmitted to the sides of the washing machine. Discuss the assumptions made in your analysis in view of what you might know about washing machines.



Solution: The transmitted force is given by $F_T = \sqrt{k^2 + c^2\omega_r^2}$ where

$$c = 2\zeta\omega_n,\quad \omega_n = \sqrt{\frac{k}{m}} = 7.071 \text{ rad/s},\ \omega_r = 300\frac{2\pi}{60} = 31.42 \text{ rad/s},$$

and X is given by equation (2.84) as

$$X = \frac{m_0 e}{m}\frac{r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$

Since $r = \dfrac{\omega_r}{\omega_n} = 4.443$, then X = 0.1317 m and

$$F_T = (0.1317)\sqrt{(1000)^2 + [2(0.01)(20)(7.071)]^2(31.42)^2} = 132.2 \text{ N}$$

Two important assumptions have been made:

i) The out-of-balance mass is concentrated at a point and

ii) The mass is constant and distributed evenly (keep in mind that water enters and leaves) so that the mass actually changes.

5.9 Referring to Problem 5.8, let the spring constant and damping rate become variable. The quantities m, m_b, e and ω are all fixed by the previous design of the washing machine. Design the isolation system (i.e., decide on which value of k and c to use) so that the force transmitted to the side of the washing machine (considered as ground) is less than 100N.

Solution:

The force produced by the unbalance is $F_r = m_b a$ where a is given by the magnitude of equation (2.81):

$$F_r = m_0\left|\ddot{x}_r\right| = em_0\omega_r^2 = (0.25)(10)\left[300\left(\frac{2\pi}{60}\right)\right]^2 = 2467.4 \text{ N}$$

Since $F_T < 100$ N,

$$\text{T.R.} = \frac{F_T}{F_r} = \frac{100}{2467.4} = 0.0405$$

If the damping ratio is kept at 0.01, this becomes

$$\text{T.R.} = 0.0405 = \sqrt{\frac{1+\left[2(0.01)r\right]^2}{\left(1-r^2\right)^2+\left[2(0.01)r\right]^2}}$$

Solving for r yields $r = 5.079$.

Since $r = \dfrac{\omega_r}{\sqrt{k/m}}$,

$$k = \frac{m\omega_r^2}{r^2} = \frac{(20)\left[300\left(\frac{2\pi}{60}\right)\right]^2}{5.079^2} = 765 \text{ N/m}$$

and

$$c = 2\zeta\sqrt{km} = 2(0.01)\sqrt{(765)(20)} = 2.47 \text{ kg/s}$$

5.10 A harmonic force of maximum value of 25 N and frequency of 180 cycles/min acts on a machine of 25 kg mass. Design a support system for the machine (i.e., choose *c*, *k*) so that only 10% of the force applied to the machine is transmitted to the base supporting the machine.

Solution: From equation (5.7),

$$\text{T.R.} = 0.1 = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}} \tag{1}$$

If we choose $\zeta = 0.1$, then solving the equation (1) numerically yields $r = 3.656$. Since $r = \dfrac{\omega}{\sqrt{k/m}}$ then:

$$k = \frac{m\omega^2}{r^2} = \frac{(25)\left[180\left(\frac{2\pi}{60}\right)\right]^2}{3.656^2} = 665 \text{ N/m}$$

and

$$c = 2\zeta\sqrt{km} = 2(0.1)\sqrt{(665)(25)} = 25.8 \text{ kg/s}$$

5.11 Consider a machine of mass 70 kg mounted to ground through an isolation system of total stiffness 30,000 N/m, with a measured damping ratio of 0.2. The machine produces a harmonic force of 450 N at 13 rad/s during steady-state operating conditions. Determine (a) the amplitude of motion of the machine, (b) the phase shift of the motion (with respect to a zero phase exciting force), (c) the transmissibility ratio, (d) the maximum dynamic force transmitted to the floor, and (e) the maximum velocity of the machine.

Solution:

(a) The amplitude of motion can be found from Window 5.2:

$$X = \frac{F_0 / m}{\left[\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2\right]^{1/2}}$$

where $\omega_n = \sqrt{\dfrac{k}{m}}$ = 20.7 rad/s. So,

$$X = 0.0229 \text{ m}$$

(b) The phase can also be found from Window 5.2:

$$\phi = \tan^{-1}\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2} = 22.5^\circ = 0.393 \text{ rad}$$

(c) From Eq. 5.7, with $r = \dfrac{\omega}{\omega_n}$ =0.628

$$\text{T.R.} = \sqrt{\frac{1 + \left(2\zeta r\right)^2}{\left(1 - r^2\right)^2 + \left(2\zeta r\right)^2}} = 1.57$$

(d) The magnitude of the force transmitted to the ground is

$$F_T = \left(\text{T.R.}\right)F_0 = \left(450\right)\left(1.57\right) = 707.6 \text{ N}$$

(e) The maximum velocity would be

$$\omega A_0 = \left(13\right)\left(0.0229\right) = 0.298 \text{ m/s}$$

5.12 A small compressor weighs about 70 lb and runs at 900 rpm. The compressor is mounted on four supports made of metal with negligible damping.
(a) Design the stiffness of these supports so that only 15% of the harmonic force produced by the compressor is transmitted to the foundation.
(b) Design a metal spring that provides the appropriate stiffness using Section 1.5 (refer to Table 1.2 for material properties).

Solution:

(a) From Figure 5.9, the lines of 85% reduction and 900 rpm meet at a static deflection of 0.35 in. The spring stiffness is then

$$k = \frac{mg}{\delta_s} = \frac{70 \text{ lb}}{0.35 \text{ in}} = 200 \text{ lb/in}$$

The stiffness of each support should be $k/4$ = 50 lb/in.

(b) Try a helical spring given by equation (1.67):

$$k = 50 \text{ lb/in} = 8756 \text{ N/m} = \frac{Gd^4}{64nR^3}$$

Using R = 0.1 m, n = 10, and $G = 8.0 \times 10^{10}$ N/m^2 (for steel) yields

$$d = \left[\frac{64(8756)(10)(0.1)^3}{8.0 \times 10^{10}}\right]^{1/4} = 0.0163 \text{ m} = 1.63 \text{ cm}$$

5.13 Typically, in designing an isolation system, one cannot choose any continuous value of *k* and c but rather, works from a parts catalog wherein manufacturers list isolators available and their properties (and costs, details of which are ignored here). Table 5.3 lists several made up examples of available parts. Using this table, design an isolator for a 500-kg compressor running in steady state at 1500 rev/min. Keep in mind that as a rule of thumb compressors usually require a frequency ratio of r =3.

Solution:

Since $r = \dfrac{\omega}{\sqrt{k/m}}$, then

$$k = \frac{m\omega^2}{r^2} = \frac{\left(500\left[1500\left(\dfrac{2\pi}{60}\right)\right]^2\right)}{3^2} = 1371\times 10^3 \text{ N/m}$$

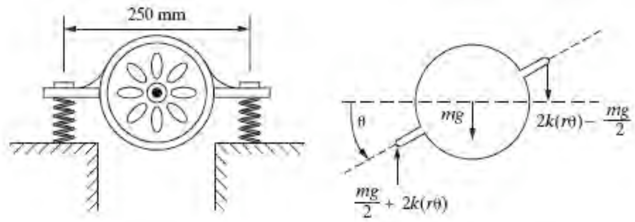
Choose isolator R-3 from Table 5.3. So, $k = 1000 \times 10^3$ N/m and $c = 1500$ N·s/m.

Check the value of r:

$$r = \frac{1500\left(\dfrac{2\pi}{60}\right)}{\sqrt{1000\times 10^3 / 500}} = 3.51$$

This is reasonably close to $r = 3$.

5.14 An electric motor of mass 10 kg is mounted on four identical springs as indicated in Figure P5.14. The motor operates at a steady-state speed of 1750 rpm. The radius of gyration (see Example 1.4.6 for a definition) is 100 mm. Assume that the springs are undamped and choose a design (i.e., pick k) such that the transmissibility ratio in the vertical direction is 0.0194. With this value of k, determine the transmissibility ratio for the torsional vibration (i.e., using θ rather than x as the displacement coordinates).



Solution:

TABLE 5.3 Catalog values of stiffness and damping properties of various off-the-shelf isolators

Part No.[a]	R-1	R-2	R-3	R-4	R-5	M-1	M-2	M-3	M-4	M-5
k(10^3N/m)	250	500	1000	1800	2500	75	150	250	500	750
c(N·s/m)	2000	1800	1500	1000	500	110	115	140	160	200

[a]The "R" in the part number designates that the isolator is made of rubber, and the "M" designates metal. In general, metal isolators are more expensive than rubber isolators.

With no damping, the transmissibility ratio is

$$\text{T.R.} = \frac{1}{r^2 - 1}$$

where

$$r = \frac{\omega}{\sqrt{4k/m}} = \frac{1750\left(\frac{2\pi}{60}\right)}{\sqrt{4k/10}} = \frac{579.5}{\sqrt{4k}}$$

$$0.0194 = \frac{1}{\frac{(579.5)^2}{4k} - 1}$$

$$4k = 6391 \text{ N/m}$$

For each spring, $k = 1598$ N/m.

For torsional vibration, the equation of motion is

$$I\ddot{\theta} = -\left[\frac{mg}{2} + 2kr\theta\right]r - \left[2kr\theta - \frac{mg}{2}\right]r$$

where $r = \dfrac{0.250\text{ m}}{2}$ = 0.125 m and from the definition of the radius of gyration and the center of percussion (see Example 1.4.6):

$$I = mk_0^2 = (10)(0.1)^2 = 0.1\,\text{kg}\cdot\text{m}^2$$

So,

$$0.1\ddot{\theta} + 4(1598)(0.125)^2\,\theta = 0$$

$$\ddot{\theta} + 998.6\theta = 0$$

The frequency ratio, r, is now

$$r = \frac{1750\left(\dfrac{2\pi}{60}\right)}{\sqrt{998.6}} = 5.80$$

$$\text{T.R.} = \frac{1}{r^2 - 1} = 0.0306$$

5.15 A large industrial exhaust fan is mounted on a steel frame in a factory. The plant manager has decided to mount a storage bin on the same platform. Adding mass to a system can change its dynamics substantially and the plant manager wants to know if this is a safe change to make. The original design of the fan support system is not available. Hence measurements of the floor amplitude (horizontal motion) are made at several different motor speeds in an attempt to measure the system dynamics. No resonance is observed in running the fan from zero to 500 rpm. Deflection measurements are made and it is found that the amplitude is 10 mm at 500 rpm and 4.5 mm at 400 rpm. The mass of the fan is 50 kg and the plant manager would like to store up to 50 kg on the same platform. The best operating speed for the exhaust fan is between 400 and 500 rpm depending on environmental conditions in the plant.

Solution:

A steel frame would be very lightly damped, so

$$\frac{X}{Y} = \frac{1}{1-r^2}$$

Since no resonance is observed between 0 and 500 rpm, $r < 1$.

When $\omega = 500\left(\frac{2\pi}{60}\right) = 52.36$ rad/s, X = 10 mm, so

$$10 = \frac{Y}{1-\left(\frac{52.36}{\omega_n}\right)^2}$$

Also, at $\omega = 400\left(\frac{2\pi}{60}\right) = 41.89$ rad/s, X = 4.5 mm, so

$$4.5 = \frac{Y}{1-\left(\frac{41.89}{\omega_n}\right)^2}$$

Solving for ω_n and Y yields

ω_n = 59.57 rad/s
Y = 2.275 mm
The stiffness is $k = m\omega_n^2 = (50)(59.57)^2$ = 177,453 N/m. If an additional 50 kg is added so that m = 100 kg, the natural frequency becomes

$$\omega_n = \sqrt{\frac{177{,}453}{100}} = 42.13 \text{ rad/s} = 402.3 \text{ rpm}$$

This would not be advisable because the normal operating range is 400 rpm to 500 rpm, and resonance would occur at 402.3 rpm.

5.16 A 350-kg rotating machine operates at 800 cycles/min. It is desired to reduce the transmissibility ratio by one-fourth of its current value by adding a rubber vibration isolation pad. How much static deflection must the pad be able to withstand?

Solution:

From equation (5.12), with $R = 0.25$:

$$r = \sqrt{\frac{2-0.25}{1-0.25}} = 1.528 = \frac{\omega}{\sqrt{k/m}} = \frac{800\left(\frac{2\pi}{60}\right)}{\sqrt{k/350}}$$

$$k = 1.053\times 10^{6} \text{ N/m}$$

The static deflection is

$$\delta_s = \frac{mg}{k} = \frac{(350)(9.81)}{1.053\times 10^{6}} = 3.26 \text{ mm}$$

5.17 A 68-kg electric motor is mounted on an isolator of mass 1200 kg. The natural frequency of the entire system is 160 cycles/min and has a measured damping ratio of $\zeta = 1$. Determine the amplitude of vibration and the force transmitted to the floor if the out-of-balance force produced by the motor is $F(t) = 100 \sin(31.4t)$ in newtons.

Solution:

The amplitude of vibration is given in Window 5.2 as

$$A_0 = \frac{F_0/m}{\left[\left(\omega_n^2-\omega^2\right)^2+\left(2\zeta\omega_n\omega\right)^2\right]^{1/2}}$$

where $F_0 = 100$ N, $m = 1268$ kg, $\omega = 31.4$ rad/s, and $\omega_n = 160\left(\frac{2\pi}{60}\right) = 16.76$ rad/s. So,

$$X = 6.226\times 10^{-5} \text{ m}$$

The transmitted force is given by Eq. (5.6), with $r = \frac{31.4}{16.76} = 1.874$

$$F_T = F_0\sqrt{\frac{1+\left(2\zeta r\right)^2}{\left(1-r^2\right)^2+\left(2\zeta r\right)^2}} = 85.97 \text{ N}$$

5.18 The force exerted by an eccentric (e = 0.22 mm) flywheel of 1000 kg, is 600 cos(52.4t) in newtons. Design a mounting to reduce the amplitude of the force exerted on the floor to 1% of the force generated. Use this choice of damping to ensure that the maximum force transmitted is never greater than twice the generated force.

Solution:

Two conditions are given. The first is that T.R. = 2 at resonance (r = 1), and the second is that T.R. = 0.01 at the driving frequency. Use the first condition to solve for ζ. From equation (5.7),

$$T.R. = 2 = \left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2}$$

$$\zeta = 0.2887$$

At the frequency, $r = \dfrac{52.4}{\sqrt{k/1000}}$, so

$$T.R. = 0.01 = \left[\frac{1+\left[2\left(0.2887\right)r\right]^2}{\left(1-r^2\right)^2+\left[2\left(0.2887\right)r\right]^2}\right]$$

$$r = 57.78 = \frac{52.4}{\sqrt{k/1000}}$$

$$k = 822.6 \text{ N/m}$$

Also,

$$c = 2\zeta\sqrt{km} = 523.6 \text{ kg/s}$$

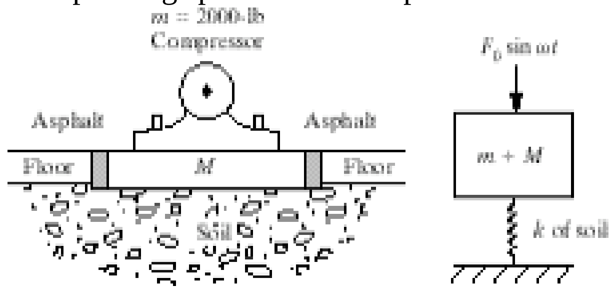
5.19 A rotating machine weighing 4000 lb has an operating speed of 2000 rpm. It is desired to reduce the amplitude of the transmitted force by 80% using isolation pads. Calculate the stiffness required of the isolation pads to accomplish this design goal.

Solution:

Using Figure 5.9, the lines of 2000 rpm and 80% reduction meet at $\delta_s = 0.053$ in. The spring stiffness should be

$$k = \frac{mg}{\delta_s} = \frac{4000 \text{ lb}}{0.053 \text{ in}} = 75,472 \text{ lb/in}$$

5.20 The mass of a system may be changed to improve the vibration isolation characteristics. Such isolation systems often occur when mounting heavy compressors on factory floors. This is illustrated in Figure P5.20. In this case the soil provides the stiffness of the isolation system (damping is neglected) and the design problem becomes that of choosing the value of the mass of the concrete block/compressor system. Assume that the stiffness of the soil is about $k = 2.0 \times 10^7$ N/m and design the size of the concrete block (i.e., choose m) such that the isolation system reduces the transmitted force by 75%. Assume that the density of concrete is $\rho = 23{,}000$ N/m^3. The surface area of the cement block is 4 m^2. The steady-state operating speed of the compressor is 1800 rpm.



Solution:

Using Figure 5.9, the lines of 75% reduction and 1800 rpm cross at $\delta_s = 0.053$ in = 0.1346 cm. Thus the weight of the block should be

$$W_T = (m + M)g = k\delta_s = 2.0\times10^7\left(0.1346\times10^{-2}\right) = 26{,}924 \text{ N}$$

The compressor weights mg = (2000 lb)(4.448222 N/lb) = 8896.4 N. The concrete block should weight $W = W_T - 8896.4 = 18{,}028$ N. The volume of the block needs to be

$$V = \frac{W}{\rho} = \frac{18{,}028}{23{,}000} = 0.7838 \text{ m}^2$$

Assume the surface area is part exposed to the surface. Let the top be a meters on each side (square) and b meters deep. The volume and surface area equations are

$$A = 4\text{m}^2 = a^2$$

$$V = 0.7838 \text{ m}^3 = a^2 b$$

Solving for a and b yields

$$a = 2 \text{ m}$$

$$b = 0.196 \text{ m}$$

5.21 The instrument board of an aircraft is mounted on an isolation pad to protect the panel from vibration of the aircraft frame. The dominant vibration in the aircraft is measured to be at 2000 rpm. Because of size limitation in the aircraft's cabin, the isolators are only allowed to deflect 1/8 in. Find the percent of motion transmitted to the instrument pane if it weights 50 lb.

Solution:

From equation (2.71), with negligible damping,

$$\frac{X}{Y} = \frac{1}{\sqrt{\left(1 - r^2\right)^2}}$$

This is the same as the equation that yields Figure 5.9. The lines of 2000 rpm and δ_s = 0.125 in meet at 93%. So only 7% of the plane's motion is transmitted to the instrument panel.

5.22 Design a base isolation system for an electronic module of mass 5 kg so that only 10% of the displacement of the base is transmitted into displacement of the module at 50 Hz. What will the transmissibility be if the frequency of the base motion changes to 100 Hz? What if it reduces to 25 Hz?

Solution: Using Figure 5.9, the lines of 90% reduction and ω = (50 Hz)(60) = 3000 rpm meet at δ_s = 0.042 in = 0.1067 cm. The spring stiffness is then

$$k = \frac{mg}{\delta_s} = \frac{(5)(9.81)}{0.001067} = 45{,}979 \text{ N/m}$$

The natural frequency is $\omega = \sqrt{k/m}$ = 95.89 rad/s.

At ω = 100 Hz, $r = \dfrac{100(2\pi)}{95.89}$ = 6.552, so the transmissibility ratio is

$$T.R. = \frac{1}{r^2 - 1} = 0.0238$$

At ω = 25 Hz, $r = \dfrac{100(2\pi)}{95.89}$ = 1.638, so the transmissibility ratio is

$$T.R. = \frac{1}{r^2 - 1} = 0.594$$

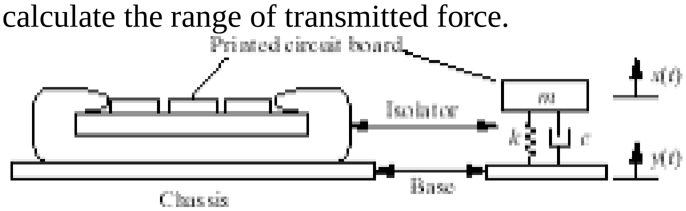
5.23 Redesign the system of Problem 5.22 such that the smallest transmissibility ratio possible is obtained over the range 50 to 75 Hz.

Solution:

If the deflection is limited, say 0.1 in, then the smallest transmissibility ratio in the frequency range of 50 to 75 Hz (3000 to 4500 rpm) would be 0.04 (96% reduction). The stiffness would be

$$k = \frac{mg}{\delta_s} = \frac{(5)(9.81)}{(0.1)(2.54)(0.01)} = 19{,}311 \text{ N/m}$$

5.24 A 2-kg printed circuit board for a computer is to be isolated from external vibration of frequency 3 rad/s at a maximum amplitude of 1 mm, as illustrated in Figure P5.24. Design an undamped isolator such that the transmitted displacement is 10% of the base motion. Also calculate the range of transmitted force.



Solution:

Using Figure 5.9, the lines of 90% reduction and ω = 3(2π)(60)=1131 rpm meet at δ_s = 0.3 in = 0.762 cm. The stiffness is

$$k = \frac{mg}{\delta_s} = \frac{(2)(9.81)}{0.00762} = 2574.8 \text{ N/m}$$

From Window 5.1, the transmitted force would be

$$F_T = kYr^2\left(\frac{1}{1-r^2}\right)$$

Since Y = 0.001 m and $r = \dfrac{3}{\sqrt{2574.8/2}} = 0.08361$

$$F_T = 0.0181 \text{ N}$$

5.25 Change the design of the isolator of Problem 5.24 by using a damping material with damping value ζ chosen such that the maximum T.R. at resonance is 2.

Solution:

At resonance, r = 1 and T.R. = 2, so

$$2 = \left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2}$$

Solving for ζ yields ζ = 0.2887. Also T.R. = 0.01 at ω = 3 rad/s, so

$$0.01 = \left[\frac{1+0.3333r^2}{\left(1-r^2\right)^2+0.3333r^2}\right]$$
$$r = 6.134$$

Solving for k,

$$k = \frac{m\omega^2}{r^2} = \frac{(2)(3)^2}{6.134^2} = 0478 \text{ N/m}$$

The damping constant is

$$c = 2\zeta\sqrt{km} = 0.565 \text{ kg/s}$$

5.26 Calculate the damping ratio required to limit the displacement transmissibility to 4 at resonance for any damped isolation system.

Solution:

At resonance r = 1, so

$$T.R. = 4 = \left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2}$$
$$\zeta = 0.129$$