**Problems and Solutions Section 5.3 (5.27 through 5.36)**

**5.27** A motor is mounted on a platform that is observed to vibrate excessively at an operating speed of 6000 rpm producing a 250-N force. Design a vibration absorber (undamped) to add to the platform. Note that in this case the absorber mass will only be allowed to move 2 mm because of geometric and size constraints.

**Solution:**

The amplitude of the absorber mass can be found from equation (5.22) and used to solve for $k_a$:

$$X_a = 0.002 \text{ m} = \frac{F_0}{k_a} = \frac{250}{k_a}$$

$$k_a = 125{,}000 \text{ N/m}$$

From equation (5.21),

$$\omega^2 = \frac{k_a}{m_a}$$

$$m_a = \frac{k_a}{\omega^2} = \frac{125{,}000}{\left[6000\left(\frac{2\pi}{60}\right)\right]^2} = 0.317 \text{ kg}$$

**5.28** Consider an undamped vibration absorber with β = 1 and μ = 0.2. Determine the operating range of frequencies for which $\left|Xk / F_0\right| \leq 0.5$.

**Solution:**

From equation (5.24), with β = $\frac{\omega_a}{\omega_p} = 1$(i.e., $\omega_a = \omega_p$) and $\mu = 0.2$,

$$\frac{Xk}{F_0} = \frac{1-\left(\frac{\omega}{\omega_a}\right)}{\left[1+0.2(1)^2-\left(\frac{\omega}{\omega_a}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]-0.2(1)^2}$$

$$= \frac{1-\left(\frac{\omega}{\omega_a}\right)^2}{\left(\frac{\omega}{\omega_a}\right)^4 - 2.2\left(\frac{\omega}{\omega_a}\right)^2 + 1}$$

For $\frac{Xk}{F_0}$ = 0.5, this yields

$$0.5\left(\frac{\omega}{\omega_a}\right)^4 - 0.1\left(\frac{\omega}{\omega_a}\right)^2 - 0.5 = 0$$

Solving for the physical solution gives

$$\left(\frac{\omega}{\omega_a}\right) = 1.051$$

Solving for $\left(\frac{\omega}{\omega_a}\right)$ gives

$$\left(\frac{\omega}{\omega_a}\right) = 0.955,\ 1.813$$

Comparing this to the sketch in Figure 5.15, the values for which $\left|\frac{Xk}{F_0}\right| \leq 5$ are

$$0.955\omega_a \leq \omega \leq 1.051\omega_a \quad \text{and} \quad \omega \geq 1.813\omega_a$$

**5.29** Consider an internal combustion engine that is modeled as a lumped inertia attached to ground through a spring. Assuming that the system has a measured resonance of 100 rad/s, design an absorber so that the amplitude is 0.01 m for a (measured) force input of $10^2$ N.

**Solution:**

The amplitude of the absorber mass can be found from equation (5.22) and used to solve for $k_a$:

$$X_a = 0.01\text{m} = \frac{F_0}{k_a} = \frac{100}{k_a}$$

$$k_a = 10{,}000 \text{ N/m}$$

Choose $\omega = 2\omega_n = 200$ rad/s. From equation (5.21),

$$m_a = \frac{k_a}{\omega^2} = \frac{10{,}000}{200^2} = 0.25 \text{ kg}$$

**5.30** A small rotating machine weighing 50 lb runs at a constant speed of 6000 rpm. The machine was installed in a building and it was discovered that the system was operating at resonance. Design a retrofit undamped absorber such that the nearest resonance is at least 20% away from the driving frequency.

**Solution:**

By observing Figure 5.15, the values of $\mu = 0.25$ and $\beta = 1$ result in the combined system's natural frequencies being 28.1% above the driving frequency and 21.8% below the driving frequency (since $\beta = \frac{\omega_a}{\omega_p} = 1$ and $\omega = \omega_p$). So the absorber should weigh

$$m_a = \mu m = (0.25)(50 \text{ lb}) = 12.5 \text{ lb}$$

and have stiffness

$$k_a = m_a\omega_a^2 = m_a\omega^2 = (12.5 \text{ lb})(4.448222 \text{ N/lb})\left(\frac{1}{9.81}\right)(6000)^2\left(\frac{2\pi}{60}\right)^2$$

$$k_a = 2.24\times 10^6 \text{ N/m} = 12{,}800 \text{ lb/in}$$

**5.31** A 3000-kg machine tool exhibits a large resonance at 120 Hz. The plant manager attaches an absorber to the machine of 600 kg tuned to 120 Hz. Calculate the range of frequencies at which the amplitude of the machine vibration is less with the absorber fitted than without the absorber.

**Solution:**

For $\dfrac{Xk}{F_0}$ = 1, equation (5.24) yields

$$\left[1+\mu\left(\frac{\omega_a}{\omega_p}\right)^2-\left(\frac{\omega}{\omega_a}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]-\mu\left(\frac{\omega_a}{\omega_p}\right)^2=1-\left(\frac{\omega}{\omega_a}\right)^2$$

Since $\mu=\dfrac{m_a}{m}=\dfrac{600}{3000}=0.2,$ this becomes $\left(\dfrac{\omega}{\omega_a}\right)=0,\ 1.0954.$

For $\dfrac{Xk}{F_0}$ = -1, equation (5.24) yields

$$\left[1+\mu\left(\frac{\omega_a}{\omega_p}\right)^2-\left(\frac{\omega}{\omega_p}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]-\mu\left(\frac{\omega_a}{\omega_p}\right)^2=\left(\frac{\omega}{\omega_a}\right)^2-1$$

$$\left(\frac{\omega_a}{\omega_p}\right)^2\left(\frac{\omega}{\omega_a}\right)^4-\left[2+(\mu+1)\left(\frac{\omega_a}{\omega_p}\right)^2\right]\left(\frac{\omega}{\omega_a}\right)^2+2=0$$

Since $\omega_a=\omega_p$,

$$\left(\frac{\omega}{\omega_a}\right)^4-3.2\left(\frac{\omega}{\omega_a}\right)^2+2=0$$

$$\left(\frac{\omega}{\omega_a}\right)=0.9229,1.5324$$

The range of frequencies at which $\left|\dfrac{Xk}{F_0}\right|$ > 1 is

$$0<\omega<0.9229\omega_a \text{ and } 1.0954\omega_a<\omega<1.5324\omega_a$$

Since $\omega_a = \omega_p$,

$$0 < \omega < 695.8 \text{ rad/s and } 825.9 < \omega < 1155.4 \text{ rad/s}$$

**5.32** A motor-generator set is designed with steady-state operating speed between 2000 and 4000 rpm. Unfortunately, due to an imbalance in the machine, a large violent vibration occurs at around 3000 rpm. An initial absorber design is implemented with a mass of 2 kg tuned to 3000 rpm. This, however, causes the combined system natural frequencies that occur at 2500 and 3000 rpm. Redesign the absorber so that $\omega_1 < 2000$ rpm and $\omega_2 > 4000$ rpm, rendering the system safe for operation.

**Solution:** The mass of the primary system can be computed from equation (5.25). Since $\beta = \dfrac{\omega_a}{\omega_p} = 1$ and $\left(\dfrac{\omega_1}{\omega_a}\right)^2 = \left(\dfrac{2500}{3000}\right)^2 = 0.6944$, then

$$(1)^2(0.6944)^2 - \left[1 + (1)^2(1+\mu)\right](0.6944) + 1 = 0$$
$$\mu = 0.1344$$
$$m = \frac{m_a}{\mu} = \frac{2}{0.1344} = 14.876 \text{ kg}$$

By increasing μ to 0.55 and decreasing β to 0.89, the design goal can be achieved. The mass and stiffness of the absorber should be

$$m_a = \mu m = (0.55)(14.876) = 8.18 \text{ kg}$$
$$k_a = m_a\omega_a^2 = m_a\beta^2\omega_p^2 = (8.18)(0.89)^2\left[3000\left(\frac{2\pi}{60}\right)\right]^2 = 639{,}600 \text{ N/m}$$

**5.33** A rotating machine is mounted on the floor of a building. Together, the mass of the machines and the floor is 2000 lb. The machine operates in steady state at 600 rpm and causes the floor of the building to shake. The floor-machine system can be modeled as a spring-mass system similar to the optical table of Figure 5.14. Design an undamped absorber system to correct this problem. Make sure you consider the bandwidth.

**Solution:** To minimize the transmitted force, let $\omega_a = \omega = 600$ rpm. Also, since the floor shakes at 600 rpm, it is assumed that $\omega_p = 600$ rpm so that $\beta = 1$. Using equation (5.26) with $\mu = 0.1$ yields

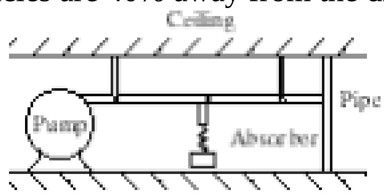
$$\frac{\omega_n}{\omega_a} = 0.8543,\ 1.1705$$

So the natural frequencies of the combined system are $\omega_1 = 512.6$ rpm and $\omega_2 = 702.3$ rpm. These are sufficiently enough away from 600 rpm to avoid problems. Therefore the mass and stiffness of the absorber are

$$m_a = \mu m = (0.1)(2000 \text{ lbm}) = 200 \text{ lbm}$$

$$k_a = m_a \omega_a^2 = (200 \text{ lbm})\left(\frac{\text{slug}}{32.1174 \text{ lbm}}\right)\left[600\left(\frac{2\pi}{60}\right)\right]^2 = 25{,}541 \text{ lb/ft}$$

**5.34** A pipe carrying steam through a section of a factory vibrates violently when the driving pump hits a speed of 300 rpm (see Figure P5.34). In an attempt to design an absorber, a trial 9-kg absorber tuned to 300 rpm was attached. By changing the pump speed it was found that the pipe-absorber system has a resonance at 207 rpm. Redesign the absorber so that the natural frequencies are 40% away from the driving frequency.




**Solution:**

The driving frequency is 300 rpm. 40% above and below this frequency is 180 rpm and 420 rpm. This is the design goal.

The mass of the primary system can be computed from equation (5.25). Since $\beta = \frac{\omega_a}{\omega_p} = 1$ and $\left(\frac{\omega_1}{\omega_a}\right)^2 = \left(\frac{207}{300}\right)^2 = 0.4761,$ then

$$(1)^2(0.4761)^2 - \left[1 + (1)^2(1+\mu)\right](0.4761) + 1 = 0$$

$$\mu = 0.5765$$

$$m = \frac{m_a}{\mu} = \frac{9}{0.5765} = 15.611 \text{ kg}$$

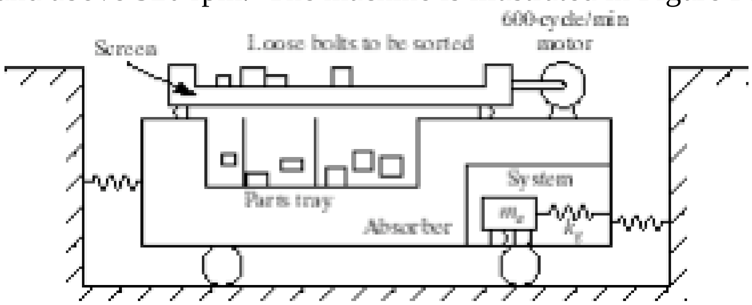
By increasing μ to 0.9 and decreasing β to 0.85, the design goal can be achieved. The mass and stiffness of the absorber should be

$$m_a = \mu m = (0.9)(15.611) = 14.05 \text{ kg}$$

$$\mathrm{k_a} = m_a\omega_a^2 = m_a\beta^2\omega_p^2 = (14.05)(0.85)^2\left[300\left(\frac{2\pi}{60}\right)\right]^2 = 10{,}020 \text{ N/m}$$

Note that μ is very large, which means a poor design.

**5.35** A machine sorts bolts according to their size by moving a screen back and forth using a primary system of 2500 kg with a natural frequency of 400 cycle/min. Design a vibration absorber so that the machine-absorber system has natural frequencies below 160 cycles/min and above 320 rpm. The machine is illustrated in Figure P5.35.




**Solution:**

Using Equation (5.26), and choose (by trial and error) β = 0.4 and μ = 0.01, the design goal of $\omega_1 < 160$ rpm and $\omega_2 > 320$ rpm can be achieved. The actual values are $\omega_1 = 159.8$ rpm and $\omega_2 = 400.4$ rpm. The mass and stiffness of the absorber should be

$$m_a = \mu m = (0.01)(2500) = 25 \text{ kg}$$

$$k_a = m_a \omega_a^2 = m_a \beta^2 \omega_p^2 = (25)(0.2)^2 \left[400\left(\frac{2\pi}{60}\right)\right]^2 = 1754.6 \text{ N/m}$$

**5.36** A dynamic absorber is designed with μ = 1/4 and $\omega_a = \omega_p$. Calculate the frequency range for which the ratio $\left|Xk / F_0\right| < 1$.

**Solution:**

From Equation (5.24), with $\beta = \dfrac{\omega_a}{\omega_p} = 1$ and $\mu = 0.25$,

$$\frac{Xk}{F_0} = \frac{1 - \left(\frac{\omega}{\omega_a}\right)^2}{\left[1 + 0.25\left(1^2\right) - \left(\frac{\omega}{\omega_a}\right)^2\right]\left[1 - \left(\frac{\omega}{\omega_a}\right)^2\right] - 0.25\left(1\right)^2}$$

$$= \frac{1 - \left(\frac{\omega}{\omega_a}\right)^2}{\left(\frac{\omega}{\omega_a}\right)^4 - 2.25\left(\frac{\omega}{\omega_a}\right)^2 + 1}$$

For $\dfrac{Xk}{F_0} = 1$, this yields

$$\left(\frac{\omega}{\omega_a}\right)^4 - 1.25\left(\frac{\omega}{\omega_a}\right)^2 = 0$$

$$\left(\frac{\omega}{\omega_a}\right) = 0,\ 1.118$$

For $\dfrac{Xk}{F_0}$ = 1, this yields

$$-\left(\frac{\omega}{\omega_a}\right)^4 + 3.25\left(\frac{\omega}{\omega_a}\right)^2 - 2 = 0$$

$$\left(\frac{\omega}{\omega_a} = 0.9081,\ 1.557\right)$$

Comparing this to the sketch in Figure 5.15, the values for which $\left|\dfrac{Xk}{F_0}\right| < 1$ are

$$0.9081\omega_a < \omega < 1.118\omega_a \text{ and } \omega > 1.557\omega_a$$