Problems and Solutions Section 5.6 (5.67 through 5.73)

5.67 Compare the resonant amplitude at steady state (assume a driving frequency of 100 Hz) of a piece of nitrite rubber at 50°F versus the value at 75°F. Use the values for η from Table 5.2.

Solution:

From equation (5.63),

$$X = \frac{F_0}{k(1+\eta j) - m\omega^2}$$

At resonance $\omega = \sqrt{\frac{k}{m}}$ so

$$X = \frac{F_0}{k(1-\eta j)-1} = \frac{F_0}{k\eta j}$$

The magnitude is

$$|X| = \frac{1}{\eta}\left(\frac{F_0}{k}\right)$$

At 50°, η = 0.5 and at 75°, η = 0.28, so

$$|X|_{50^\circ} = \frac{2F_0}{k}$$

$$|X|_{75^\circ} = \frac{3.57F_0}{k}$$

5.68 Using Equation (5.67), calculate the new modulus of a 0.05 × 0.01 × 1, piece of pinned-pinned aluminum covered with a 1-cm-thick piece of nitrite rubber at 75°F driven at 100 Hz.

Solution:

From Table 1.2, $E_1 = 7.1 \times 10^{10}$ N/m^2 for aluminum. From Table 5.2, $E_2 = 2.758 \times 10^7$ N/m^2 for nitrate rubber, Also,

$$I = I_1 = \frac{1}{3}(0.05)(1)^3 = 0.01667 \text{ m}^4$$

$$e_2 = \frac{E_2}{E_1} = \frac{2.758 \times 10^7}{7.1 \times 10^{10}} = 3.885 \times 10^{-4}$$

$$h_2 = \frac{H_2}{H_1} = \frac{0.01}{0.01} = 1$$

From Equation (5.67),

$$E = \frac{E_1 I_1}{I}\left[1 + e_2 h_2^2 + 3(1 + h_2)^2 \frac{e_2 h_2}{1 + e_2 j_2}\right] = 7.136 \times 10^{10} \text{ N/m}^2$$

5.69 Calculate Problem 5.68 again at 50°F. What percent effect does this change in temperature have on the modulus of the layered material?

Solution:

From Problem 5.68, with $E_2 = 4.137 \times 10^7$ N/m^2,

$$I = I_1 = 0.01667 \text{ m}^4$$
$$e_2 = \frac{E_2}{E_1} = \frac{4.137 \times 10^7}{7.1 \times 10^{10}} = 5.827 \times 10^{-4}$$
$$h_2 = \frac{H_2}{H_1} = \frac{0.01}{0.01} = 1$$

From Equation (5.67),

$$E = \frac{E_1 I_1}{I}\left[1 + e_2 h_2^2 + 3\left(1 + h_2\right)^2 \frac{e_2 h_2}{1 + e_2 h_2}\right] = 7.154 \times 10^{10}\ \text{N/m}^2$$

This is an increase of 0.25% of the layered material's modulus.

5.70 Repeat the design of Example 5.6.1 by
(a) changing the operating frequency to 1000 Hz, and
(b) changing the operating temperature to 50°F.
Discuss which of these designs yields the most favorable system.

Solution:

From Ex. 5.6.1, $E_1 = 7.1\times 10^{10}$ N/m^2 and $h_2 = 1$.

(a) 75°, 1000 Hz

$$\eta_2 = 0.55$$
$$E_2 = 4.826\times 10^7 \text{ N/m}^2$$
$$e_2 = \frac{E_2}{E_1} = 6.797\times 10^{-4}$$

From Equation (5.68),

$$\eta = \frac{e_2 h_2\left(3 + 6h_2 + 4h_2^2 + 2e_2 h_2^2 + e_2^2 h_2^4\right)}{\left(1 + e_2 h_2\right)\left(1 + 4e_2 h_2 + 6e_2 h_2^2 + 4e_2 h_2^3 + e_2^2 h_2^4\right)}\eta_2 = 0.00481$$

(b) 50°, 1000 Hz

$$\eta_2 = 0.5$$
$$E_2 = 4.137\times 10^7 \text{ N/m}^2$$
$$e_2 = \frac{E_2}{E_1} = \frac{4.137\times 10^7}{7.1\times 10^{10}} = 5.827\times 10^{-4}$$

From Equation (5.68),

$$\eta = 0.00375$$

Increasing the driving frequency results in a higher loss factor compared to the effects of lowering the temperature.

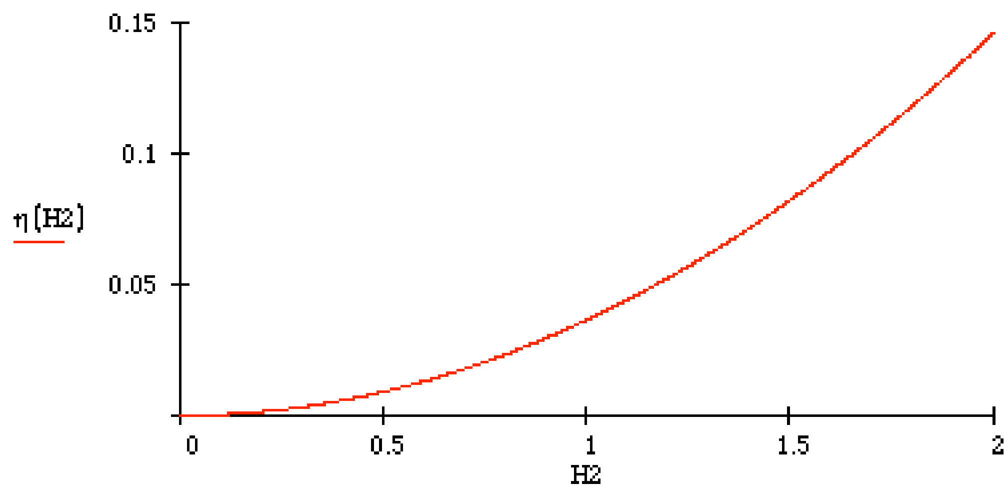
5.71 Reconsider Example 5.6.2. Make a plot of thickness of the damping treatment versus loss factor.

Solution:

From Ex. 5.6.2, $\eta_2 = 0.261$, $e_2 = 0.01$, and $H_1 = 1$ cm. So, from Equation (5.69),

$$\eta = 14 e_2 \frac{H_2^2}{H_1^2} \eta_2 = 0.03654 H_2^2 \qquad \left(H_2 \text{ in cm}\right)$$

$$\eta(H2) := 0.03654 \cdot H2^2$$



A plot of η versus H_2 in centimeters

5.72 Calculate the maximum transmissibility coefficient of the center of the shelf of Example 5.6.1. Make a plot of the maximum transmissibility ratio for this system frequency, using Table 5.2 for each temperature.

Solution: If the system is modeled as shown in Figure 5.18, then the maximum transmissibility occurs at (from Equation (5.50)),

$$\left(\frac{Xk}{F_0}\right)_{\max} = 1 + \frac{2}{\mu}$$

where μ is found from Equation (5.49) as the solution to

$$\zeta = \frac{1}{\sqrt{2(\mu+1)(\mu+2)}}$$

The value of ζ is $\frac{\eta}{2}$ at resonance. So, at 75° and 100 Hz,

$$\zeta = \frac{\eta}{2} = \frac{0.00151}{2} = 0.000755 = \frac{1}{\sqrt{2(\mu+1)(\mu+2)}}$$

$$\Rightarrow \mu = 935$$

$$\frac{Xk}{F_0} = 1 + \frac{2}{935} = 1.002$$

For 50° and 100 Hz, η = 0.00375 (from Problem 5.70), so

$$\zeta = \frac{\eta}{2} = \frac{0.00375}{2} = 0.001875 = \frac{1}{\sqrt{2(\mu+1)(\mu+2)}}$$

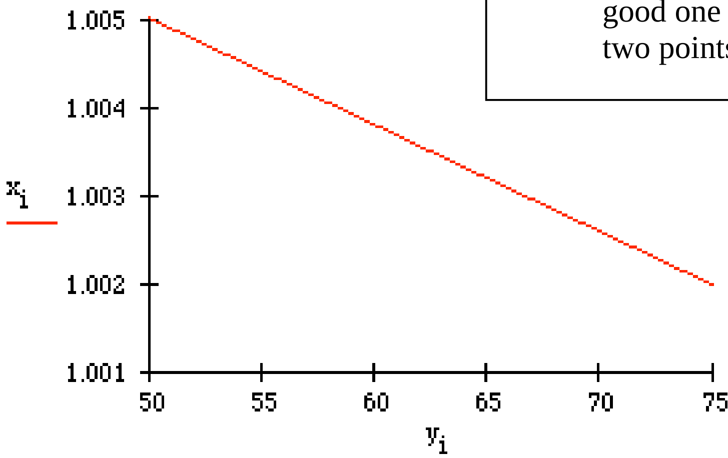
$$\mu = 375.6$$

$$\frac{Xk}{F_0} = 1 + \frac{2}{375.6} = 1.005$$

$i := 1 .. 2$ $x_1 := 1.002$ $x_2 := 1.005$

$y_1 := 75$ $y_2 := 50$

This gives some idea of the relationship, but not a very good one as it includes only two points



5.73 The damping ratio associated with steel is about $\zeta = 0.001$. Does it make any difference whether the shelf in Example 5.6.1 is made out of aluminum or steel? What percent improvement in damping ratio at resonance does the rubber layer provide the steel shelf?

Solution:

If the shelf in Ex. 5.6.1 is made out of steel, $E_1 = 2.0 \times 10^{11}$ N/m^2. Therefore,

$$e_2 = \frac{E_2}{E_1} = \frac{2.758 \times 10^7}{2.0 \times 10^{11}} = 0.0001379$$

Also, $\eta_2 = 0.55$ and $h_2 = 1$. From Equation (5.68),

$$\eta = \frac{e_2 h_2 \left(3 + 6h_2 + 4h_2^2 + 2e_2 h_2^2 + e_2^2 H_2^4\right)}{\left(1 + e_2 h_2\right)\left(1 + 4e_2 h_2 + 6e_2 h_2^3 + 4e_2 h_2^3 + e_2 h_2^4\right)} \eta_2 = 0.0005$$

At resonance, $\zeta = \dfrac{\eta}{2} = 0.00025.$ The rubber actually reduced the damping of the steel shelf by 75%.