Problems and Solution Section 5.7 (5.74 through 5.80)

5.74 A 100-kg compressor rotor has a shaft stiffness of 1.4×10^7 N/m. The compressor is designed to operate at a speed of 6000 rpm. The internal damping of the rotor shaft system is measured to be $\zeta = 0.01$.

(a) If the rotor has an eccentric radius of 1 cm, what is the rotor system's critical speed?

(b) Calculate the whirl amplitude at critical speed. Compare your results to those of Example 5.7.1.

Solution:

(a) The critical speed is the rotor's natural frequency, so

$$\omega_c = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.4 \times 10^7}{100}} = 374.2 \text{ rad/s} = 3573 \text{ rpm}$$

(b) At critical speed, $r = 1$, so from Equation (5.81),

$$X = \frac{\alpha}{2\zeta} = \frac{0.01}{2(0.01)} = 0.5 \text{ m}$$

So a system with higher eccentricity and lower damping has a greater whirl amplitude (see Example 5.7.1).

5.75 Redesign the rotor system of Problem 5.74 such that the whirl amplitude at critical speed is less than 1 cm by changing the mass of the rotor.

Solution: From Problem 5.74, $k = 1.4 \times 10^7$ N/m, $m = 100$ kg, $\zeta = 0.01$, and $\alpha = 0.01$m. Since the whirl amplitude at critical speed must be less than 0.01 m, the value of ζ that would satisfy this is, from equation (5.81),

$$X = \frac{\alpha}{2\zeta}$$

$$\zeta = \frac{\alpha}{2X} = \frac{0.01}{2(0.01)} = 0.5$$

The original damping ratio was 0.01, so the value of c is

$$c = 2\zeta m\omega = 2(0.01)(100)\sqrt{\frac{1.4 \times 10^7}{100}} = 784.33 \text{ kg/s}$$

So, the new mass should be, with $\zeta = 0.5$,

$$748.33 = 2(0.5)m\sqrt{\frac{k}{m}} = \sqrt{km} = \sqrt{1.4\times10^{7}\,m}$$

$$\Rightarrow m = 0.04\text{ kg}$$

This is not practical.

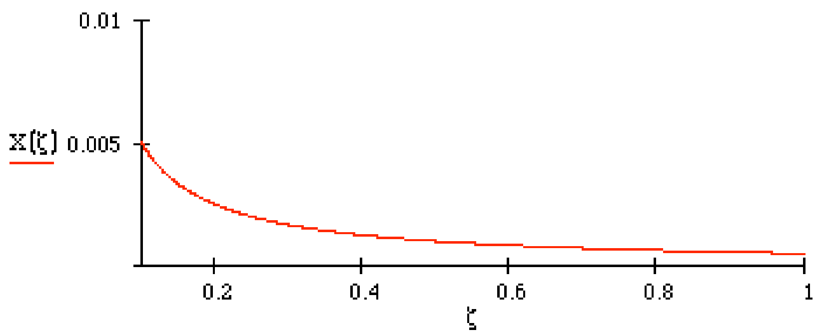
5.76 Determine the effect of the rotor system's damping ratio on the design of the whirl amplitude at critical speed for the system of Example 5.7.1 by plotting X at $r = 1$ for ζ between $0 < \zeta < 1$.

Solution:

From Example 5.7.1, with $r = 1$ and $\alpha = 0.001$ m,

$$X = \frac{0.001}{2\zeta} = \frac{0.0005}{\zeta}$$

$$X(\zeta) := \frac{0.0005}{\zeta}$$



5.77 The flywheel of an automobile engine has a mass of about 50 kg and an eccentricity of about 1 cm. The operating speed ranges from 1200 rpm (idle) to 5000 rpm (red line). Choose the remaining parameters so that whirling amplitude is never more than 1 mm.

Solution:

From Equation (5.81),

$$X = 0.001 = \frac{0.01r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$

Choosing ζ = 0.1, the physical solution is

$$r = 0.3018$$

By observing Figure 5.34, r = 0.3018 is the maximum value of r. So at $\left(\omega_r\right)_{\max}$ = 500 rpm, the stiffness must be

$$r = 0.3018 = \frac{5000\left(\dfrac{2\pi}{60}\right)}{\sqrt{k/50}}$$

$$k = 1.505 \times 10^8 \text{ N/m}$$

5.78 Consider the design of the compressor rotor system of Example 5.7.1. The amplitude of the whirling motion depends on the parameters α, ζ, *m*, *k* and the driving frequency. Which parameter has the greatest effect on the amplitude? Discuss your results.

Solution:

From Example 5.7.1, $\alpha = 0.001$ m, $\zeta = 0.05$, $m = 55$ kg, $\omega_r = 6000$ rpm, and $k = 1.4 \times 10^7$ N/m. To find out what effect each parameter has on this system, each value will be varied by 10%.

The original system has $r = 1.2454$ and $X = 0.002746$ m.

$$
\begin{array}{llllll}
0.9a = 0.009\text{m} & \rightarrow & r = 1.2454 & \rightarrow & X = 0.002471 \text{ m } (\text{-}10.0\%) \\
1.1a = 0.0011 \text{ m} & \rightarrow & r = 1.2454 & \rightarrow & X = 0.003020 \text{ m } \left(+10.0\%\right) \\
1.9\zeta = 0.045 & \rightarrow & r = 1.2454 & \rightarrow & X = 0.002759 \text{ m } \left(+0.465\%\right) \\
1.1\zeta = 0.055 & \rightarrow & r = 1.2454 & \rightarrow & X = 0.002732 \text{ m } \left(\text{-}0.507\%\right) \\
0.9m = 49.5 \text{ kg} & \rightarrow & r = 1.1815 & \rightarrow & X = 0.003379 \text{ m } \left(+23.1\%\right) \\
1.1m = 60.5 \text{ kg} & \rightarrow & r = 1.3062 & \rightarrow & X = 0.002376 \text{ m } \left(\text{-}13.5\%\right) \\
0.9k = 1.26 \times 10^7 \text{ N/m} & \rightarrow & r = 1.3127 & \rightarrow & X = 0.002344 \text{ m } \left(\text{-}14.6\%\right) \\
1.1k = 1.54 \times 10^7 \text{ N/m} & \rightarrow & r = 1.1874 & \rightarrow & X = 0.003304 \text{ m } \left(+20.3\%\right)
\end{array}
$$

The mass and stiffness values have the greatest effect on the amplitude, while the damping ratio has the smallest effect.

5.79 At critical speed the amplitude is determined entirely by the damping ratio and the eccentricity. If a rotor has an eccentricity of 1 cm, what value of damping ratio is required to limit the deflection to 1 cm?

Solution:

Since X = 0.01 m, a = 0.01 m, and at critical speed r = 1, then from Equation (5.81),

$$X = 0.01 \text{ m } = \frac{a}{2\zeta} = \frac{0.01}{2\zeta}$$

$$\zeta = 0.5$$

5.80 A rotor system has damping limited by $\zeta < 0.05$. What is the maximum value of eccentricity allowable in the rotor design if the maximum amplitude at critical speed must be less than 1 cm?

Solution:

Since X = 0.01 m, $\zeta < 0.05$, and at critical speed r = 1, then from Equation (5.81),

$$X = 0.01 \text{ m } = \frac{a}{2\zeta} = \frac{a}{2(0.05)}$$

$$a = 0.001 \text{ m } = 1 \text{ mm}$$