Problems and Solutions Section 5.8 (5.81 through 5.85)

5.81 Recall the definitions of settling time, time to peak, and overshoot given in Example 3.2.1 and illustrated in Figure 3.6. Consider a single-degree-of-freedom system with mass m = 2 kg, damping coefficient c = 0.8 N·s/m, and stiffness 8 N/m. Design a PD controller such that the settling time of the closed-loop system is less than 10 s.

Solution: The settling time is

$$t_s = \frac{3}{\zeta\omega}$$

Since t_s = 10 s,

$$\zeta\omega = 0.3$$

The equation of motion with a PD controller is

$$m\ddot{x} + \left(c + g_2\right)\dot{x} + \left(k + g_1\right)x = 0$$

So,

$$\omega = \sqrt{\frac{k + g_1}{m}} = \sqrt{\frac{8 + g_1}{2}}$$

$$\zeta = \frac{c + g_2}{2m\omega} = \frac{0.8 + g_2}{2\left(2\right)\omega}$$

Therefore,

$$\zeta\omega = \left(\frac{0.8 + g_2}{4\omega}\right)\omega = 0.3$$

$$g_2 = 0.4 \text{ N} \cdot \text{s/m}$$

The gain g_1 can take on any value (including 0).

5.82 Redesign the control system given in Example 5.8.1 if the available internal damping is reduced to 50 N·s/m.

Solution: If the value of c is limited to 50 N·s/m, then g_2 becomes

$$g_2 = 180 - c = 180 - 50 = 130 \text{ N} \cdot \text{s/m}$$

5.83 Consider the compressor rotor-shaft system discussed in Problem 5.74. Modern designers have considered using electromagnetic bearings in such rotor systems to improve their design. Use a derivative feedback control law on the design of this compressor to increase the effective damping ratio to $\zeta = 0.5$. Calculate the required gain. How does this affect the answer to parts (a) and (b) of Problem 5.74?

Solution: From Problem 5.74, $m = 100$ kg, $k = 1.4 \times 10^7$ N/m, $a = 0.01$, $\zeta_{old} = 0.01$. The value of c is

$$c = 2\zeta_{old}\sqrt{km} = 2(0.01)\sqrt{(1.4\times 10^7)(100)} = 748.3 \text{ kg/s}$$

With derivative feedback, the coefficient of $\dot{x}$ in the equation of motion is $c + g_2$. For $\zeta = 0.5$,

$$c + g_2 = 748.3 + g_2 = 2(0.5)\sqrt{(1.4\times 10^7)(100)} = 37{,}416.6$$

$$g_2 = 36{,}668.2 \text{ kg/s}$$

(a) The rotor's critical speed remains the same because it is only dependent upon the mass stiffness.

(b) The whirl amplitude becomes

$$X = \frac{a}{2\zeta} = \frac{0.01}{2(0.5)} = 0.01 \text{ m}$$

It is reduced by 80% because of the increased damping.

5.84 Calculate the magnitude of the force required of the actuator used in the feedback control system of Example 5.8.1. See if you can find a device that provides this much force.

Solution: The magnitude of the actuator force would be

$$F = g_2|\dot{x}| = g_2\omega_n X$$

where X is, from Equation (2.26), at steady-state,

$$X = \frac{F_0 / m}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega\ )^2}}$$

A large value of X would occur at resonance, for example, where $\omega = \omega_{dr} = 10$ rad/s, so

$$F = (80)(10)\frac{F_0 / 10}{2(0.9)(10)(10)} = 0.444F_0$$

5.85 In some cases the force actuator used in a control system also introduces dynamics. In this case a system of the form given in Equation (5.27) may result where m_a, c_a and k_a are values associated with the actuator (rather than an absorber). In this case the absorber system indicated in Figure 5.18 can be considered as the control system and the motion of the mass m is the object of the control system. Let m = 10 kg, k = 100 N/m, and c = 0. Choose the feedback control law to be

$$u = -g_1 x - g_2 \dot{x}$$

and assume that c_a = 20 N·s/m, k_a = 100 N/m and m_a = 1 kg. Calculate g_1 and g_2 so that x is as small as possible for a driving frequency of 5 rad/s. [Hint: Replace k with $k + g$, and c with $c + g$ in Equation (5.27)]

Solution:

Let the control law be called position feedback, applied to the mass m. The equation of motion then becomes Equation (5.27) with k replaced by $k + g_1$. Then the amplitude X can be expressed as Equation (5.35) with k replaced by $k + g_1$ and given values of m, m_a, k_a and c_a. This yields

$$\frac{X^2}{F_0^2} = \frac{(100-25)^2 + (25)(400)}{\left\{\left[100 + g_1 - (10)(25)\right]\left[100-25\right] - 2500\right\}^2 + \left[100 - (11)(25)\right]^2 (25)(400)}$$

$$\frac{X^2}{F_0^2} = \frac{2.78}{g_1^2 - 366.7g_1 + 88{,}055.6}$$

Clearly X is a minimum if g_1^2 -366.7g_1 + 88,055.6 is a minimum. Thus consider the derivatives of the quadratic form with respect to g_1 to find the max value per the discussion on the top of page 265.

$$\frac{d}{dg_1}\left(g_1 - 366.7g_1 + 88{,}055.6\right) = 2g_1 - 366.7 = 0$$

so that $g_1 = 183.35$

Note that $d^2 / dg_1^2 = 2 > 0$ so that this is a maximum and X is a minimum for this gain.