Problems and Solutions Section 7.2 (7.1-7.5)

7.1 A low-frequency signal is to be measured by using an accelerometer. The signal is physically a displacement of the form 5 sin (0.2*t*). The noise floor of the accelerometer (i.e. the smallest magnitude signal it can detect) is 0.4 volt/g. The accelerometer is calibrated at 1 volt/g. Can the accelerometer measure this signal?

Solution:

From the problem statement:

$$x(t) = 0.5\sin(0.2t) \quad \text{m}$$
$$\dot{x}(t) = 0.1\cos(0.2t) \quad \text{m/s}$$
$$\ddot{x}(t) = -0.02\sin(0.2t) \quad \text{m/s}^2$$

The peak acceleration is:

$$\pm 0.2\ \text{m/s}^2 \left[\frac{1g}{9.8\ \text{m/s}^2}\right] = \pm 0.0204g$$

Accelerometer calibration is $1\text{V}/g$, therefore the peak output of the accelerometer is:

$$\pm 0.0204g\left[\frac{1\text{V}}{g}\right] = \pm 0.0204\text{V}$$

Since the noise floor on the accelerometer is 0.4 V, then this acceleration cannot be measured.

7.2 Referring to Chapter 2, calculate the response of a single-degree-of-freedom system to a unit impulse and then to a unit triangle input lasting T second. Compare the two responses. The differences correspond to the differences between a "perfect" hammer hit and a more realistic hammer hit, as indicated in Figure 7.2. Use ζ = 0.01 and ω = 4 rad/s for your model.

Solution:

System: $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = f(t)$ (Letting m = 1)

(i) $f(t) = \delta(t)$, a unit impulse

$x(t) = e^{-\zeta\omega_n t}\sin(\omega_d t)$ $\qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$

(ii) $f(t) = \dfrac{t}{T}u(t) - \dfrac{2}{T}(t-T)u(t-T) + \dfrac{1}{T}(t-2T)u(t-2T)$

$u(t\text{-}a)$ = unit step at $t = a$.

$x(t) = \dfrac{1}{T}\{r(t) - 2r(t-T) + r(t-2T)\}$

From table of Laplace transforms:

$$r(t) = \frac{1}{\omega_n^2}\left\{t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_d}e^{-\zeta\omega_n t}\sin(\omega_d t + \theta)\right\}u(t) \qquad \cos\theta = 2\zeta^2 - 1$$

$$x(t) \approx \frac{1}{T\omega_n^3}\Big\{\Big[\omega_n t - 2\zeta - e^{-\zeta\omega_n t}\sin(\omega_d t\Big]u(t)$$
$$-2\Big[\omega_n(t-T) - 2\zeta - e^{-\zeta\omega_n(t-T)}\sin(\omega_d(t-T))\Big]u(t-T)$$
$$+\Big[\omega_n(t-2T) - 2\zeta - e^{-\zeta\omega_n(t-2T)}\sin(\omega_d(t-2T))\Big]u(t-2T)\Big\}$$

since $\omega_n \approx \omega_d$ and $\theta \approx \pi/2$

7.3 Compare the Laplace transform of $\delta(t)$ with the Laplace transform of the triangle input of Figure 7.2 and Problem 7.2.

Solution:

(i) $f(t) = \delta(t)$, unit impulse

$F(s) = 1$

(ii) $f(t) = \frac{t}{T}u(t) - \frac{2}{T}(t-T)u(t-T) + \frac{1}{T}(t-2T)u(t-2T)$, unit triangle with period T.

$$F(s) = \frac{1}{T}\left\{\int_0^\infty te^{-st}dt - 2\int_T^\infty (t-T)e^{-st}dt + \int_{2T}^\infty (t-2T)e^{-st}dt\right\}$$

$$F(s) = \frac{1}{Ts^2}\left\{1 + e^{-sT} + e^{s2T}\right\}$$

7.4 Plot the error in measuring the natural frequency of a single-degree-of-freedom system of mass 10 kg and stiffness 350 N/m if the mass of the excitation device (shaker) is included and varies from 0.5 to 5 kg.

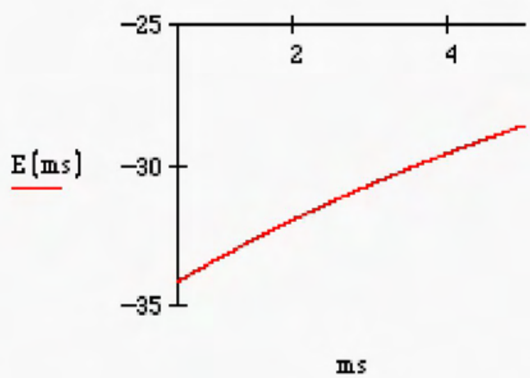
Solution:

$m = 10$ kg
$k = 350$ N/m
$0.5 \leq m_s \leq 5.0$ kg

$$\text{Error} = \sqrt{\frac{k}{m + m_s}} - \sqrt{\frac{k}{m}}$$

$k := 350 \qquad m := 10$

$$E(ms) := \sqrt{\frac{k}{m + ms}} - \sqrt{\frac{k}{m}}$$



7.5 Calculate the Fourier transform of $f(t)$ = 3 sin 2t – 2 sin t – cos t and plot the spectral coefficients.

Solution:

$F(t) = 3\sin(2t) - 2\sin(t) - \cos(t)$

$\omega_T = 1$ rad/sec

$a_1 = -1 \qquad b_1 = -2 \qquad b_2 = 3$

$a_n = 0,\ n = 2, 3, \ldots \quad b_n = 0,\ n = 3, 4, 5, \ldots$