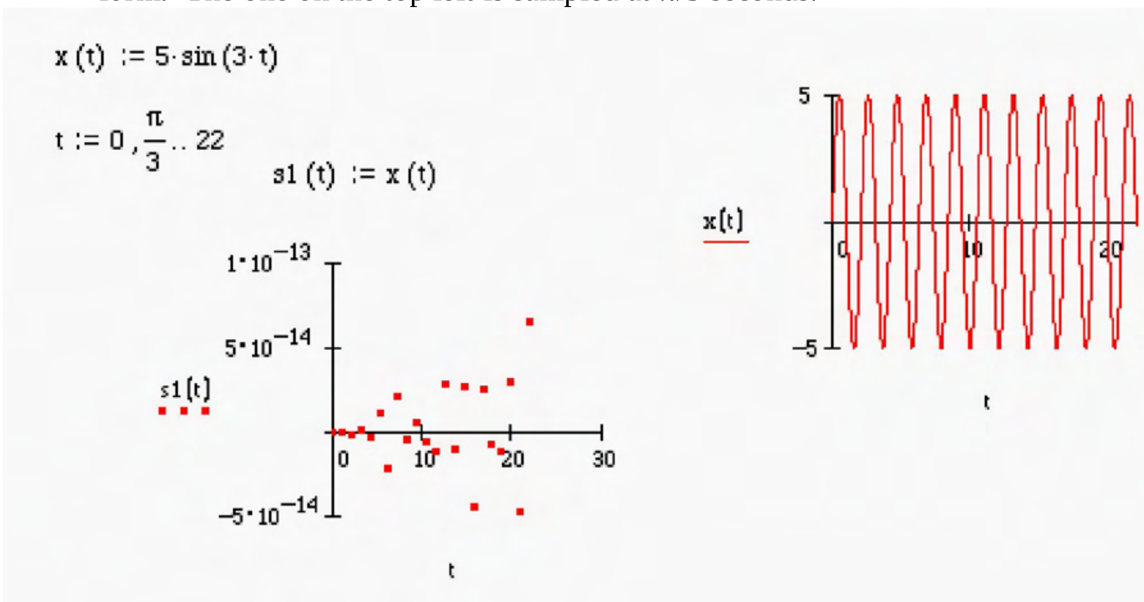
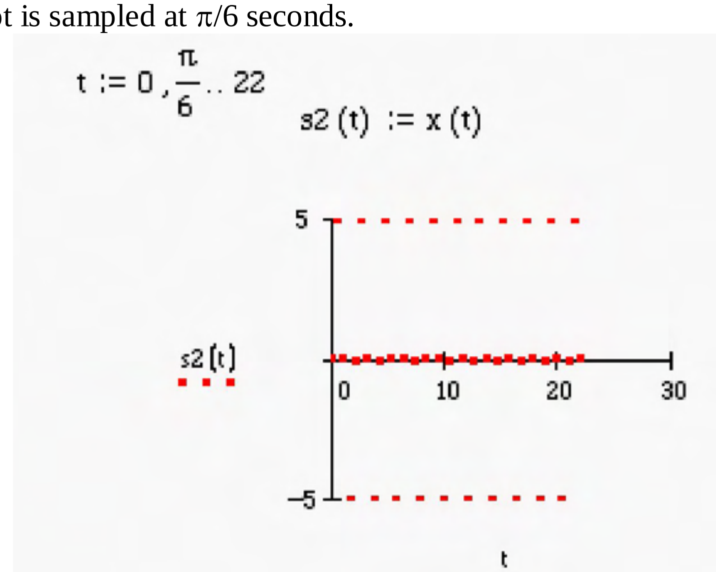
Problems and Solutions Section 7.3 (7.6-7.9)

7.6 Represent 5 sin 3*t* as a digital signal by sampling the signal at π/3, π/6 and π/12 seconds. Compare these three digital representations.

Solution: Four plots are shown. The one at the top far right is the exact wave form. The one on the top left is sampled at π/3 seconds.

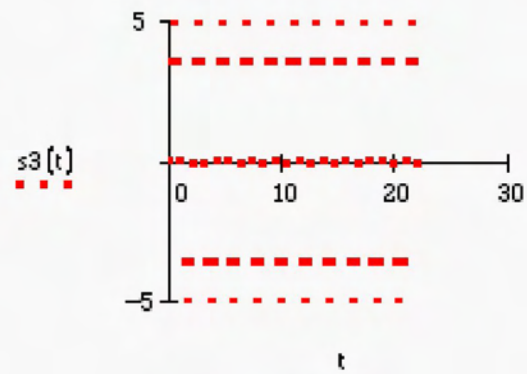


The next plot is sampled at π/6 seconds.



The next plot is sampled at π/12 seconds.

$$t := 0, \frac{\pi}{12} .. 22 \qquad s3(t) := x(t)$$



None of the plots give the shape of a sine wave. However if the s3 is connected by lines, the wave shape is close.

7.7 Compute the Fourier coefficient of the signal |1120 sin (120 πt)|.

Solution:

$f(t)$ = $|120 sin(120\pi t)|$ (absolute value of the sine wave)

To calculate the Fourier series:

T = 1/120 sec ω_T = 240 π rad/sec

$$a_o = 240 \int_0^{1/120} 120 \sin(120\pi t) dt$$

$$a_o = 480/\pi$$

$$a_n = 240 \int_0^{1/120} 120 \sin(120\pi t) \cos(240\pi n t) dt$$

$$a_n = \frac{480}{\pi(1-4n^2)}$$

$$b_n = 240 \int_0^{1/120} 120 \sin(120\pi t) \sin(240\pi n t) dt$$

$$b_n = 0$$

$$f(t) = \frac{240}{\pi}\left\{1 + \sum_{n=1}^{\infty} \frac{2}{1-4n^2} \cos(240\pi n)t\right\}$$

7.8 Consider the periodic function

$$x(t) = \begin{cases} -5 & 0 < t < \pi \\ 5 & \pi < t < 2\pi \end{cases}$$

and $x(t) = (t + 2\pi)$. Calculate the Fourier coefficients. Next plot $x(t)$: $x(t)$ represented by the first term in the Fourier series, $x(t)$ represented by the first two terms of the series, and $x(t)$ represented by the first three terms of the series. Discuss your results.

Solution: For the Fourier Series: $T = 2\pi$ $\omega_T = 1$

$$a_0 = 0$$

$$a_n = \frac{2}{2\pi}\left\{\int_0^{\pi} -5\cos(nt)dt + \int_{\pi}^{2\pi} 5\cos(nt)dt\right\}$$

$$\Rightarrow a_n = 0$$

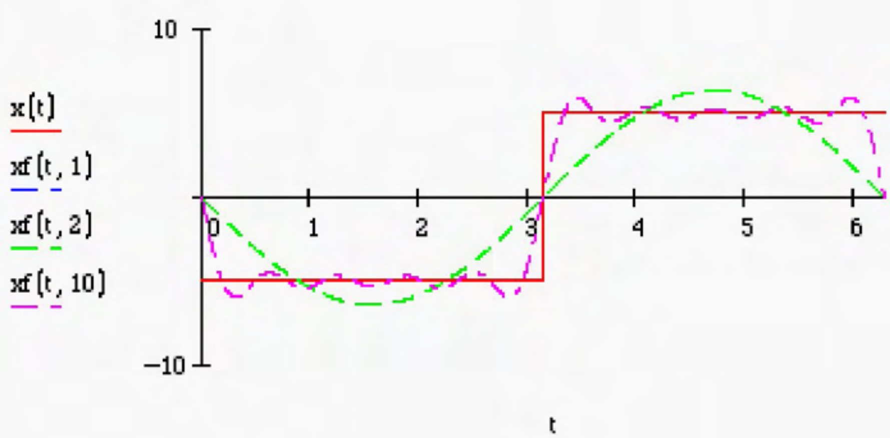
$$b_n = \frac{2}{2\pi}\left\{\int_0^{\pi} -5\sin(nt)dt \int_{\pi}^{2\pi} 5\sin(nt)dt\right\} \Rightarrow$$

$$b_n = \frac{-5}{\pi n}[1 - 2\cos(n\pi) + \cos(2n\pi)]$$

$$x(t) = -\frac{5}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(1 - 2\cos(n\pi) + \cos(2n\pi)\sin(nt)$$

$x(t) := -5 + 10 \cdot \Phi(t - \pi)$ The funtion for one cycle (0<t<2pi)

$$xf(t, N) := \frac{-5}{\pi} \cdot \sum_{n=1}^{N} \frac{(-2 \cdot \cos(n \cdot \pi) + 1 + \cos(2 \cdot n \cdot \pi))}{n} \cdot \sin(n \cdot t)$$



7.9 Consider a signal $x(t)$ with maximum frequency of 500 Hz. Discuss the choice of record length and sampling interval.

Solution:

For a signal with maximum frequency of 500 Hz, the sampling rate, f_s, should be

$$f_s > 2(500) = 1000 \text{ Hz}$$

Due to Shannon's sampling theorem. A better choice would be

$$f_s = 2.5(500) = 1250 \text{ Hz}$$

Thus, the minimum sampling rate is 0.001 sec. and the suggested rate is 0.0008 sec.
Lower sampling rates will produce aliasing.

The record length N is usually a power of 2, such as 512, 1024, 2048, etc.
Windowing is performed to reduce leakage.