Problems and Solutions Section 8.5 (8.44 through 8.49)

8.44 Derive a consistent-mass matrix for the system of Figure 8.9. Compare the natural frequencies of this system with those calculated with the lumped-mass matrix computed in Section 8.5.

Solution: Using the vibration toolbox

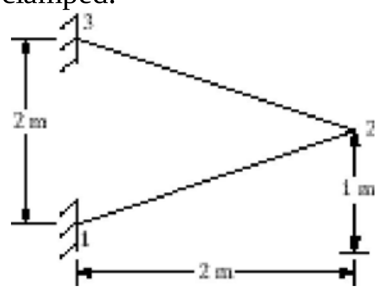
$$M = \rho A l \begin{bmatrix} .6857 & 0 \\ 0 & .7238 \end{bmatrix}$$

The natural frequencies are then

$$\omega_1 = .8311\frac{1}{l}\sqrt{\frac{E}{\rho}} \text{ and } \omega_1 = 1.479\frac{1}{l}\sqrt{\frac{E}{\rho}}$$

These are higher than those predicted with the inconsistent mass matrix

8.45 Consider the two beam system of Figure P8.45. Use VTB8_1 to create a two-element, rod/beam element model and compute the first three natural frequencies. Use $A = 0.0004$ m^2, $I = 1.33 \times 10^{-8}$ m^4, and the properties of aluminum. Assume that nodes 1 and 3 are clamped.



Solution:

```
%scipt file for problem 8.45
node=[0 0;1 .5;2 1;1  1.5;0 2];
ncon=[1 2 69e10 .004 1.33e-8 0 2700;
      2 3 69e10 .004 1.33e-8 0 2700;
      3 4 69e10 .004 1.33e-8 0 2700;
      4 5 69e10 .004 1.33e-8 0 2700];
zero=[1 1;
      1 2;
      1 3;
      5 1;
      5 2;
      5 3];
conm=[];
force=[];
save VTB8_45.con
```

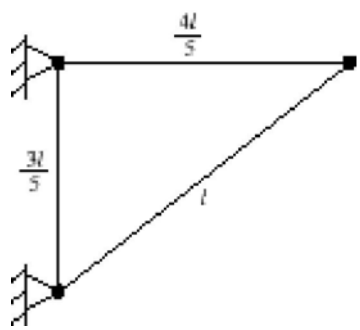
Running this yields that the first three natural frequencies are given as 377.5, 8763.7 and 10951.2 rad/s.

8.46 Follow the procedure of Problem 8.45 using two elements for each beam. Compare the natural frequencies and mode shapes of the four element model produced here to those of the two-element model of Problem 8.45. State which model is better and why.

Solution: Use the script file from 8.45 ending in VTB8_46.con
The first five natural frequencies are 286.8, 419.1, 1074.5, 1510.8, and 2838.9 rad/s. The result from the four element model is probably better because the additional elements allow the first few modes to be found in more detail. Notice the difference in the result for the first mode. The first mode is primarily a rotation of the joint between the two beams. The two element model shows this to be the only significant motion (load the .out data file to observe the mode shape vector). The four element model shows that the middle of each beam displaces and rotates as well.

The eight element model predicts the first five natural frequencies to be 284.3, 413.0 ,925.6, 1147.3, and 1959.7 rad/s, the first four of which agree well with the four element model results.

8.47 Determine a finite element model of the three-bar truss of Figure P8.47 using a lumped-mass matrix.



Solution:

Using VTB8_1

$$K = \frac{EA}{l}\begin{bmatrix} 1.89 & .48 \\ .48 & .36 \end{bmatrix}$$

The inconsistent mass matrix is

$$M = \rho Al\begin{bmatrix} .9 & 0 \\ 0 & .9 \end{bmatrix}$$

8.48 Determine a finite element model for the three-bar truss of Figure P8.47 using a consistent-mass matrix.

Solution:

Using VTB8_1 the consistent mass matrix is

$$M = \rho Al\begin{bmatrix} .6137 & -.0183 \\ -.0183 & .6549 \end{bmatrix}$$

However, this mass matrix is created using beam/rod elements. Using simple rod elements gives a consistent mass matrix

$$M = \rho Al\begin{bmatrix} .48 & .16 \\ .16 & .12 \end{bmatrix}$$

8.49 Compare the frequencies obtained for the system of Problem 8.48 with those of Figure P8.47.

Solution:

The natural frequencies using the consistent mass matrix are

$\omega_1 = 1.7321 \quad \omega_2 = 2.1651$

The natural frequencies using the inconsistent mass matrix are

$\omega_1 =.4966 \quad \omega_2 = 1.5012$

These results are terribly inconclusive, but since we have seen in previous examples that the consistent mass matrix generally yields the better results, one would expect the same to be true in this case.