Problems and Solutions Section 8.6 (8.50 through 8.54)

8.50 Consider the machine punch of Figure P8.15. Recalculate the fundamental natural frequency by reducing the model obtained in Problem 8.16 to a single degree of freedom using Guyan reduction.

Solution:

From the results of 8.16

$$K = \begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix} \times 10^8, \quad M = \begin{bmatrix} .052 & .013 \\ .013 & .026 \end{bmatrix}$$

From (8.104)

$$Q^T MQ = .052 + .013 + .013 + .026 = .104$$

From (8.105)

$$Q^T KQ = (4-2) \times 10^8 = 2 \times 10^8$$

$$\omega = \sqrt{\frac{2 \times 10^8}{.104}} = 43852.9 \text{ rad/s}$$

which is a poor prediction of the first natural frequency. If we reorder *K* and *M* (reducing to coordinate 2) we get

$$Q^T MQ = .026 + .013 + .013 = .052$$

$$Q^T KQ = (2-1) \times 10^8 = 1 \times 10^8$$

$$\omega = 43852.9 \text{ rad/s}$$

which is the same result as reducing to coordinate 1.

8.51 Compute a reduced-order model of the three-element model of a cantilevered bar given in Example 8.3.2 by eliminating u_2 and u_3 using Guyan reduction. Compare the frequencies of each model to those of the distributed model given in Window 8.1.

Solution:

$$M = \frac{\rho A l}{18}\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$K = \frac{3EI}{l}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Let $\tilde{M}$ and $\tilde{K}$ be the matrices with the coefficients factored out.

$$\tilde{M}_{11} = 4,\ \tilde{M}_{21} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \tilde{M}_{12}^T,\ M_{22} = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\tilde{K}_{11} = 2,\ \tilde{K}_{21} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \tilde{K}_{11}^T,\ K_{22} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

Using equations (8.104) and (8.105)

$$\tilde{M}_r = Q^T M Q = 14$$

$$\tilde{K}_r = Q^T K Q = 1$$

and

$$\omega_n = \sqrt{\frac{\frac{3EA}{l}}{\frac{14\rho A l}{18}}} = 1.964\frac{1}{l}\sqrt{\frac{E}{\rho}}$$

as compared to the distributed model value of

$$\omega_1 = 1.57\frac{1}{l}\sqrt{\frac{E}{\rho}}$$

8.52 Consider the system defined by the matrices

$$M = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad K = \begin{bmatrix} 20 & -1 & 0 & 0 \\ -1 & 20 & -3 & 0 \\ 0 & -3 & 20 & -17 \\ 0 & 0 & 17 & 17 \end{bmatrix}$$

Use mass condensation to reduce this to a two-degree-of-freedom system with a nonsingular mass matrix.

Solution:

Following the same procedure as example 8.6.1

$$M_r = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \text{ and } K_r = \begin{bmatrix} 19.95 & -.15 \\ -.15 & 36.55 \end{bmatrix}$$

8.53 Recall the punch press problem modeled in Figure 4.28 and treated in Example 4.8.3. The mass and stiffness matrices are given by

$$M = \begin{bmatrix} 0.4 \times 10^3 & 0 & 0 \\ 0 & 2.0 \times 10^3 & 0 \\ 0 & 0 & 8.0 \times 10^3 \end{bmatrix} \quad K = \begin{bmatrix} 30 \times 10^4 & 30 \times 10^4 & 0 \\ 30 \times 10^4 & 38 \times 10^4 & 8 \times 10^4 \\ 0 & 8 \times 10^4 & 88 \times 10^4 \end{bmatrix}$$

Recalling that the only external force acting on the machine is at the $x_1(t)$ coordinate, reduce this to a single-degree-of-freedom system using Guyan reduction to remove x_2 and x_3. Compare this single frequency with those of Example 4.8.3.

Solution:

Following the same procedure as example 8.6.1

$M_r = 1.7385 \times 10^3, K_r = 5.8537 \times 10^4$ and the natural frequency is

$$\omega_n = \sqrt{\frac{K_r}{M_r}} = 5.803 \text{rad/s}$$

Example 4.8.3 gave the first natural frequency as $\omega_1 = 5.387$ rad/s which is within 10% of the Guyan reduced prediction.

8.54. Consider the beam example given in Example 7.6.2. Using the values given there (An aluminum beam: 0.5128 m x 25.5 mm x 3.2 mm, $E = 6.9\times10^{10}$ N/m^2 , ρ = 2715 kg/m^3, A = 8.16 m^2 and $I = 6.96\times10^{-11}$ m^4), compute the first 4 natural frequencies as accurately as possible and compare them to both the analytical values and the measured values.