

# Propulsion

## 1 Engine Thrust and Heat

Let's look at an engine flying at a velocity  $V_0$  [ $m/s$ ] (with respect to the air). The air enters the engine with a velocity  $V_0$ . Suppose it exits the engine with a velocity  $V_j$  [ $m/s$ ]. Also suppose that the mass of the air that goes through the engine every second (the **mass flow**) is  $m$  [ $kg/s$ ]. The **thrust** can now be found using the change in momentum:

$$T = I_{out} - I_{in} = m(V_j - V_0) \quad (1)$$

Now suppose the engine uses  $m_f$  [ $kg/s$ ] kilograms of fuel per second. The **fuel flow**  $F$  [ $N/s$ ] is now defined as:

$$F = m_f \cdot g \quad (2)$$

If we know the **heating value**  $H$  [ $J/kg$ ] (the amount of joules in one kilogram of fuel), we can also find the added energy  $Q$  [ $J/s$ ] every second:

$$Q = m_f \cdot H = \frac{F \cdot H}{g} \quad (3)$$

And if we want to find the heat  $q_{in}$  [ $J/kg$ ] added to one kilogram of air, we can calculate it using:

$$q_{in} = \frac{Q}{m} = H \frac{m_f}{m} \quad (4)$$

## 2 Engine Power and Efficiency

When we know the velocity of the air prior to and after the engine, and if we know the mass flow of the air, we can calculate the energy put into the air every second - the **jet power**  $P_j$  [ $J/s$ ]:

$$P_j = \frac{1}{2}mV_j^2 - \frac{1}{2}mV_0^2 = \frac{1}{2}m(V_j^2 - V_0^2) = \frac{1}{2}m(V_j - V_0)(V_j + V_0) = \frac{1}{2}T(V_j + V_0) \quad (5)$$

The **power available**  $P_a$  [ $J/s$ ], which is the work added every second, can be calculated using:

$$P_a = T \cdot V_0 = m_f \cdot V_0(V_j - V_0) \quad (6)$$

Using these equations for the power, we can calculate various efficiencies. The **propulsive efficiency**  $\eta_j$  is defined as:

$$\eta_j = \frac{P_a}{P_j} = \frac{T \cdot V_0}{\frac{1}{2}T(V_j + V_0)} = \frac{2V_0}{V_j + V_0} = \frac{2}{1 + \frac{V_j}{V_0}} \quad (7)$$

The **thermal efficiency**  $\eta_{th}$  is defined as:

$$\eta_{th} = \frac{P_j}{Q} = \frac{\frac{1}{2}m(V_j^2 - V_0^2)}{m_f \cdot H} = \frac{V_j^2 - V_0^2}{2q_{in}} \quad (8)$$

The **total efficiency**  $\eta_{tot}$  is defined as:

$$\eta_{tot} = \frac{P_a}{Q} = \frac{T \cdot V_0}{Q} = \eta_j \cdot \eta_{th} \quad (9)$$

There are multiple ways to increase efficiency. To increase propulsive efficiency, it is necessary to keep  $V_j$  close to  $V_0$  (instead of giving a little bit of air a lot of acceleration, give a lot of air a little bit of acceleration). To increase thermal efficiency, the **pressure ratio**  $\varepsilon$  (the ratio of the pressure before and after the compressor) should be high. So the air should be compressed as much as possible.

Using the equation  $Q = m q_{in}$ , the thrust can also be rewritten as:

$$T = Q \frac{\eta_{tot}}{V_0} = m q_{in} \frac{2}{V_0 + V_j} \eta_{th} \quad (10)$$

This equation also shows that if the thrust increases (thus the engine gets a higher RPM and thus a higher compression ratio) also the thermal efficiency increases.

### 3 The Perfect Jet Engine

The **perfect jet engine** is a jet engine at which most steps are isentropic. Only when the air goes through the combustion chamber, the entropy rises. At that step, the temperature rises isobaric. From this fact the entropy after the combustion chamber can be found (by following the isobar lines in a Mollier diagram, for example). It is also assumed that all the energy taken out by the turbine is put back in the system by the compressor.

The perfect jet engine consists of 5 parts. So when air goes through the jet engine, it has 6 consecutive states. These states change as follows:

- The inlet (1-2):  $w_{in} = q_{in} = 0$   $V_2 \approx 0$   $\Rightarrow \Delta h = \frac{1}{2} V_1^2$
- The compressor (2-3):  $q_{in} = 0$   $V_3 \approx V_2 \approx 0$   $\Rightarrow \Delta h = w_{in}$
- The combustion chamber (3-4):  $w_{in} = 0$   $V_4 \approx V_3 \approx 0$   $\Rightarrow \Delta h = q_{in}$
- The turbine (4-5):  $w_{out} = -w_{in}$   $q_{in} = 0$   $V_5 \approx V_4 \approx 0$   $\Rightarrow \Delta h = -w_{in}$
- The exhaust (5-6):  $w_{in} = q_{in} = 0$   $V_5 \approx 0$   $\Rightarrow \Delta h = -\frac{1}{2} V_6^2$

Note that the free stream velocity is  $V_0 = V_1$  and the exhaust velocity is  $V_j = V_6$ .

### 4 The Turbo-Prop Engine

The **turbo-prop engine** is very similar to the jet engine. The turbo-prop is in fact a jet engine with a propeller in front of it. This propeller accelerates air just slightly, giving the engine a higher propulsive efficiency. An indication for this is the by-pass ratio, defined as:

$$\lambda = \frac{M_{cold}}{M_{hot}} \quad (11)$$

where  $M_{cold}$  is the cold airflow (passing through the propeller) and  $M_{hot}$  is the hot airflow (passing through the compressor). In a turbo-prop engine, there is one additional step. Between the (first) turbine and the exhaust, there is a second turbine. This second turbine is connected to the propeller. However, since the turbo-prop engine isn't a jet engine, there also isn't a jet power. Instead, there is the shaft power (break power)  $P_{br}$  [J/s]. The propulsive efficiency  $\eta_p$  and the thermal efficiency  $\eta_{th}$  are now defined as:

$$\eta_p = \frac{P_a}{P_{br}} \quad \eta_{th} = \frac{P_{br}}{Q} \quad (12)$$

The propeller in a turbo-prop engine doesn't give the air a lot of acceleration. So let's assume that  $V = V_j$ . The thermal efficiency now is:

$$\eta_{th} = \frac{P_{br}}{Q} = \frac{m(h_5 - h_6)}{Q} = \frac{m(h_5 - h_{exhaust}) - \frac{1}{2}mV^2}{Q} \quad (13)$$

For a jet engine, the thermal efficiency can be expressed as:

$$\eta_{th} = \frac{P_j}{Q} = \frac{\frac{1}{2}mV_j^2 - \frac{1}{2}mV_0^2}{Q} = \frac{m(h_5 - h_{exhaust}) - \frac{1}{2}mV^2}{Q} \quad (14)$$

These equations are equal! (Note that for a jet engine  $h_{exhaust} = h_6$  but for a turbo-prop engine  $h_{exhaust} = h_7$ , due to the extra step in a turbo-prop engine.)

There is another similarity between the turbo-prop engine and the jet engine. For both engines the efficiency decreases if the temperature increases, or if the aircraft flies higher (lower density).

## 5 Nozzles

When engines fly at (nearly) supersonic speeds, their engines often have a nozzle with a throat. For the sonic regions  $V \uparrow$  as  $A \downarrow$ , where  $A$  is the cross-section of the engine. However, for supersonic speed  $V \uparrow$  as  $A \uparrow$ . The throat (where  $A$  is smallest) is the point at which the transition from sonic to supersonic velocity occurs. Thus  $M_t = 1 \Leftrightarrow V_t = a$  at the throat.

The nozzle should be made in such a way that at the exhaust the pressure in the engine  $p_e$  is equal to the outside pressure  $p_0$ . If  $p_e > p_0$  (the airplane is flying at a height higher than it was designed for) the exhausted air continues to expand and accelerate after it has exited the nozzle, but this acceleration does not contribute to the aircraft thrust. So there is a waste of energy. If  $p_e < p_0$  (the airplane is flying at a height lower than it was designed for) the exhausted air causes shock waves, which also mean a waste of energy.