1. Let $y(x)=\sum_{n=0}^{\infty} c_n x^n$. Then $y'(x)=\sum_{n=1}^{\infty} nc_n x^{n-1}$ and the given equation, $y'-y=0$, becomes $\sum_{n=1}^{\infty} nc_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$. Replacing n by $n+1$ in the first sum gives $\sum_{n=0}^{\infty}(n+1)c_{n+1}x^n - \sum_{n=0}^{\infty} c_n x^n = 0$, so $\sum_{n=0}^{\infty}\left[(n+1)c_{n+1}-c_n\right]x^n=0$. Equating coefficients gives $(n+1)c_{n+1}-c_n=0$, so the recursion relation is $c_{n+1}=\frac{c_n}{n+1}$, $n=0,1,2,\ldots$. Then $c_1=c_0$, $c_2=\frac{1}{2}c_1=\frac{c_0}{2}$, $c_3=\frac{1}{3}c_2=\frac{1}{3}\cdot\frac{1}{2}c_0=\frac{c_0}{3!}$, $c_4=\frac{1}{4}c_3=\frac{c_0}{4!}$, and in general, $c_n=\frac{c_0}{n!}$. Thus, the solution is

$$y(x)=\sum_{n=0}^{\infty} c_n x^n=\sum_{n=0}^{\infty}\frac{c_0}{n!}x^n=c_0\sum_{n=0}^{\infty}\frac{x^n}{n!}=c_0e^x$$

2. Let $y(x)=\sum_{n=0}^{\infty} c_n x^n$. Then $y'=xy \Rightarrow y'-xy=0 \Rightarrow \sum_{n=1}^{\infty} nc_n x^{n-1} - x\sum_{n=0}^{\infty} c_n x^n = 0$ or $\sum_{n=1}^{\infty} nc_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^{n+1}=0$. Replacing n with $n+1$ in the first sum and n with $n-1$ in the second gives $\sum_{n=0}^{\infty}(n+1)c_{n+1}x^n - \sum_{n=1}^{\infty} c_{n-1}x^n=0$ or $c_1+\sum_{n=1}^{\infty}(n+1)c_{n+1}x^n-\sum_{n=1}^{\infty}c_{n-1}x^n=0$. Thus, $c_1+\sum_{n=1}^{\infty}\left[(n+1)c_{n+1}-c_{n-1}\right]x^n=0$. Equating coefficients gives $c_1=0$ and $(n+1)c_{n+1}-c_{n-1}=0$. Thus, the recursion relation is $c_{n+1}=\frac{c_{n-1}}{n+1}$, $n=1,2,\ldots$. But $c_1=0$, so $c_3=0$ and $c_5=0$ and in general $c_{2n+1}=0$. Also, $c_2=\frac{c_0}{2}$, $c_4=\frac{c_2}{4}=\frac{c_0}{4\cdot 2}=\frac{c_0}{2^2\cdot 2!}$, $c_6=\frac{c_4}{6}=\frac{c_0}{6\cdot 4\cdot 2}=\frac{c_0}{2^3\cdot 3!}$ and in general $c_{2n}=\frac{c_0}{2^n\cdot n!}$. Thus, the solution is

$$y(x)=\sum_{n=0}^{\infty} c_n x^n=\sum_{n=0}^{\infty} c_{2n}x^{2n}=\sum_{n=0}^{\infty}\frac{c_0}{2^n\cdot n!}x^{2n}=c_0\sum_{n=0}^{\infty}\frac{\left(x^2/2\right)^n}{n!}=c_0e^{x^2/2}$$

3. Assuming $y(x)=\sum_{n=0}^{\infty} c_n x^n$, we have $y'(x)=\sum_{n=1}^{\infty} nc_n x^{n-1}=\sum_{n=0}^{\infty}(n+1)c_{n+1}x^n$ and $-x^2y=-\sum_{n=0}^{\infty} c_n x^{n+2}=-\sum_{n=2}^{\infty} c_{n-2}x^n$. Hence, the equation $y'=x^2y$ becomes $\sum_{n=0}^{\infty}(n+1)c_{n+1}x^n-\sum_{n=2}^{\infty}c_{n-2}x^n=0$ or $c_1+2c_2x+\sum_{n=2}^{\infty}\left[(n+1)c_{n+1}-c_{n-2}\right]x^n=0$. Equating coefficients gives $c_1=c_2=0$ and

$c_{n+1}=\dfrac{c_{n-2}}{n+1}$ for $n=2,3$, … . But $c_1=0$, so $c_4=0$ and $c_7=0$ and in general $c_{3n+1}=0$. Similarly $c_2=0$ so $c_{3n+2}=0$. Finally $c_3=\dfrac{c_0}{3}$, $c_6=\dfrac{c_3}{6}=\dfrac{c_0}{6\cdot 3}=\dfrac{c_0}{3^2\cdot 2!}$, $c_9=\dfrac{c_6}{9}=\dfrac{c_0}{9\cdot 6\cdot 3}=\dfrac{c_0}{3^3\cdot 3!}$, … , and $c_{3n}=\dfrac{c_0}{3^n\cdot n!}$. Thus, the solution is

$$y(x)=\Sigma_{n=0}^{\infty}c_n x^n=\Sigma_{n=0}^{\infty}c_{3n}x^{3n}=\Sigma_{n=0}^{\infty}\frac{c_0}{3^n\cdot n!}x^{3n}=c_0\Sigma_{n=0}^{\infty}\frac{x^{3n}}{3^n \text{n!}}=c_0\Sigma_{n=0}^{\infty}\frac{\left(x^3/3\right)^n}{n!}=c_0e^{x^3/3}$$

4. Let $y(x)=\Sigma_{n=0}^{\infty}c_n x^n\Rightarrow y'(x)=\Sigma_{n=1}^{\infty}nc_n x^{n-1}=\Sigma_{n=0}^{\infty}(n+1)c_{n+1}x^n$. Then the differential equation becomes $(x-3)\Sigma_{n=0}^{\infty}(n+1)c_{n+1}x^n+2\Sigma_{n=0}^{\infty}c_n x^n=0\Rightarrow$ $\Sigma_{n=0}^{\infty}(n+1)c_{n+1}x^{n+1}-3\Sigma_{n=0}^{\infty}(n+1)c_{n+1}x^n+2\Sigma_{n=0}^{\infty}c_n x^n=0\Rightarrow\Sigma_{n=1}^{\infty}nc_n x^n-\Sigma_{n=0}^{\infty}3(n+1)c_{n+1}x^n+\Sigma_{n=0}^{\infty}2c_n x^n=0$ $\Rightarrow\Sigma_{n=0}^{\infty}\left[(n+2)c_n-3(n+1)c_{n+1}\right]x^n=0\left(\text{since}\Sigma_{n=1}^{\infty}nc_n x^n=\Sigma_{n=0}^{\infty}nc_n x^n\right)$. Equating coefficients gives $(n+2)c_n-3(n+1)c_{n+1}=0$, thus the recursion relation is $c_{n+1}=\dfrac{(n+2)c_n}{3(n+1)}$, $n=0,1,2,\ldots$. Then $c_1=\dfrac{2c_0}{3}$, $c_2=\dfrac{3c_1}{3(2)}=\dfrac{3c_0}{3^2}$, $c_3=\dfrac{4c_2}{3(3)}=\dfrac{4c_0}{3^3}$, $c_4=\dfrac{5c_3}{3(4)}=\dfrac{5c_0}{3^4}$, and in general, $c_n=\dfrac{(n+1)c_0}{3^n}$. Thus the solution is $y(x)=\Sigma_{n=0}^{\infty}c_n x^n=c_0\Sigma_{n=0}^{\infty}\dfrac{n+1}{3^n}x^n$. $\left[\text{Note that } c_0\Sigma_{n=0}^{\infty}\dfrac{n+1}{3^n}x^n=\dfrac{9c_0}{(3-x)^2}\text{ for}|x|<3.\right]$

5. Let $y(x)=\Sigma_{n=0}^{\infty}c_n x^n\Rightarrow y'(x)=\Sigma_{n=1}^{\infty}nc_n x^{n-1}$ and $y''(x)=\Sigma_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n$. The differential equation becomes $\Sigma_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n+x\Sigma_{n=1}^{\infty}nc_n x^{n-1}+\Sigma_{n=0}^{\infty}c_n x^n=0$ or $\Sigma_{n=0}^{\infty}\left[(n+2)(n+1)c_{n+2}+nc_n+c_n\right]x^n\left(\text{since}\Sigma_{n=1}^{\infty}nc_n x^n=\Sigma_{n=0}^{\infty}nc_n x^n\right)$. Equating coefficients gives $(n+2)(n+1)c_{n+2}+(n+1)c_n=0$, thus the recursion relation is $c_{n+2}=\dfrac{-(n+1)c_n}{(n+2)(n+1)}=-\dfrac{c_n}{n+2}$, $n=0,1,2,\ldots$.

Then the even coefficients are given by $c_2=-\dfrac{c_0}{2}$, $c_4=-\dfrac{c_2}{4}=\dfrac{c_0}{2\cdot 4}$, $c_6=-\dfrac{c_4}{6}=-\dfrac{c_0}{2\cdot 4\cdot 6}$, and in general,

$c_{2n}=(-1)^n \dfrac{c_0}{2\cdot 4\cdot \cdot \cdot \cdot \cdot 2n} = \dfrac{(-1)^n c_0}{2^n \text{n!}}$. The odd coefficients are $c_3=-\dfrac{c_1}{3}$, $c_5=-\dfrac{c_3}{5}=\dfrac{c_1}{3\cdot 5}$,

$c_7=-\dfrac{c_5}{7}=-\dfrac{c_1}{3\cdot 5\cdot 7}$, and in general, $c_{2n+1}=(-1)^n \dfrac{c_1}{3\cdot 5\cdot 7\cdot \cdot \cdot \cdot \cdot (2n+1)} = \dfrac{(-2)^n \text{n!}\, c_1}{(2n+1)!}$. The solution is

$y(x)=c_0\sum_{n=0}^{\infty} \dfrac{(-1)^n}{2^n \text{n!}} x^{2n}+c_1\sum_{n=0}^{\infty} \dfrac{(-2)^n \text{n!}}{(2n+1)!} x^{2n+1}$.

6. Let $y(x)=\sum_{n=0}^{\infty} c_n x^n$. Then $y^{\prime\prime}(x)=\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}=\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n$. Hence, the equation $y^{\prime\prime}=y$ becomes $\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n-\sum_{n=0}^{\infty}c_n x^n=0$ or $\sum_{n=0}^{\infty}\left[(n+2)(n+1)c_{n+2}-c_n\right]x^n=0$.

So the recursion relation is $c_{n+2}=\dfrac{c_n}{(n+2)(n+1)}$, $n=0,1,\ldots$. Given c_0 and c_1 , $c_2=\dfrac{c_0}{2\cdot 1}$, $c_4=\dfrac{c_2}{4\cdot 3}=\dfrac{c_0}{4!}$,

$c_6=\dfrac{c_4}{6\cdot 5}=\dfrac{c_0}{6!}$, … , $c_{2n}=\dfrac{c_0}{(2n)!}$ and $c_3=\dfrac{c_1}{3\cdot 2}$, $c_5=\dfrac{c_3}{5\cdot 4}=\dfrac{c_1}{5\cdot 4\cdot 3\cdot 2}=\dfrac{c_1}{5!}$, $c_7=\dfrac{c_5}{7\cdot 6}=\dfrac{c_1}{7!}$, … ,

$c_{2n+1}=\dfrac{c_1}{(2n+1)!}$. Thus, the solution is

$$y(x)=\sum_{n=0}^{\infty} c_n x^n=\sum_{n=0}^{\infty} c_{2n}x^{2n}+\sum_{n=0}^{\infty} c_{2n+1}x^{2n+1}=c_0\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}+c_1\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}$$

The solution can be written as

$y(x)=c_0\cosh x+c_1\sinh x\left[\text{or}\, y(x)=c_0\dfrac{e^x+e^{-x}}{2}+c_1\dfrac{e^x-e^{-x}}{2}=\dfrac{c_0+c_1}{2}e^x+\dfrac{c_0-c_1}{2}e^{-x}\right]$.

7. Let $y(x)=\sum_{n=0}^{\infty} c_n x^n$. Then $y^{\prime\prime}=\sum_{n=0}^{\infty} n(n-1)c_n x^{n-2}$, $xy^{\prime}=\sum_{n=0}^{\infty} nc_n x^n$ and $\left(x^2+1\right)y^{\prime\prime}=\sum_{n=0}^{\infty} n(n-1)c_n x^n+\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n$. The differential equation becomes

$\sum_{n=0}^{\infty}\left[(n+2)(n+1)c_{n+2}+[n(n-1)+n-1]c_n\right]x^n=0$. The recursion relation is $c_{n+2}=-\dfrac{(n-1)c_n}{n+2}$,

$n=0,1,2,\ldots$. Given c_0 and c_1 , $c_2=\dfrac{c_0}{2}$, $c_4=-\dfrac{c_2}{4}=-\dfrac{c_0}{2^2\cdot 2!}$, $c_6=-\dfrac{3c_4}{6}=(-1)^2\dfrac{3c_0}{2^3\cdot 3!}$, … ,

$c_{2n}=(-1)^{n-1}\dfrac{1\cdot 3\cdot\cdots\cdot(2n-3)\,c_0}{2^n n!}=(-1)^{n-1}\dfrac{(2n-3)!\,c_0}{2^n 2^{n-2} n!(n-2)!}=(-1)^{n-1}\dfrac{(2n-3)!\,c_0}{2^{2n-2} n!(n-2)!}$ for $n=2,3,\ldots$.

$c_3=\dfrac{0\cdot c_1}{3}=0 \Rightarrow c_{2n+1}=0$ for $n=1,2,\ldots$. Thus the solution is

$y(x)=c_0+c_1x+c_0\dfrac{x^2}{2}+c_0\sum_{n=2}^{\infty}\dfrac{(-1)^{n-1}(2n-3)!}{2^{2n-2}n!(n-2)!}x^{2n}$.

8. Assuming $y(x)=\sum_{n=0}^{\infty}c_nx^n$, $y''(x)=\sum_{n=2}^{\infty}n(n-1)c_nx^{n-2}=\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n$ and $-xy(x)=-\sum_{n=0}^{\infty}c_nx^{n+1}=-\sum_{n=1}^{\infty}c_{n-1}x^n$. The equation $y''=xy$ becomes $\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n-\sum_{n=1}^{\infty}c_{n-1}x^n=0$ or $2c_2+\sum_{n=1}^{\infty}\left[(n+2)(n+1)c_{n+2}-c_{n-1}\right]x^n=0$. Equating coefficients gives $c_2=0$ and $c_{n+2}=\dfrac{c_{n-1}}{(n+2)(n+1)}$ for $n=1,2$, … . Since $c_2=0$, $c_{3n+2}=0$ for $n=0,1,2,\ldots$. Given c_0 , $c_3=\dfrac{c_0}{3\cdot 2}$, $c_6=\dfrac{c_3}{6\cdot 5}=\dfrac{c_0}{6\cdot 5\cdot 3\cdot 2}$, … , $c_{3n}=\dfrac{c_0}{3n(3n-1)(3n-3)(3n-4)\cdot\cdots\cdot 6\cdot 5\cdot 3\cdot 2}$. Given c_1 , $c_4=\dfrac{c_1}{4\cdot 3}$, $c_7=\dfrac{c_4}{7\cdot 6}=\dfrac{c_1}{7\cdot 6\cdot 4\cdot 3}$, … , $c_{3n+1}=\dfrac{c_1}{(3n+1)3n(3n-2)(3n-3)\ldots 7\cdot 6\cdot 4\cdot 3}$. The solution can be written as

$$y(x)=c_0\sum_{n=0}^{\infty}\frac{(3n-2)(3n-5)\cdot\cdots\cdot 7\cdot 4\cdot 1}{(3n)!}x^{3n}+c_1\sum_{n=0}^{\infty}\frac{(3n-1)(3n-4)\cdot\cdots\cdot 8\cdot 5\cdot 2}{(3n+1)!}x^{3n+1}$$

9. Let $y(x)=\sum_{n=0}^{\infty}c_nx^n$. Then $-xy'(x)=-x\sum_{n=1}^{\infty}nc_nx^{n-1}=-\sum_{n=1}^{\infty}nc_nx^n=-\sum_{n=0}^{\infty}nc_nx^n$, $y''(x)=\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n$, and the equation $y''-xy'-y=0$ becomes $\sum_{n=0}^{\infty}\left[(n+2)(n+1)c_{n+2}-nc_n-c_n\right]x^n=0$. Thus, the recursion relation is $c_{n+2}=\dfrac{nc_n+c_n}{(n+2)(n+1)}=\dfrac{c_n(n+1)}{(n+2)(n+1)}=\dfrac{c_n}{n+2}$ for $n=0,1,2$, … . One of the given conditions is $y(0)=1$. But $y(0)=\sum_{n=0}^{\infty}c_n(0)^n=c_0+0+0+\cdots=c_0$, so $c_0=1$. Hence, $c_2=\dfrac{c_0}{2}=\dfrac{1}{2}$, $c_4=\dfrac{c_2}{4}=\dfrac{1}{2\cdot 4}$, $c_6=\dfrac{c_4}{6}=\dfrac{1}{2\cdot 4\cdot 6}$, … , $c_{2n}=\dfrac{1}{2^n n!}$. The other given condition is $y'(0)=0$. But

$y'(0)=\sum_{n=1}^{\infty} nc_n(0)^{n-1}=c_1+0+0+\cdots=c_1$, so $c_1=0$. By the recursion relation, $c_3=\frac{c_1}{3}=0$, $c_5=0$, …, $c_{2n+1}=0$ for $n=0,1,2,\ldots$. Thus, the solution to the initial–value problem is

$$y(x)=\sum_{n=0}^{\infty} c_n x^n=\sum_{n=0}^{\infty} c_{2n}x^{2n}=\sum_{n=0}^{\infty}\frac{x^{2n}}{2^n n!}=\sum_{n=0}^{\infty}\frac{\left(x^2/2\right)^n}{n!}=e^{x^2/2}$$

10. Assuming that $y(x)=\sum_{n=0}^{\infty} c_n x^n$, we have $x^2y=\sum_{n=0}^{\infty} c_n x^{n+2}$ and

$$y''(x)=\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}=\sum_{n=-2}^{\infty}(n+4)(n+3)c_{n+4}x^{n+2}=2c_2+6c_3x+\sum_{n=0}^{\infty}(n+4)(n+3)c_{n+4}x^{n+2}$$

Thus, the equation $y''+x^2y=0$ becomes $2c_2+6c_3x+\sum_{n=0}^{\infty}\left[(n+4)(n+3)c_{n+4}+c_n\right]x^{n+2}=0$. So $c_2=c_3=0$ and the recursion relation is $c_{n+4}=-\frac{c_n}{(n+4)(n+3)}$, $n=0,1,2,\ldots$.
But $c_1=y'(0)=0=c_2=c_3$ and by the recursion relation, $c_{4n+1}=c_{4n+2}=c_{4n+3}=0$ for $n=0,1,2,\ldots$.
Also, $c_0=y(0)=1$, so

$c_4=-\frac{c_0}{4\cdot 3}=-\frac{1}{4\cdot 3}$, $c_8=-\frac{c_4}{8\cdot 7}=\frac{(-1)^2}{8\cdot 7\cdot 4\cdot 3}$, …, $c_{4n}=\frac{(-1)^n}{4n(4n-1)(4n-4)(4n-5)\cdot\cdots\cdot 4\cdot 3}$.
Thus, the solution to the initial–value problem is

$$y(x)=\sum_{n=0}^{\infty} c_n x^n=c_0+\sum_{n=0}^{\infty} c_{4n}x^{4n}=1+\sum_{n=1}^{\infty}(-1)^n\frac{x^{4n}}{4n(4n-1)(4n-4)(4n-5)\cdot\cdots\cdot 4\cdot 3}$$

11. Assuming that $y(x)=\sum_{n=0}^{\infty} c_n x^n$, we have $xy=x\sum_{n=0}^{\infty} c_n x^n=\sum_{n=0}^{\infty} c_n x^{n+1}$,
$x^2y'=x^2\sum_{n=1}^{\infty} nc_n x^{n-1}=\sum_{n=0}^{\infty} nc_n x^{n+1}$,

$y''(x)=\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}=\sum_{n=-1}^{\infty}(n+3)(n+2)c_{n+3}x^{n+1}$ [replace n with $n+3$] =
$2c_2+\sum_{n=0}^{\infty}(n+3)(n+2)c_{n+3}x^{n+1}$,
and the equation $y''+x^2y'+xy=0$ becomes $2c_2+\sum_{n=0}^{\infty}\left[(n+3)(n+2)c_{n+3}+nc_n+c_n\right]x^{n+1}=0$.
So

$c_2=0$ and the recursion relation is $c_{n+3}=\frac{-nc_n-c_n}{(n+3)(n+2)}=-\frac{(n+1)c_n}{(n+3)(n+2)}$, $n=0,1,2$, … .
But $c_0=y(0)=0=c_2$ and by the recursion relation, $c_{3n}=c_{3n+2}=0$ for $n=0$, 1 , 2 , … .
Also, $c_1=y'(0)=1$, so

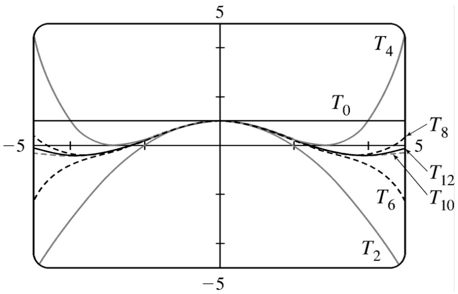
$c_4=-\frac{2c_1}{4\cdot 3}=-\frac{2}{4\cdot 3}$, $c_7=-\frac{5c_4}{7\cdot 6}=(-1)^2\frac{2\cdot 5}{7\cdot 6\cdot 4\cdot 3}=(-1)^2\frac{2^2 5^2}{7!}$, … , $c_{3n+1}=(-1)^n\frac{2^2 5^2\cdot\cdots\cdot(3n-1)^2}{(3n+1)!}$.
Thus, the solution is

$$y(x)=\sum_{n=0}^{\infty}c_n x^n=x+\sum_{n=1}^{\infty}\left[(-1)^n\frac{2^2 5^2\cdot\cdots\cdot(3n-1)^2 x^{3n+1}}{(3n+1)!}\right]$$

12. **(a)** Let $y(x)=\sum_{n=0}^{\infty}c_n x^n$. Then $x^2y''(x)=\sum_{n=2}^{\infty}n(n-1)c_n x^n=\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^{n+2}$,
$xy'(x)=\sum_{n=1}^{\infty}nc_n x^n=\sum_{n=-1}^{\infty}(n+2)c_{n+2}x^{n+2}=c_1x+\sum_{n=0}^{\infty}(n+2)c_{n+2}x^{n+2}$, and the equation
$x^2y''+xy'+x^2y=0$ becomes $c_1x+\sum_{n=0}^{\infty}\left\{[(n+2)(n+1)+(n+2)]c_{n+2}+c_n\right\}x^{n+2}=0$. So $c_1=0$ and the recursion relation is $c_{n+2}=-\frac{c_n}{(n+2)^2}$, $n=0,1,2$, … . But $c_1=y'(0)=0$ so $c_{2n+1}=0$ for $n=0,1,2,\ldots$. Also,
$c_0=y(0)=1$, so $c_2=-\frac{1}{2^2}$, $c_4=-\frac{c_2}{4^2}=(-1)^2\frac{1}{4^2 2^2}=(-1)^2\frac{1}{2^4(2!)^2}$, $c_6=-\frac{c_4}{6^2}=(-1)^3\frac{1}{2^6(3!)^2}$, … ,
$c_{2n}=(-1)^n\frac{1}{2^{2n}(n!)^2}$. The solution is

$$y(x)=\sum_{n=0}^{\infty}c_n x^n=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{2^{2n}(n!)^2}$$

(b) The Taylor polynomials T_0 to T_{12} are shown in the graph. Because T_{10} and T_{12} are close together throughout the interval $[-5,5]$, it is reasonable to assume that T_{12} is a good approximation to the Bessel function on that interval.


5
−5
5
−5
T_4
T_0
T_8
T_{12}
T_{10}
T_6
T_2