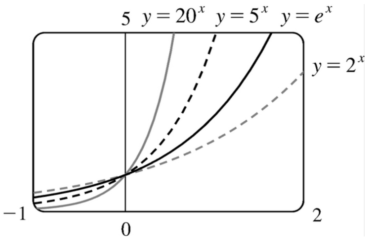
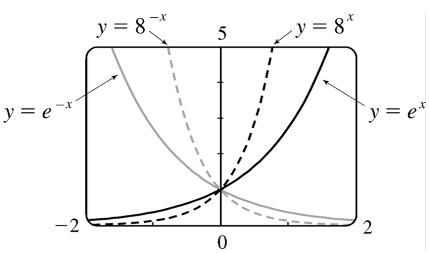
1. **(a)** $f(x)=a^x$, $a>0$
(b) R
(c) $(0,\infty)$
(d) See Figures (c), (b), and (a), respectively.

2. **(a)** The number e is the value of a such that the slope of the tangent line at $x=0$ on the graph of $y=a^x$ is exactly 1 .
(b) $e\approx 2.71828$
(c) $f(x)=e^x$

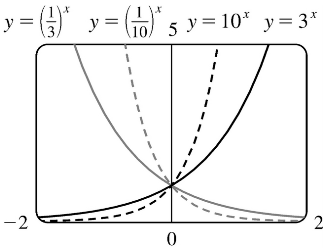
3. All of these graphs approach 0 as $x\to -\infty$, all of them pass through the point $(0,1)$, and all of them are increasing and approach ∞ as $x\to\infty$. The larger the base, the faster the function increases for $x>0$, and the faster it approaches 0 as $x\to -\infty$.



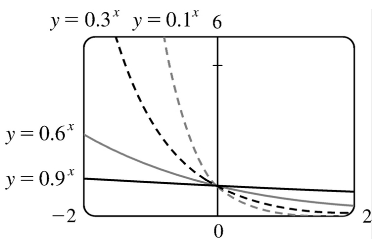
4. The graph of e^{-x} is the reflection of the graph of e^x about the $y-$ axis, and the graph of 8^{-x} is the reflection of that of 8^x about the y –axis. The graph of 8^x increases more quickly than that of e^x for $x>0$, and approaches 0 faster as $x\to -\infty$.



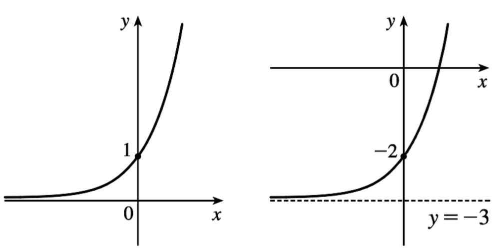
5. The functions with bases greater than 1 (3^x and 10^x) are increasing, while those with bases less than 1 $\left[\left(\frac{1}{3}\right)^x \text{and} \left(\frac{1}{10}\right)^x\right]$ are decreasing. The graph of $\left(\frac{1}{3}\right)^x$ is the reflection of that of 3^x about the $y-$ axis, and the graph of $\left(\frac{1}{10}\right)^x$ is the reflection of that of 10^x about the $y-$ axis. The graph of 10^x increases more quickly than that of 3^x for $x>0$, and approaches 0 faster as $x\to -\infty$.



6. Each of the graphs approaches ∞ as $x \to -\infty$, and each approaches 0 as $x \to \infty$. The smaller the base, the faster the function grows as $x \to -\infty$, and the faster it approaches 0 as $x \to \infty$.

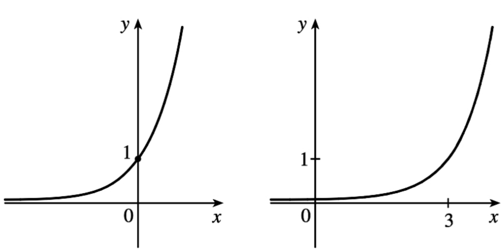


7. We start with the graph of $y=4^x$ (Figure 3) and then shift 3 units downward. This shift doesn't affect the domain, but the range of $y=4^x-3$ is $(-3,\infty)$. There is a horizontal asymptote of $y=-3$.



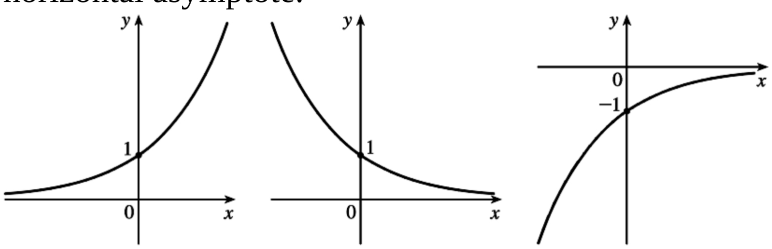
$y=4^x$ $y=4^x-3$

8. We start with the graph of $y=4^x$ (Figure 3) and then shift 3 units to the right. There is a horizontal asymptote of $y=0$.



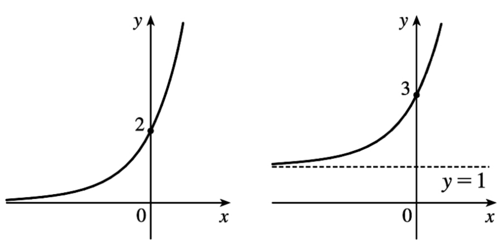
$y=4^x$ $y=4^{x-3}$

9. We start with the graph of $y=2^x$ (Figure 2), reflect it about the y –axis, and then about the x –axis (or just rotate 180° to handle both reflections) to obtain the graph of $y=-2^{-x}$. In each graph, $y=0$ is the horizontal asymptote.



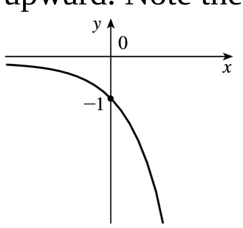
$y=2^x$ $y=2^{-x}$ $y=-2^{-x}$

10. We start with the graph of $y=e^x$ (Figure 13), vertically stretch by a factor of 2, and then shift 1 unit upward. There is a horizontal asymptote of $y=1$.

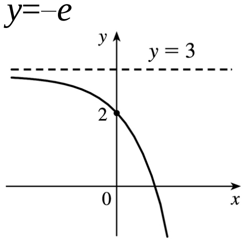


$y=2e^x$ $y=1+2e^x$

11. We start with the graph of $y=e^x$ (Figure 13), reflect it about the x –axis, and then shift 3 units upward. Note the horizontal asymptote of $y=3$.



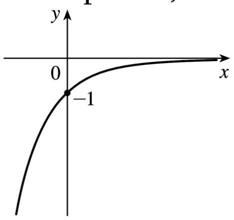
$y=-e^x$



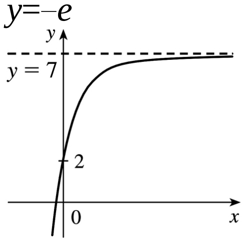
$y=3-e^x$

12. We start with the graph of $y=e^x$ (Figure 13), reflect it about the y –axis, and then about the x –axis

(or just rotate 180° to handle both reflections) to obtain the graph of $y=-e^{-x}$. Now shift this graph 1 unit upward, vertically stretch by a factor of 5 , and then shift 2 units upward.



$y=-e^{-x}$



$y=2+5(1-e^{-x})$

13. **(a)** To find the equation of the graph that results from shifting the graph of $y=e^x$ 2 units downward, we subtract 2 from the original function to get $y=e^x-2$.
(b) To find the equation of the graph that results from shifting the graph of $y=e^x$ 2 units to the right, we replace x with $x-2$ in the original function to get $y=e^{(x-2)}$.
(c) To find the equation of the graph that results from reflecting the graph of $y=e^x$ about the x –axis, we multiply the original function by -1 to get $y=-e^x$.
(d) To find the equation of the graph that results from reflecting the graph of $y=e^x$ about the y –axis, we replace x with $-x$ in the original function to get $y=e^{-x}$.
(e) To find the equation of the graph that results from reflecting the graph of $y=e^x$ about the x –axis and then about the y –axis, we first multiply the original function by -1 (to get $y=-e^x$) and then replace x with $-x$ in this equation to get $y=-e^{-x}$.

14. **(a)** This reflection consists of first reflecting the graph about the x –axis (giving the graph with equation $y=-e^x$) and then shifting this graph $2\cdot 4=8$ units upward. So the equation is $y=-e^x+8$.
(b) This reflection consists of first reflecting the graph about the y –axis (giving the graph with equation $y=e^{-x}$) and then shifting this graph $2\cdot 2=4$ units to the right. So the equation is $y=e^{-(x-4)}$.

15. **(a)** The denominator $1+e^x$ is never equal to zero because $e^x>0$, so the domain of $f(x)=1/(1+e^x)$ is R .

(b) $1-e^x=0 \Leftrightarrow e^x=1 \Leftrightarrow x=0$, so the domain of $f(x)=1/(1-e^x)$ is $(-\infty,0)\cup(0,\infty)$.

16. **(a)** The sine and exponential functions have domain R , so $g(t)=\sin(e^{-t})$ also has domain R .
(b) The function $g(t)=\sqrt{1-2^t}$ has domain $\{t\,|\,1-2^t\geq 0\}=\{t\,|\,2^t\leq 1\}=\{t\,|\,t\leq 0\}=(-\infty,0]$.

17. Use $y=Ca^x$ with the points $(1,6)$ and $(3,24)$. $6=Ca^1 \left[C=\frac{6}{a} \right]$ and $24=Ca^3 \Rightarrow 24=\left(\frac{6}{a}\right)a^3 \Rightarrow$ $4=a^2 \Rightarrow a=2$ [since $a>0$] and $C=\frac{6}{2}=3$. The function is $f(x)=3\cdot 2^x$.

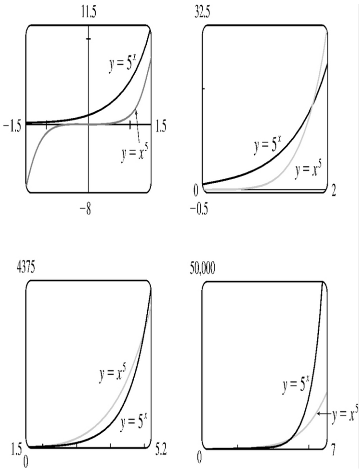
18. Given the y –intercept $(0,2)$, we have $y=Ca^x=2a^x$. Using the point $\left(2,\frac{2}{9}\right)$ gives us $\frac{2}{9}=2a^2 \Rightarrow$ $\frac{1}{9}=a^2 \Rightarrow a=\frac{1}{3}$ [since $a>0$]. The function is $f(x)=2\left(\frac{1}{3}\right)^x$ or $f(x)=2(3)^{-x}$.

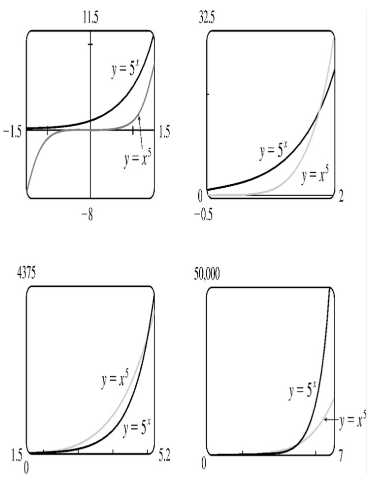
19. If $f(x)=5^x$, then $\frac{f(x+h)-f(x)}{h}=\frac{5^{x+h}-5^x}{h}=\frac{5^x5^h-5^x}{h}=\frac{5^x\left(5^h-1\right)}{h}=5^x\left(\frac{5^h-1}{h}\right)$.

20. Suppose the month is February. Your payment on the 28th day would be $2^{28-1}=2^{27}=134,217,728$ cents, or \$1 , 342 , 177 . 28 . Clearly, the second method of payment results in a larger amount for any month.

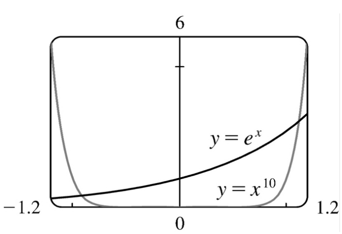
21. 2 ft =24 in, $f(24)=24^2$ in =576 in =48 ft. $g(24)=2^{24}$ in $=2^{24}/(12\cdot 5280)$ mi ≈ 265 mi

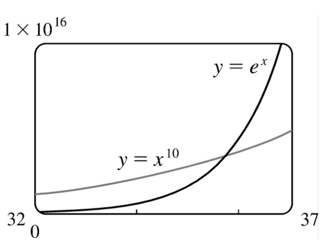
22. We see from the graphs that for x less than about 1.8 , $g(x)=5^x>f(x)=x^5$, and then near the point $(1.8,17.1)$ the curves intersect. Then $f(x)>g(x)$ from $x\approx 1.8$ until $x=5$. At $(5,3125)$ there is another point of intersection, and for $x>5$ we see that $g(x)>f(x)$. In fact, g increases much more rapidly than f beyond that point.





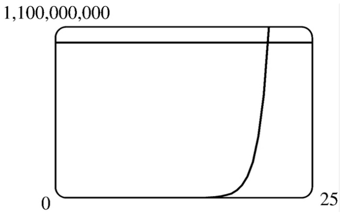
23. The graph of g finally surpasses that of f at $x \approx 35.8$.



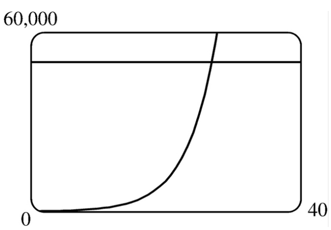


24. We graph $y=e^x$ and $y=1,000,000,000$ and determine where $e^x=1\times 10^9$. This seems to be true at $x\approx 20.723$, so

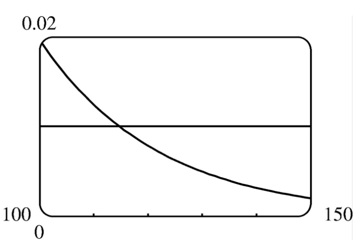
$e^x > 1 \times 10^9$ for $x > 20.723$.



25. **(a)** Fifteen hours represents 5 doubling periods (one doubling period is three hours).
$100 \cdot 2^5 = 3200$
(b) In t hours, there will be $t/3$ doubling periods. The initial population is 100 , so the population y at time t is $y = 100 \cdot 2^{t/3}$.
(c) $t = 20 \Rightarrow y = 100 \cdot 2^{20/3} \approx 10\,,159$
(d) We graph $y_1 = 100 \cdot 2^{x/3}$ and $y_2 = 50\,,000$. The two curves intersect at $x \approx 26.9$, so the population reaches 50 , 000 in about 26.9 hours.

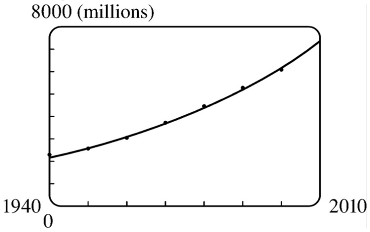


26. **(a)** Sixty hours represents 4 half–life periods. $2 \cdot \left(\frac{1}{2} \right)^4 = \frac{1}{8}$ g
(b) In t hours, there will be $t/15$ half–life periods. The initial mass is 2 g, so the mass y at time t is $y = 2 \cdot \left(\frac{1}{2} \right)^{t/15}$.
(c) 4 days $= 4 \cdot 24 = 96$ hours. $t = 96 \Rightarrow y = 2 \cdot \left(\frac{1}{2} \right)^{96/15} \approx 0.024$ g
(d) $y = 0.01 \Rightarrow t \approx 114.7$ hours



27. An exponential model is

$y=ab^t$, where $a=3.154832569\times 10^{-12}$ and $b=1.017764706$. This model gives $y(1993)\approx 5498$ million and $y(2010)\approx 7417$ million.



28. An exponential model is $y=ab^t$, where $a=1.9976760197589\times 10^{-9}$ and $b=1.0129334321697$. This model gives $y(1925)\approx 111$ million, $y(2010)\approx 330$ million, and $y(2020)\approx 375$ million.